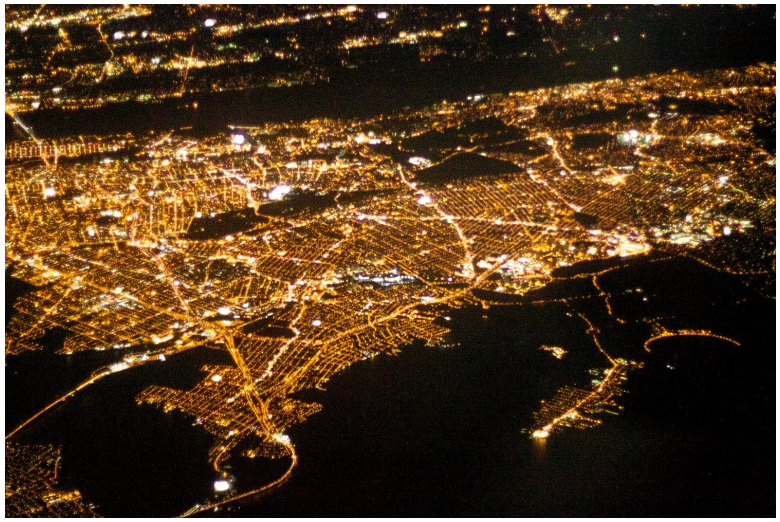
Coordinate geometry

IGCSE Mathematics

This handout covers Topic 3, Coordinate geometry. Parts marked **(Extended)** are only tested on the Extended papers; everything else is for both levels.

Coordinates

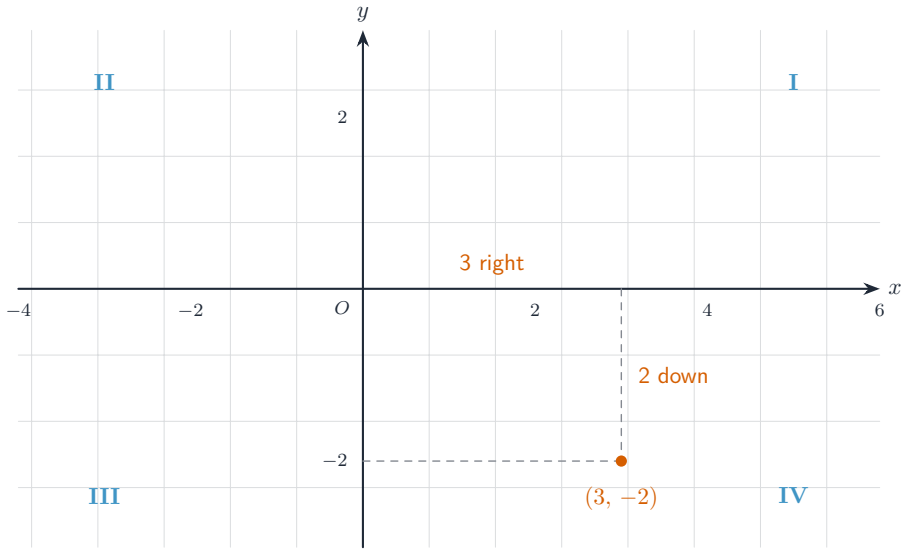


A city street grid: every place is fixed by its coordinates.

A point on a graph is described by its **coordinates**, sometimes called Cartesian coordinates, written (x, y). The first number is the across value and the second is the up value.

- The two number lines are the **axes**: the **horizontal** x-axis and the **vertical** y-axis.
- They cross at the **origin**, the point $(0, 0)$.
- The axes split the grid into four **quadrants**.

So the point $(3, -2)$ is found by going 3 to the right and 2 down.



The axes meet at the origin O and split the plane into four quadrants; $(3, -2)$ means 3 right then 2 down.

The equation of a straight line

Every non-vertical straight line can be written as

$$y = mx + c,$$

where m is the **gradient** (steepness) and c is the **intercept** – the y-value where the line crosses the y-axis.

