

E1.7 Indices I

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS indices • laws of indices • factor • power
• prime factor

Objective

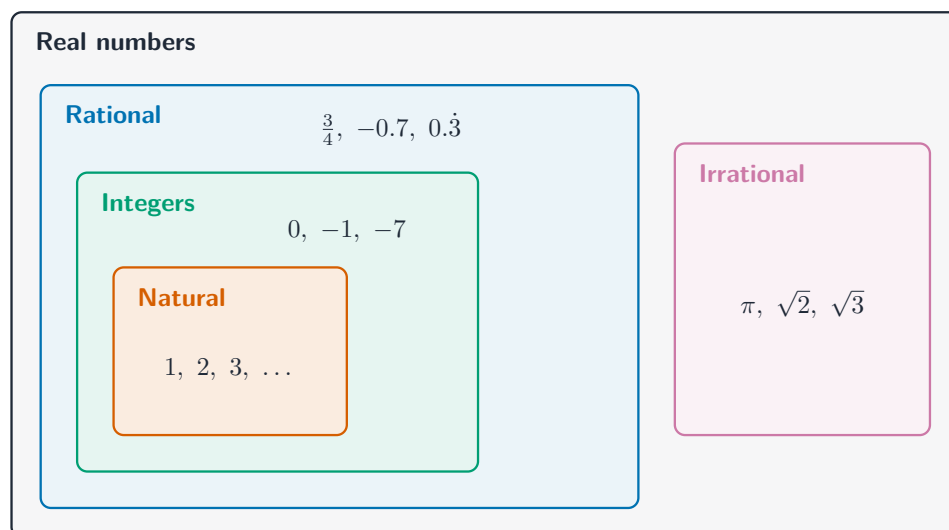
Build the skills to answer Extended exam questions on **indices** , and **zero, negative and fractional** , , – the **laws of** powers.

You must be able to:

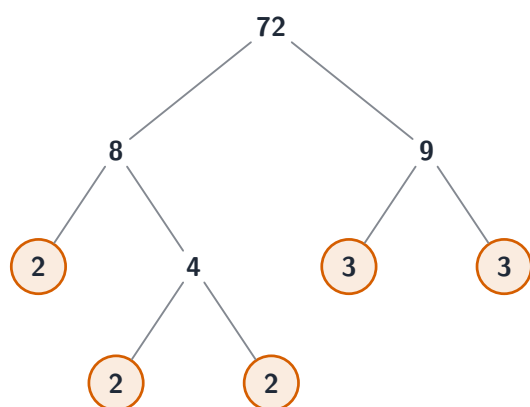
- use $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$
- evaluate a^0 , a^{-n} and $a^{\frac{m}{n}}$
- simplify expressions with indices

1 Key knowledge and worked examples

■ Laws of indices



Add powers to multiply; subtract to divide



$$72 = 2^3 \times 3^2$$

Indices appear when a number is written in prime factors

$$a^m \times a^n = a^{m+n} \quad a^m \div a^n = a^{m-n} \quad (a^m)^n = a^{mn}$$

$$a^0 = 1 \quad a^{-n} = \frac{1}{a^n} \quad a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

- **Worked:** $2^{-3} \times 2^4 = 2^1 = 2$; $(2^3)^2 = 2^6 = 64$; $8^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{4}$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Simplify $x^5 \times x^3$. [1]

2.2 Work out the value of 2^{-3} . [1]

2.3 Work out the value of $81^{\frac{1}{2}}$. [1]

3 Exam-style questions

3.1 Simplify $(2a^2)^3$. [2]

3.2 Work out the value of $27^{\frac{2}{3}}$. [2]

3.3 Work out the value of $16^{-\frac{1}{2}}$. [2]

3.4 (a) Simplify, leaving your answers in index form: [4]

(i) $\frac{x^7 \times x^{-3}}{x^2}$ (ii) $(2y^3)^4$

(b) Evaluate without a calculator, showing each step: [4]

(i) $27^{\frac{2}{3}}$ (ii) $\left(\frac{25}{4}\right)^{-\frac{1}{2}}$

(c) Solve $2^{x+1} = \frac{1}{32}$. [3]

(d) Explain, using the laws of indices, why $a^0 = 1$ for any non-zero a . [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- add the indices: $x^5 \times x^3 = \mathbf{x^8}$

2.2

- $2^{-3} = \frac{1}{2^3} = \frac{\mathbf{1}}{\mathbf{8}}$

2.3

- $81^{\frac{1}{2}} = \sqrt{81} = \mathbf{9}$

3.1

- $(2a^2)^3 = 2^3 a^6$
- $\mathbf{8a^6}$

3.2

- $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2$
- $\mathbf{9}$

3.3

- $16^{-\frac{1}{2}} = \frac{1}{\sqrt{16}}$
- $\frac{\mathbf{1}}{\mathbf{4}}$

3.4

(a)

(i)

- $\frac{x^7 \times x^{-3}}{x^2} = \frac{x^4}{x^2} = \mathbf{x^2}$

(ii)

- $(2y^3)^4 = 2^4 y^{12} = \mathbf{16y^{12}}$

(b)

(i)

- $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = \mathbf{9}$

(ii)

- $\left(\frac{25}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{25}\right)^{\frac{1}{2}} = \frac{\mathbf{2}}{\mathbf{5}}$

(c)

- $\frac{1}{32} = 2^{-5}$
- $x + 1 = -5$
- $\mathbf{x = -6}$

(d)

- $\frac{a^n}{a^n} = a^{n-n} = a^0$ by the division law
- any non-zero number divided by itself is 1, so $\mathbf{a^0 = 1}$