

E6.4 Trigonometric functions

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS exact values

• trigonometric equation

• sine and cosine rules

• trigonometric functions

• sine

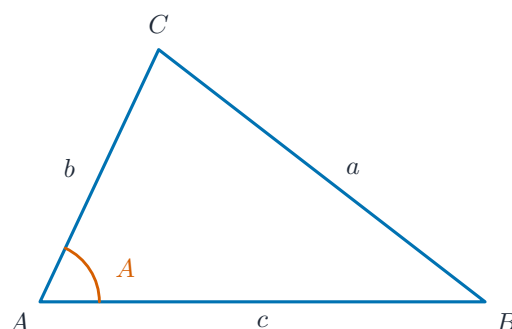
Objective

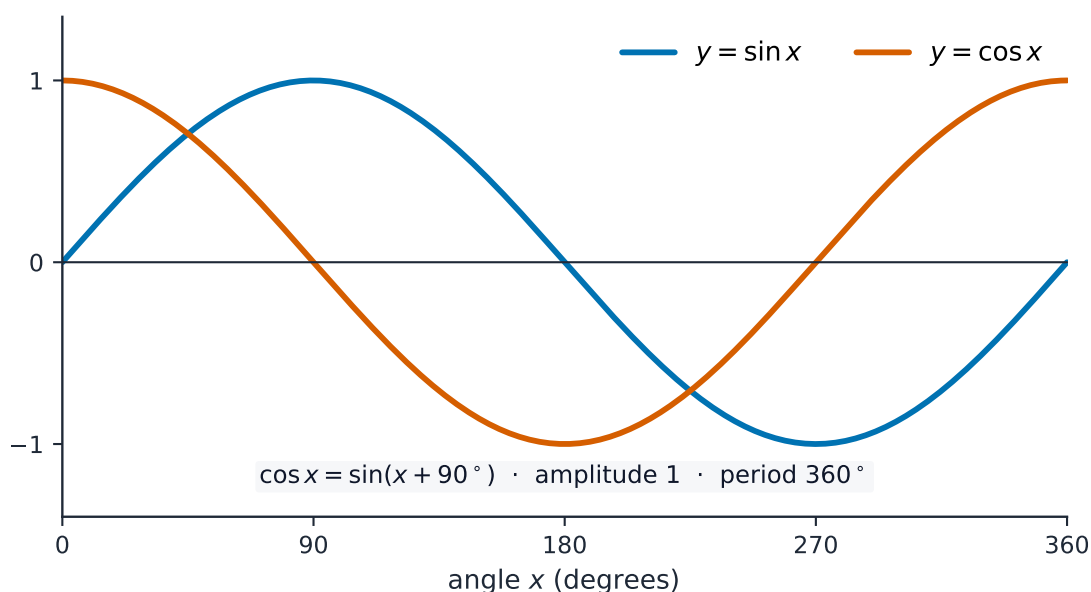
Build the skills to answer Extended exam questions on **trigonometric functions**– **exact values**, solving **trigonometric equations**, and the **sine and cosine rules**.**You must be able to:**

- recall exact values of $\sin$, $\cos$, $\tan$ for $0-90^\circ$
- use the cosine rule and the sine rule in any triangle
- find the area of a triangle with $\frac{1}{2}ab \sin C$

1 Key knowledge and worked examples

■ The sine and cosine rules

*For any triangle, not just right-angled ones*



The sine and cosine function graphs

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area} = \frac{1}{2}ab \sin C$$

- **Cosine rule (two sides + included angle):** $b = 7$, $c = 8$, $A = 40^\circ \rightarrow a^2 = 49 + 64 - 112 \cos 40^\circ = 27.2 \rightarrow a = 5.2$ cm.
- **Exact:** $\sin 60^\circ = \frac{\sqrt{3}}{2}$; solve $\sin x = \frac{\sqrt{3}}{2}$ ($0-360^\circ$) $\rightarrow x = 60^\circ$ or 120° .

