E7.3 Magnitude of a vector

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS magnitude • column vector • component • scalar multiple • exact form

Objective

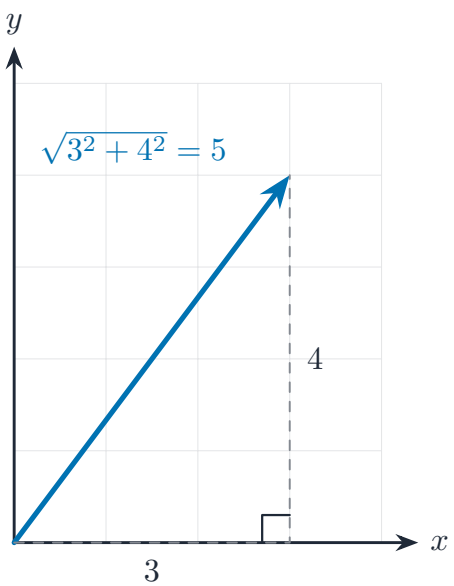
Build the skills to answer Extended exam questions on the **magnitude of a vector** – finding $|\mathbf{a}|$ from a **column vector**, combining vectors first and then taking the magnitude, and working backwards from a given magnitude.

You must be able to:

- find the magnitude of a column vector using Pythagoras
- add, subtract and multiply column vectors before finding a magnitude
- find a missing component when the magnitude is given

1 Key knowledge and worked examples

■ Magnitude is Pythagoras on the two components



The two components are the short sides; the magnitude is the hypotenuse

- **Formula:** if $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ then $|\mathbf{a}| = \sqrt{x^2 + y^2}$.
- **The signs vanish**, because both components are squared. $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$ has magnitude 10, the same as $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$.
- **Combine FIRST.** $|\mathbf{a} + \mathbf{b}|$ means add the vectors and then take the magnitude; it is NOT $|\mathbf{a}| + |\mathbf{b}|$.
- **Scalar multiple:** $2\mathbf{a}$ multiplies BOTH components by 2.

Answer shapes (marked by **method marks** – M1 for a correct method, A1 for the correct answer): the combined column vector, and the square root expression, are separate method marks.

2 Practice

2.1 $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Find $|\mathbf{a}|$. [2]

2.2 $\mathbf{b} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$. Find $|\mathbf{b}|$. [2]

2.3 $\mathbf{c} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$. Find $|\mathbf{c}|$. [2]

3 Exam-style questions

3.1 $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

(a) Find $\mathbf{a} + \mathbf{b}$ as a column vector. [1]

(b) Find $|\mathbf{a} + \mathbf{b}|$, giving your answer in exact form. [2]

(c) Find $|2\mathbf{a} - \mathbf{b}|$, correct to 3 significant figures. [3]

3.2 The vector $\begin{pmatrix} k \\ 8 \end{pmatrix}$ has magnitude 17. Find the positive value of k. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $\sqrt{3^2 + 4^2} = \sqrt{25}$

$$= \mathbf{5}$$

2.2

- $\sqrt{(-5)^2 + 12^2} = \sqrt{169}$

$$= \mathbf{13}$$

2.3

- $\sqrt{6^2 + (-8)^2} = \sqrt{100}$

$$= \mathbf{10}$$

3.1

(a)

- add the components separately

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

(b)

- $\sqrt{3^2 + 2^2} = \sqrt{13}$

$$= \sqrt{\mathbf{13}}$$

- 13 has no square factor, so this is already exact

(c)

- $2\mathbf{a} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$, so $2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 9 \\ -11 \end{pmatrix}$
- $\sqrt{81 + 121} = \sqrt{202}$

$$= 14.212\ldots = \mathbf{14.2}$$

3.2

- $k^2 + 8^2 = 17^2$
- $k^2 = 289 - 64 = 225$

$$k = \mathbf{15}$$