Exercise walk-through

E7.2
Vectors in two dimensions

IGCSE Mathematics

You must be able to

- add, subtract and multiply column vectors by a scalar
- find the magnitude of a vector
- use $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

1 The method

Method — Working with vectors

- **Add/scale:** $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

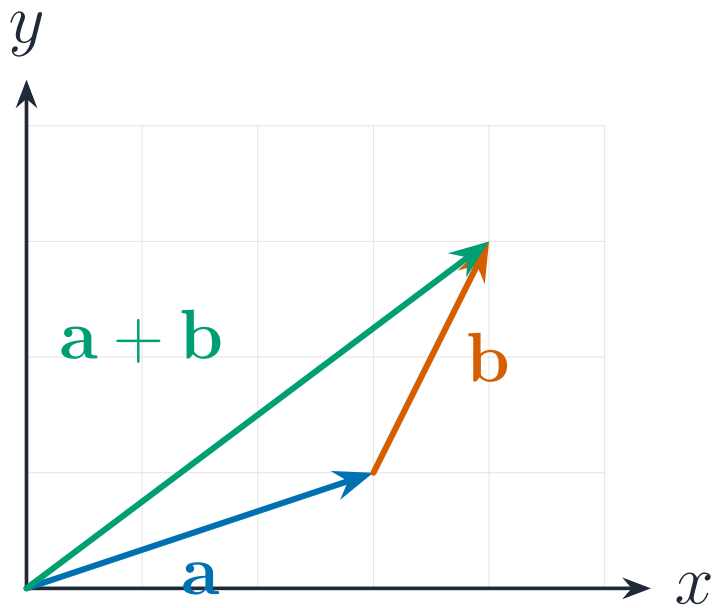
  $\rightarrow \mathbf{a} + \mathbf{b} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$; $3\mathbf{a} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$.

- **Magnitude** : $\left| \begin{pmatrix} x \\ y \end{pmatrix} \right| = \sqrt{x^2 + y^2}$

  $\rightarrow \left| \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right| = 5.$

- **Position vectors:** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

Answer shapes: **M1** a correct method, **A1** the correct answer.



Exercises 2.1·2.2

2.1 $\mathbf{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$. Find $\mathbf{a} + \mathbf{b}$. [1]

2.2 Find $2\mathbf{a}$ for $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. [1]

Exercises 2.1·2.2

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2.1

- $\begin{pmatrix} 4+1 \\ 2-3 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

2.2 Find $2\mathbf{a}$ for $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. [1]

2.2

- $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$

Exercises 2.3·3.1

2.3 Find the magnitude of $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$. [2]

3.1 $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$. Find $\mathbf{a} - \mathbf{b}$. [2]

Exercises 2.3·3.1

2.3 Find the magnitude of $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$. [2]

2.3

$$\sqrt{6^2 + 8^2} = \sqrt{100} = \mathbf{10}$$

3.1 $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$. Find $\mathbf{a} - \mathbf{b}$. [2]

3.1

- $\begin{pmatrix} 5-1 \\ 2-6 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$

Exercise 3.2

3.2 The position vector of A is $\begin{pmatrix}1\\3\end{pmatrix}$ and of B is $\begin{pmatrix}7\\11\end{pmatrix}$. Find $\overrightarrow{AB}$ and its magnitude. [3]

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3.2

- $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix}6\\8\end{pmatrix}$
- magnitude $\sqrt{6^2 + 8^2} = \mathbf{10}$

Exercise 3.3 (a)

3.3 $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

(a) Work out $2\mathbf{a} - \mathbf{b}$ as a column vector. [2]

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(a)

- $2\mathbf{a} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$
- $2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 6 - (-2) \\ -2 - 4 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

Exercise 3.3 (b)

3.3 $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

(b) Find $|\mathbf{a}|$, giving your answer in surd form and to 2 decimal places. [3]

Exercise 3.3 (b)

3.3 $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

(b) Find $|\mathbf{a}|$, giving your answer in surd form and to 2 decimal places. [3]

(b)

- $|\mathbf{a}| = \sqrt{3^2 + (-1)^2}$
- $\sqrt{\mathbf{10}} = \mathbf{3.16}$ to 2 d.p.

Exercise 3.3 (c)

3.3 $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

(c) $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$. Write $\overrightarrow{AC}$ and $\overrightarrow{OB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

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(c)

- $\overrightarrow{AC} = -\mathbf{a} + \mathbf{b}$
- in a parallelogram $\overrightarrow{OB} = \mathbf{a} + \mathbf{b}$

Exercise 3.3 (d)

3.3 $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

(d) M is the midpoint of AB. Show that $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}\mathbf{b}$, and hence write $\overrightarrow{OM}$ as a column vector. [4]

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(d)

- $\overrightarrow{AB} = \mathbf{b}$ so $\overrightarrow{AM} = \frac{1}{2}\mathbf{b}$
- $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \mathbf{a} + \frac{1}{2}\mathbf{b}$
- $= \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$