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Write down the exact value of $\cos 30^\circ$. [1]

2.2 Solve $\sin x = \frac{1}{2}$ for $0^\circ \leq x \leq 360^\circ$. [2]

2.3 Find the area of a triangle with sides $a = 6$ cm, $b = 9$ cm and included angle $C = 50^\circ$. [2]

3 Exam-style questions

3.1 A triangle has $b = 5$ cm, $c = 7$ cm and the angle between them $A = 60^\circ$. Find side a . [3]

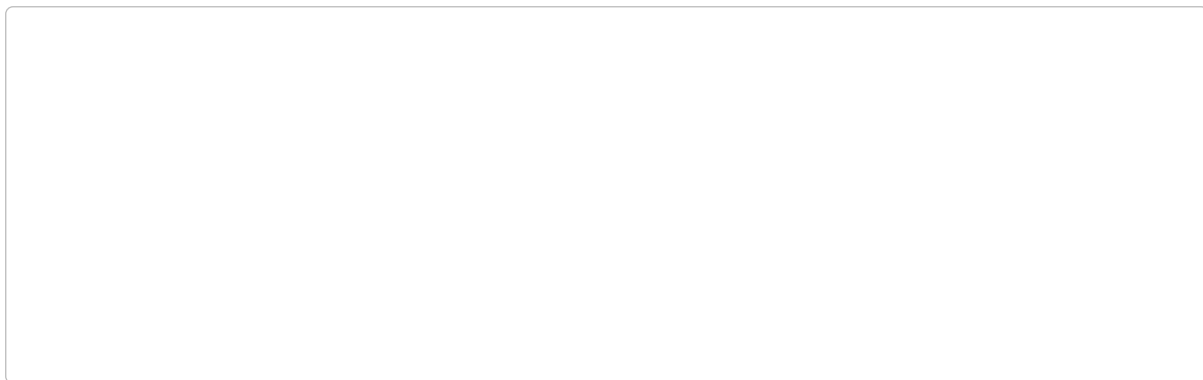
3.2 In a triangle, $a = 8$ cm, angle $A = 40^\circ$ and angle $B = 75^\circ$. Use the sine rule to find side b . [3]

3.3 (a) Write down the exact values of $\sin 30^\circ$, $\cos 45^\circ$ and $\tan 60^\circ$. [3]

(b) Solve $2 \sin x = 1$ for $0^\circ \leq x \leq 360^\circ$. [3]

(c) Solve $\cos x = -0.5$ for $0^\circ \leq x \leq 360^\circ$. [3]

(d) Sketch $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$, mark its maximum and minimum, and use the sketch to explain why $\sin x = 1.4$ has no solution. [4]



Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $\cos 30^\circ = \frac{\sqrt{3}}{2}$

2.2

- $x = 30^\circ$
- sine is also positive in the second quadrant: $x = 180 - 30 = \mathbf{150^\circ}$

2.3

- $\text{area} = \frac{1}{2}ab \sin C$
- $\frac{1}{2}(6)(9) \sin 50^\circ = 27 \times 0.766 = \mathbf{20.7 \text{ cm}^2}$

3.1

- cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$
- $a^2 = 25 + 49 - 70(0.5) = 39$
- $a = \sqrt{39} = \mathbf{6.24 \text{ cm}}$

3.2

- sine rule: $\frac{b}{\sin 75^\circ} = \frac{8}{\sin 40^\circ}$

$$b = \frac{8 \sin 75^\circ}{\sin 40^\circ} = \frac{8 \times 0.966}{0.643} = \mathbf{12.0 \text{ cm}}$$

3.3

(a)

- $\sin 30^\circ = \frac{1}{2}$, $\cos 45^\circ = \frac{\sqrt{2}}{2}$, $\tan 60^\circ = \sqrt{3}$

(b)

- $\sin x = \frac{1}{2}$
- first solution $x = 30^\circ$
- second quadrant: $x = 180 - 30 = \mathbf{150^\circ}$

(c)

- related acute angle $\cos^{-1} 0.5 = 60^\circ$
- cosine is negative in the second and third quadrants
- $\mathbf{x = 120^\circ}$ and $\mathbf{x = 240^\circ}$

(d)

- a correct sine curve: $(0, 0)$, $(90, 1)$, $(180, 0)$, $(270, -1)$, $(360, 0)$
- maximum and minimum marked
- the curve never rises above $y = 1$, so $\sin x = 1.4$ has **no solution**