- A line like $x = k$ (for example $x = 3$) is **vertical**.
- A line like $y = k$ (for example $y = 3$) is **horizontal**.

Extended: a line may also be given as $ax + by = c$. Rearrange it into $y = mx + c$ to read off the gradient and intercept.

Worked example (Extended). Find the gradient and y-intercept of $5x + 4y = 8$.

$$4y = -5x + 8 \;\Rightarrow\; y = -\tfrac{5}{4}x + 2.$$

So the gradient is $-\frac{5}{4}$ and the y-intercept is 2.

Gradient

A steep mountain road: gradient measures steepness as rise over run.

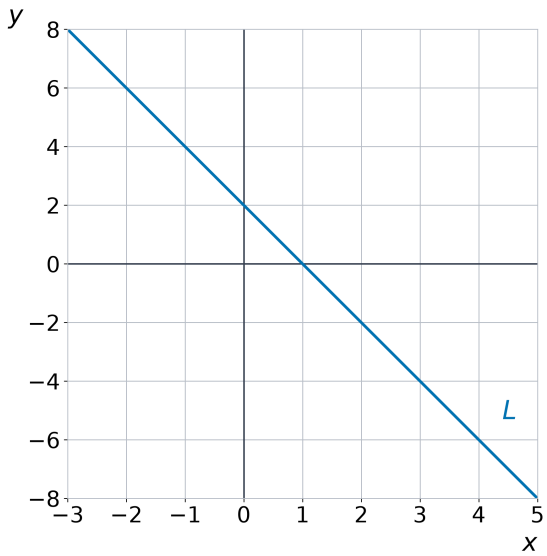
The **gradient** measures how steep a line is:

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}.$$

A positive gradient goes up to the right; a negative gradient goes down to the right. Horizontal lines have gradient zero; vertical lines have undefined gradient because the horizontal change is zero.

Core and Extended: from a grid. Choose two clear intersections on the drawn line, preferably far apart. Draw a right-angled triangle and read its horizontal and signed vertical changes in axis units. Check both numbered scales: counting squares alone may be wrong. Core gradient questions use a grid; calculation from two given coordinate pairs without a grid is an Extended demand.

Independent grid task. Use L below. Draw a large gradient triangle, read its changes in axis units and calculate the gradient. Then read the y-intercept and find the simplified equation of the line. Check a different grid point. Explain the trap caused by the two scales.

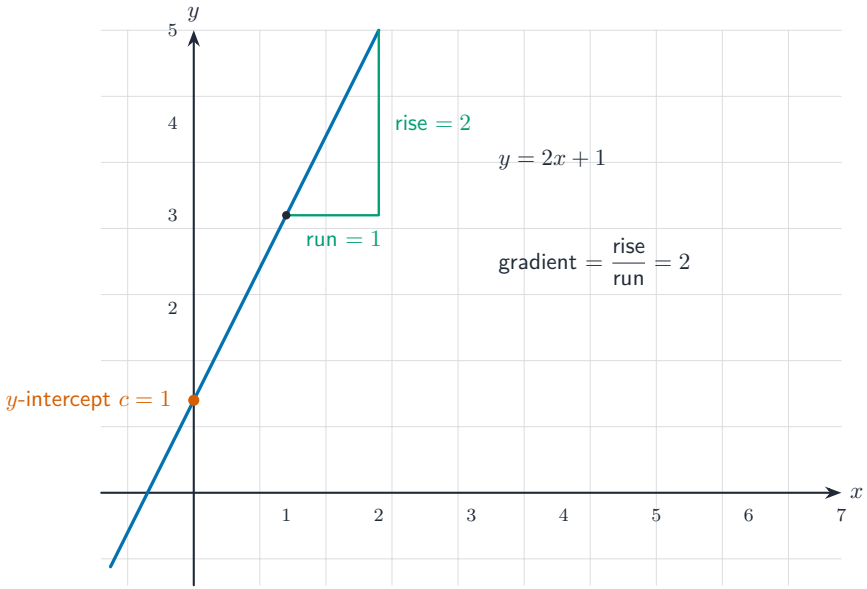


Read the numbered scales, then choose your own triangle. No equation is supplied.

