E2.4 Indices II

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS indices • equation • indices in algebra • laws of indices • base

Objective

Build the skills to answer Extended exam questions on **indices in algebra** – applying the **laws of indices** to algebraic terms, and solving simple index equations.

You must be able to:

- simplify products, quotients and powers of algebraic terms
- handle negative powers in algebra
- solve $a^x = b$ by writing both sides with the same base

1 Key knowledge and worked examples

■ Indices with letters

Index laws combine exponents of the same non-zero base.

$$a^m \times a^n = a^{m+n} \qquad a^m \div a^n = a^{m-n} \qquad (a^m)^n = a^{mn}$$

- **Worked:** $(5x^3)^2 = 25x^6$; $12a^5 \div 3a^{-2} = 4a^7$; $6x^7y^4 \times 5x^{-5}y = 30x^2y^5$.
- **Index equation:** $2^x = 32 = 2^5 \to x = 5$.

Answer shapes: M1 a correct method, **A1** the correct answer.

■ Worked application: A negative index changes location, not sign

For non-zero a and b, simplify $\dfrac{6a^{-2}b^3}{3ab^{-1}}$. Divide coefficients and subtract exponents: $2a^{-3}b^4 = \dfrac{2b^4}{a^3}$. The negative exponent does not make the term negative; it puts the factor in the denominator. The restrictions $a \neq 0$ and $b \neq 0$ come from the original expression and remain relevant even when the final form looks defined for $b = 0$. For $3^{x+1} = 27 = 3^3$, equate exponents to get $x + 1 = 3$, so $x = 2$. Check by substituting into the original equation.

$$\boxed{6a^{-2}b^3/(3ab^{-1})}$$
$$\downarrow$$
$$\boxed{2a^{-3}b^4}$$
$$\downarrow$$
$$\boxed{2b^4/a^3 \text{ (positive indices)}}$$

2 Practice

2.1 Simplify $(3a^2)^3$. [2]

2.2 Simplify $10x^6 \div 2x^2$. [2]

2.3 Simplify $a^4 \times a^{-7}$. [1]

3 Exam-style questions

3.1 Simplify $(2x^2y)^3$. [2]

3.2 Simplify $\dfrac{15a^7b^2}{5a^3b^2}$. [2]

3.3 Solve $3^x = 81$. [2]

3.4 For non-zero x and y, simplify $\dfrac{12x^{-1}y^2}{4x^2y^{-2}}$ using positive indices. Then solve $2^{n+1} = 16$, showing a check. [4]