C2.4 Indices II

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS indices • equation • base • coefficient

Objective

Use indices ii to solve Core IGCSE Mathematics problems.

You must be able to:

- Apply integer index laws to algebraic products and quotients
- Simplify coefficients separately from powers
- Find an unknown integer exponent by matching powers

1 Key knowledge and worked examples

Key knowledge: Indices II

Index rules apply to letters as well as numbers. $a^m a^n = a^{m+n}$, $a^m/a^n = a^{m-n}$ and $(a^m)^n = a^{mn}$. Raise a numerical coefficient to the outside power too. Nonzero values have zero power 1 and negative powers are reciprocals. To solve a simple index equation, express both sides with the same base; logarithms are not needed.

Worked example: Indices II

coefficient: cube	index: multiply	(3x squared) cubed

Simplify $(3x^2)^3$.

- Cube both factors: $Q = 3^3(x^2)^3 = 27x^6$.
- In division, $12a^5/(3a^{-2}) = (12/3)a^{5-(-2)} = 4a^7$ for $a \neq 0$.
- Since $2^5 = 32$, the equation $2^x = 32$ gives $x = 5$.

Method check

Apply a power to both the coefficient and each factor. Division subtracts indices, including when the divisor has a negative index.

2 Practice

2.1 Simplify x^3x^4. [2]

2.2 Simplify $15a^6/(5a^2)$ for $a \neq 0$. [2]

2.3 Simplify $(2y^3)^2$. [2]

2.4 Solve $3^n = 81$. [2]

3 Exam-style questions

3.1 Apply the indices ii methods below. Show working and include units where appropriate.

(a) Simplify $4x^{-2} \times 3x^5$ for $x \neq 0$. [2]

(b) Divide your answer by $6x$ and simplify. [2]

(c) Find the resulting value when $x = -2$. [2]

3.2 For $x \neq 0$, let $E = (3x^2)^2/(9x^{-1})$.

(a) Simplify E fully and calculate it when x = -2. [3]

(b) Find the integer n for which $2^{n+1} = 64$. Explain why logarithms are unnecessary. [3]

Solutions

Award method and answer marks as indicated; accept equivalent correct methods.

2.1

- Add powers: x^7.

2.2

- Divide coefficients and subtract indices: $3a^4$.

2.3

- Square both factors: $4y^6$.

2.4

- $81 = 3^4$, so $n = 4$.

3.1

(a)

- $Q = (4 \times 3)x^{-2+5} = \mathbf{12x^3}$.

(b)

- $R = 12x^3/(6x) = \mathbf{2x^2}$.

(c)

- $R = 2(-2)^2 = 2(4) = \mathbf{8}$.

3.2

(a)

- Numerator is $9x^4$, so $E = x^{4-(-1)} = \mathbf{x^5}$.
- At $x = -2$, $E = (-2)^5 = \mathbf{-32}$.

(b)

- Rewrite $64 = 2^6$.
- Match exponents: $n + 1 = 6$, so $n = \mathbf{5}$.
- Both sides can be written with base 2.