Check after attempting: for example, $(0, 2)$ to $(2, -2)$ gives changes 2 across and -4 vertically, so $m = -2$. The intercept is 2, hence $y = -2x + 2$. The point $(1, 0)$ checks it. There are two large squares across and two down, but the vertical squares represent twice as many units. Counting squares alone would wrongly give -1.

Worked example (Extended: two given coordinate pairs). Find the gradient of the line through $(1, 2)$ and $(4, 11)$.

$$m = \frac{11 - 2}{4 - 1} = \frac{9}{3} = 3.$$



For $y = mx + c$, the line crosses the y-axis at c and the gradient m is the rise divided by the run.

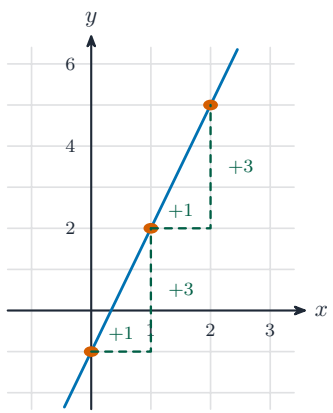
Drawing a straight-line graph

To draw $y = mx + c$, the quickest way is:

1. Mark the intercept c on the y-axis.
2. Use the gradient to step to more points (for $m = 3$, go 1 right and 3 up).
3. Join the points with a straight line.

You can also make a small **table of values**: choose at least three x-values, work out y, and plot the points. A third point checks the line through the first two. Label both axes and draw with a ruler across the requested interval, not just between nearby points.

Independent drawing task. Draw $y = -2x+3$ for $-2 \le x \le 3$, using $x = -2, 0, 1, 3$. Draw $x = -1$ on the same axes. Your table should give $y = 7, 3, 1, -3$; the second line is vertical. Check the actual drawing against all four points. **Extended:** drawing may also start from an equation such as $3x + 2y = 6$. Rearrange it to $y = -\frac{3}{2}x + 3$, or substitute values directly. Core drawing equations are normally given in $y = mx + c$ form unless a table is supplied.



drawing $y = 3x - 1$:

1. mark the intercept $c = -1$
2. gradient 3: step 1 right, 3 up, again and again
3. join the points

check with a table: $x = 0, 1, 2 \rightarrow y = -1, 2, 5$

Drawing a line fast: mark the intercept, then use the gradient to step to new points

Finding the equation of a line

If you know the gradient m and one point on the line, put the point into $y = mx + c$ to find c.

> **Worked example.** A line has gradient 3 and passes through $(1, 2)$. Find its equation.

$$y = 3x + c, \qquad 2 = 3(1) + c, \qquad c = -1.$$

So the equation is $y = 3x - 1$. (Extended: if two coordinate pairs are given without a grid, first calculate the gradient, then do this.)

For a graph-to-equation question, read c where the drawn line crosses the y-axis and use a large grid triangle to find m. Then write $y = mx + c$ and check another point. A vertical graph through $x = 4$ has equation $x = 4$, not $y = 4$.

Length and midpoint (Extended)

A **line segment** is the straight piece between two points.

