

E8.1 Introduction to probability

Name: _____ Class: _____ Date: _____

Total: 21 marks

KEY WORDS probability • probability scale • outcome
 • expected frequency • tree diagrams

Objective

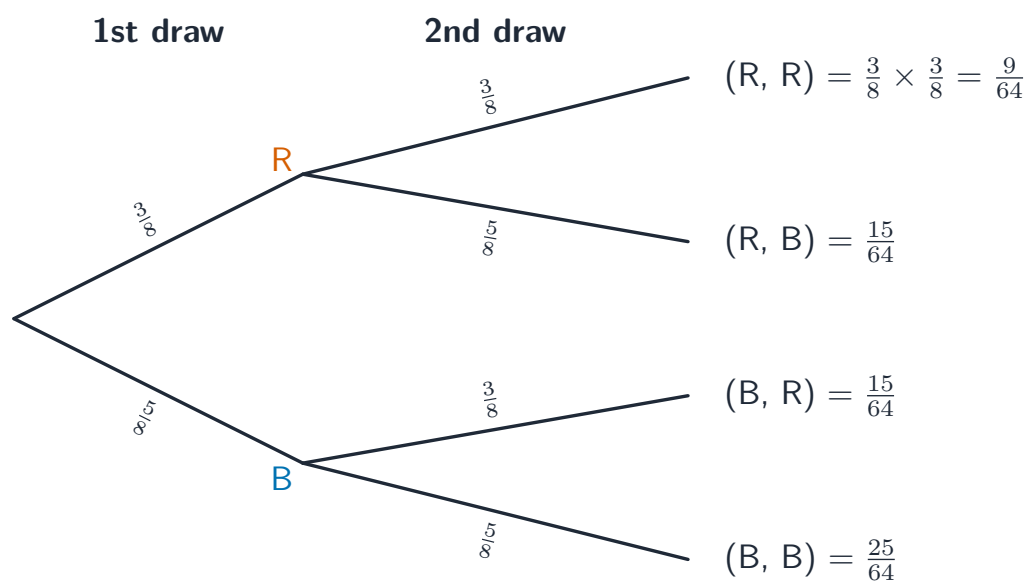
Build the skills to answer Extended exam questions on **probability** – the **probability scale**, $P(A')$, **expected frequency**, and **tree diagrams**

You must be able to:

- find a probability and use $P(A') = 1 - P(A)$
- find an expected frequency
- use a tree diagram for combined events (with and without replacement)

1 Key knowledge and worked examples

■ Probability and tree diagrams



Probability runs from 0 (impossible) to 1 (certain)

		die 2					
+		1	2	3	4	5	6
die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

the six highlighted cells all give a total of 7

A sample space lists all equally likely outcomes

$$P(\text{event}) = \frac{\text{favourable}}{\text{total}} \quad P(A') = 1 - P(A) \quad \text{expected} = P \times n$$

- **Worked (bag: 3 red, 5 blue):** $P(\text{not red}) = 1 - \frac{3}{8} = \frac{5}{8}$.
- **Tree (with replacement):** $P(\text{red, red}) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 A bag has 4 red and 6 green counters. Find the probability of drawing a red. [1]

2.2 The probability of rain is 0.3. Find the probability of no rain. [1]

2.3 The probability of a six is $\frac{1}{6}$. Find the expected number of sixes in 240 rolls. [2]

3 Exam-style questions

3.1 A bag has 3 red and 5 blue counters. Two are drawn **without replacement**. Find the probability that both are red. [3]

3.2 The probability that a train is late is 0.15. On 200 days, find the expected number of late days. [2]

3.3 A bag holds 5 red, 3 blue and 2 green counters. One counter is taken at random.

(a) Write down $P(\text{red})$ and $P(\text{not red})$, and state the connection between them. [3]

(b) The counter is replaced and the experiment is repeated 250 times. Find the expected number of blue counters. [2]

(c) Two counters are now taken **without** replacement. Draw a tree diagram for “green” and “not green”, and use it to find the probability that exactly one is green. [5]

(d) Explain why the probabilities on the second pair of branches differ from those on the first. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

$$\frac{4}{10} = \frac{\mathbf{2}}{\mathbf{5}}$$

2.2

- $1 - 0.3 = \mathbf{0.7}$

2.3

$$\frac{1}{6} \times 240 = \mathbf{40}$$

3.1

- first red $\frac{3}{8}$; after a red is taken, 2 reds out of 7 remain

$$\frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{\mathbf{3}}{\mathbf{28}}$$

3.2

- $0.15 \times 200 = \mathbf{30}$ days

3.3

(a)

- $P(\text{red}) = \frac{5}{10} = \frac{\mathbf{1}}{\mathbf{2}}$
- $P(\text{not red}) = \frac{5}{10} = \frac{\mathbf{1}}{\mathbf{2}}$
- $P(A') = 1 - P(A)$ – they add to 1

(b)

- $P(\text{blue}) = \frac{3}{10}$
- expected number $= \frac{3}{10} \times 250 = \mathbf{75}$

(c)

- first pair $\frac{2}{10}$ and $\frac{8}{10}$
- second pair $\frac{1}{9}, \frac{8}{9}$ after green and $\frac{2}{9}, \frac{7}{9}$ after not green

- exactly one green = $\left(\frac{2}{10} \times \frac{8}{9}\right) + \left(\frac{8}{10} \times \frac{2}{9}\right)$
 $= \frac{16}{90} + \frac{16}{90} = \frac{32}{90} = \frac{\mathbf{16}}{\mathbf{45}}$

(d)

- the first counter is not replaced, so only 9 remain
- the number of greens left depends on what was drawn first, so the two second-stage pairs differ