E7.4 Vector geometry

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS position vector • collinear • scalar multiple • parallelogram • midpoint

Objective

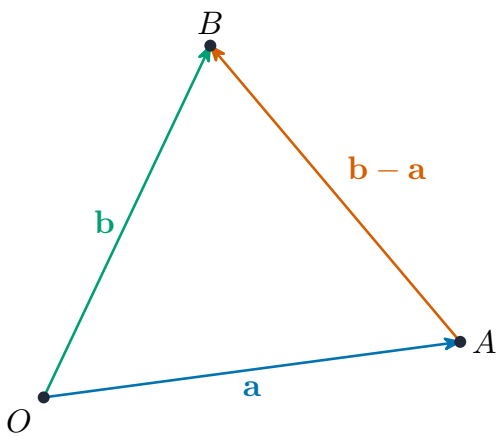
Build the skills to answer Extended exam questions on **vector geometry** – writing a journey in terms of given vectors, using **position vectors**, and proving that lines are **parallel** or that points are **collinear**.

You must be able to:

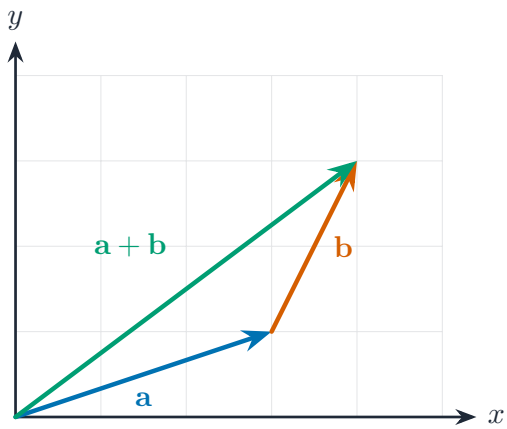
- write any journey in a diagram in terms of the given vectors
- find the position vector of a midpoint or of a point dividing a line in a given ratio
- prove two vectors are parallel, and prove three points are collinear

1 Key knowledge and worked examples

■ **Travel there by any route – the answer is the same**



Go backwards along a vector to reverse its sign



$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$$

- **The basic move:** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ when $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. "Destination minus start".
- **Midpoint:** $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{a} + \mathbf{b})$.
- **Parallel:** two vectors are parallel when one is a **scalar multiple** of the other. $4\mathbf{a} + 6\mathbf{b} = 2(2\mathbf{a} + 3\mathbf{b})$, so those two are parallel.
- **Collinear:** parallel AND sharing a point. $\overrightarrow{OQ} = 3\overrightarrow{OP}$ with the common point O means O, P and Q lie on one straight line.

Answer shapes (marked by **method marks** – M1 for a correct method, A1 for the correct answer): a valid route written down is the method mark, even before it is simplified. For a proof, the words "scalar multiple" and "common point" are the marks.

2 Practice

2.1 $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Write $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

2.2 M is the midpoint of AB. Write $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, in its simplest form. [2]

2.3 $\overrightarrow{PQ} = 2\mathbf{a} + 3\mathbf{b}$ and $\overrightarrow{RS} = 4\mathbf{a} + 6\mathbf{b}$. Show that PQ and RS are parallel. [2]

3 Exam-style questions

3.1 $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of AB.

(a) Write down $\overrightarrow{AB}$ in terms of $\mathbf{c}$. [1]

(b) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [2]

(c) N lies on OB with $\overrightarrow{ON} = \frac{2}{3}\overrightarrow{OB}$. Find $\overrightarrow{MN}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [3]

3.2 $\overrightarrow{OP} = 3\mathbf{a} - \mathbf{b}$ and $\overrightarrow{OQ} = 9\mathbf{a} - 3\mathbf{b}$. Show that O, P and Q are collinear. [3]