To find the **length** of the segment between (x_1, y_1) and (x_2, y_2), use **Pythagoras' theorem** on the horizontal and vertical gaps:

$$\text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

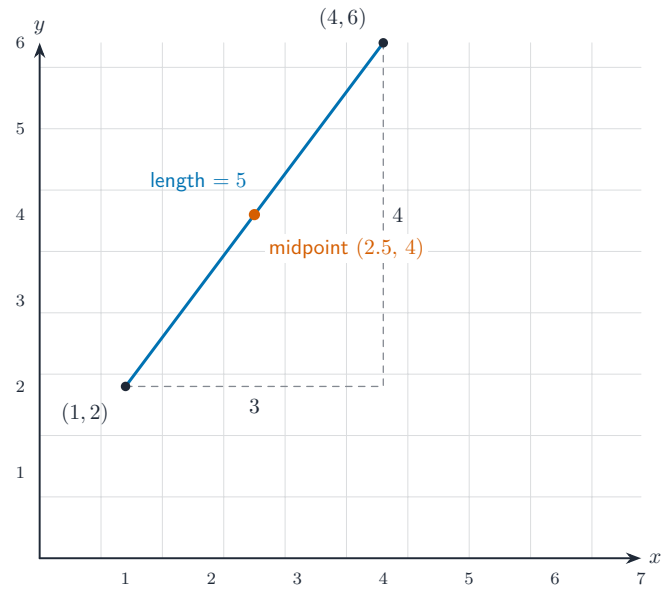
To find the **midpoint** (the point exactly in the middle), average the coordinates:

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Worked example. Find the length and midpoint of the segment from $(1, 2)$ to $(4, 6)$.

$$\text{length} = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9+16} = \sqrt{25} = 5.$$

$$\text{midpoint} = \left(\frac{1+4}{2}, \frac{2+6}{2}\right) = (2.5,\ 4).$$



The horizontal gap (3) and vertical gap (4) make a right triangle, so the length is $\sqrt{3^2+4^2} = 5$; the midpoint is the average of the coordinates.

■ Signed changes and a missing endpoint (Extended)

For $P(-4, 3)$ and $Q(2, -5)$, the coordinate changes are 6 and -8. The length is $\sqrt{6^2 + (-8)^2} = 10$ and midpoint is $(-1, -1)$. A coordinate change can be negative; a distance cannot. For a horizontal or vertical segment, use the absolute value of the one non-zero coordinate change.

If A and midpoint M are known, recover the other endpoint by $x_B = 2x_M - x_A$ and $y_B = 2y_M - y_A$. For $A(-3, 4)$ and $M(2, -1)$, $B = (7, -6)$. Check by averaging A and B. Their distance is $\sqrt{10^2 + (-10)^2} = \sqrt{200}$, about 14.1.

Parallel lines

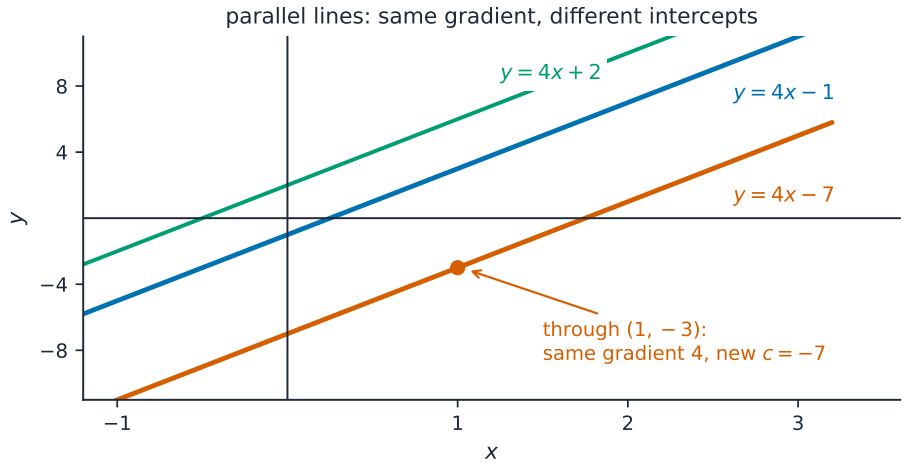
Distinct non-vertical **parallel** lines never meet and have the **same gradient**. Vertical lines are parallel too, but their gradients are undefined. Equal gradients and equal intercepts describe the same line, not two distinct parallel lines.

