C5.4 Surface area and volume

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS cylinder • cuboid • surface area • sphere • cross-section

Objective

Use surface area and volume to solve Core IGCSE Mathematics problems.

You must be able to:

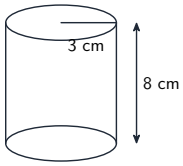
- Find prism, cylinder, cone, pyramid and sphere volumes
- Calculate surface area from the exposed faces
- Use perpendicular height for volume and slant height where area requires it

1 Key knowledge and worked examples

■ Key knowledge: Surface area and volume

Volume measures space inside; surface area totals exposed faces. Cuboid: $V = lwh$, $S = 2(lw + lh + wh)$. Prism: $V = Ah$, where A is cross-section area and h is length. Cylinder: $V = \pi r^2 h$, curved area $2\pi rh$; add circular ends when needed. Cone: $V = \pi r^2 h/3$, curved area πrl, using slant height l. Pyramid: $V = Ah/3$. Sphere: $V = 4\pi r^3/3$, $S = 4\pi r^2$.

■ Worked example: Surface area and volume



A closed cylinder has radius 3 cm and height 8 cm.

- Volume: $V = \pi r^2 h = \pi(3)^2(8) = 72\pi$ cm^3.
- Surface area: $S = 2\pi rh + 2\pi r^2 = 2\pi(3)(8) + 2\pi(3)^2 = 66\pi$ cm^2.
- A cone with $r = 3, h = 4, l = 5$ has $V = \pi(3)^2(4)/3 = 12\pi$ and total $S = \pi(3)(5) + \pi(3)^2 = 24\pi$ in the appropriate units.
- A triangular prism with cross-section base 6 cm, height 4 cm and length 9 cm has $A = bh/2 = 6(4)/2 = 12$ cm^2, then $V = Ah = 12(9) = 108$ cm^3.
- For a pyramid of base area 30 cm^2 and height 8 cm, $V = Ah/3 = 30(8)/3 = 80$ cm^3. A sphere of radius 3 cm has $V = 4\pi r^3/3 = 36\pi$ cm^3 and $S = 4\pi r^2 = 36\pi$ cm^2.

■ Method check

Draw or list the exposed faces before adding areas. A prism's volume uses its cross-section area multiplied by the distance between its ends.

2 Practice

2.1 Find the volume and surface area of a $5 \times 3 \times 2$ cm cuboid. [2]

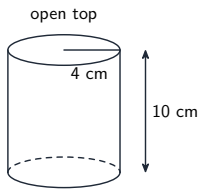
2.2 A triangular prism has cross-section base 6 cm and height 4 cm; length is 9 cm. Find its volume. [2]

2.3 A pyramid has base area 30 cm^2 and perpendicular height 8 cm. Find its volume. [2]

2.4 Find the volume and surface area of a sphere of radius 3 cm in terms of π. [2]

3 Exam-style questions

3.1 Apply the surface area and volume methods below. Show working and include units where appropriate.

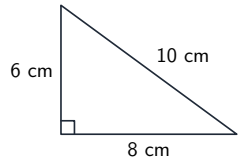


(a) An open-top cylindrical container has radius 4 cm and height 10 cm. Find its volume in terms of π. [2]

(b) Find its exposed area, including its circular base but not its open top. [2]

(c) A cone has the same radius and height. Find its volume, and compare it with the cylinder. [2]

3.2 A triangular prism is 10 cm long. Its right-triangle cross-section has perpendicular sides 6 cm and 8 cm and hypotenuse 10 cm.



cross-section; prism length 10 cm

not to scale

(a) Find the prism volume. [3]

(b) Find its total surface area, explaining how the two ends and three rectangular faces are included. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- $V = 5(3)(2) = \mathbf{30}$ cm^3; $S = 2(15 + 10 + 6) = \mathbf{62}$ cm^2.

2.2

- $A = 6(4)/2 = 12$; $V = Ah = 12(9) = \mathbf{108}$ cm^3.

2.3

- $V = Ah/3 = 30(8)/3 = \mathbf{80}$ cm^3.

2.4

- $V = 4\pi(3)^3/3 = \mathbf{36\pi}$ cm^3; $S = 4\pi(3)^2 = \mathbf{36\pi}$ cm^2.

3.1

(a)

- $V = \pi r^2 h = \pi(4)^2(10) = \mathbf{160\pi}$ cm^3.

(b)

- $S = 2\pi rh + \pi r^2 = 80\pi + 16\pi = \mathbf{96\pi}$ cm^2.

(c)

- $V_{\text{cone}} = \pi r^2 h/3 = \mathbf{160\pi/3}$ cm^3, **one third** of the cylinder volume.

3.2

(a)

- Use $A = bh/2$ for the triangular cross-section.
- $A = 6(8)/2 = 24$ cm^2.
- Then $V = Al = 24(10) = \mathbf{240}$ cm^3.

(b)

- Two ends give $S_e = 2(24) = 48$ cm^2.
- Rectangular faces give $S_r = 10(6 + 8 + 10) = 240$ cm^2.
- Total $S = S_e + S_r = \mathbf{288}$ cm^2.