Worked example. Find the equation of the line parallel to $y = 4x - 1$ that passes through $(1, -3)$.

The gradient is also 4. Put the point in:

$$-3 = 4(1) + c \ \Rightarrow\ c = -7,$$

so the line is $y = 4x - 7$.



$$y - y_1 = m(x - x_1) \ \ (\text{same } m)$$

Parallel lines: same gradient, different intercepts

Perpendicular lines (Extended)

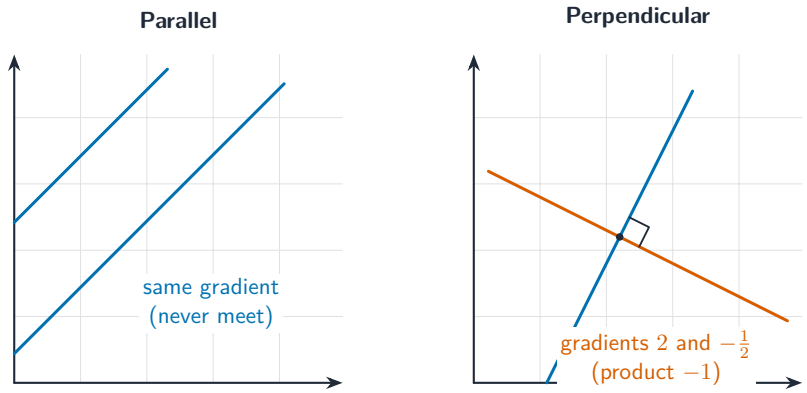
Two lines are **perpendicular** if they meet at a **right angle**. When both gradients are finite, their product is -1:

$$m_1 \times m_2 = -1, \qquad \text{so} \qquad m_2 = -\frac{1}{m_1}.$$

In words: flip the fraction and change the sign.

Worked example. Find the gradient of a line perpendicular to $2y = 3x + 1$.

Rearrange: $y = \frac{3}{2}x + \frac{1}{2}$, so the gradient is $\frac{3}{2}$. The perpendicular gradient is $-\frac{2}{3}$.



The non-vertical lines shown have equal gradients when parallel and gradient product -1 when perpendicular.

■ Perpendicular bisector

The **perpendicular bisector** of a segment cuts it in half at a right angle. To find its equation: get the midpoint, then use the perpendicular gradient through that midpoint.

> **Worked example.** Find the perpendicular bisector of the segment joining $(-3, 8)$ and $(9, -2)$.

- Midpoint: $\left(\frac{-3+9}{2}, \frac{8+(-2)}{2}\right) = (3, 3)$.
- Gradient of the segment: $\frac{-2-8}{9-(-3)} = \frac{-10}{12} = -\frac{5}{6}$.
- Perpendicular gradient: $\frac{6}{5}$.

Through $(3, 3)$: $3 = \frac{6}{5}(3) + c \Rightarrow c = 3 - \frac{18}{5} = -\frac{3}{5}$. So the bisector is

$$y = \frac{6}{5}x - \frac{3}{5}.$$

Independent bisector task (Extended). Find the perpendicular bisector of $A(-4, 1)$ and $B(2, 5)$ in $y = mx + c$ form. Check both the midpoint and the right-angle condition.

Check after attempting: midpoint $(-1, 3)$; segment gradient $2/3$; perpendicular gradient $-3/2$. Substituting the midpoint gives $c = 3/2$, so $y = -\frac{3}{2}x + \frac{3}{2}$. At $x = -1$ this gives 3, and $(2/3)(-3/2) = -1$.

Axis cases (Extended). For endpoints $(-3, 2)$ and $(5, 2)$, the segment is horizontal and its midpoint is $(1, 2)$, so its perpendicular bisector is $x = 1$. For $(4, -5)$ and $(4, 1)$, the bisector is $y = -2$. Handle these orientations directly: a vertical gradient is undefined, and there is no reciprocal of zero.