

Exercise walk-through

E7.1

Transformations

IGCSE Mathematics

You must be able to

- find the image of a point under each transformation
- describe a transformation with all the required details
- use a column vector for a translation

1

The method

Method — The four transformations

- **Reflection** : give the mirror line. $(3, 2)$ in the y -axis $\rightarrow (-3, 2)$.
- **Rotation** : give centre, angle, direction. $(3, 1)$, 90° anticlockwise about $O \rightarrow (-1, 3)$.
- **Enlargement** : give centre and scale factor k . $(1, 2)$ from O , $k = 2 \rightarrow (2, 4)$.
- **Translation** : give the column vector. $(5, 3)$ by $\begin{pmatrix} -2 \\ 4 \end{pmatrix} \rightarrow (3, 7)$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

Exercises 2.1·2.3

2.1 Reflect the point $(4, 3)$ in the x -axis.

[1]

2.2 Translate the point $(2, 1)$ by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

[1]

2.3 Enlarge the point $(2, 3)$ from the origin by scale factor 2.

[1]

Exercises 2.1·2.3

2.1 Reflect the point $(4, 3)$ in the x -axis.

[1]

2.1

- reflect in the x -axis: $(\mathbf{4}, -\mathbf{3})$

2.2 Translate the point $(2, 1)$ by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

[1]

2.2

- $(2 + 3, 1 - 2) = (\mathbf{5}, -\mathbf{1})$

2.3 Enlarge the point $(2, 3)$ from the origin by scale factor 2.

[1]

2.3

- $(2 \times 2, 3 \times 2) = (\mathbf{4}, \mathbf{6})$

Exercises 2.4·3.1

2.4 Describe fully the transformation that maps $(3, 2)$ onto $(-3, 2)$. [2]

2.5 State the two things you must give to describe a rotation fully, besides its name. [2]

3.1 The point $(4, 1)$ is rotated 90° anticlockwise about the origin. Find the image. [2]

Exercises 2.4·3.1

2.4 Describe fully the transformation that maps $(3, 2)$ onto $(-3, 2)$. [2]

2.4

- a **reflection** in the y -axis, the line $x = 0$
-

2.5 State the two things you must give to describe a rotation fully, besides its name. [2]

2.5

- the **angle of turn** and the **direction**
 - the **centre of rotation**
-

3.1 The point $(4, 1)$ is rotated 90° anticlockwise about the origin. Find the image. [2]

3.1

- $(x, y) \rightarrow (-y, x)$
- $(4, 1) \rightarrow (-1, 4)$

Exercises 3.2·3.3

3.2 A quadrilateral has vertices $(2, 1)$, $(5, 1)$, $(5, 3)$ and $(2, 3)$. After one transformation its image has vertices $(6, 3)$, $(15, 3)$, $(15, 9)$ and $(6, 9)$. State the name of this transformation and give the two further pieces of information needed to describe it fully. [3]

3.3 A shape is enlarged from the origin by scale factor $\frac{1}{2}$. The point $(6, 4)$ is on the shape. Find its image. [2]

Exercises 3.2·3.3

3.2 A quadrilateral has vertices $(2, 1)$, $(5, 1)$, $(5, 3)$ and $(2, 3)$. After one transformation its image has vertices $(6, 3)$, $(15, 3)$, $(15, 9)$ and $(6, 9)$. State the name of this transformation and give the two further pieces of information needed to describe it fully. [3]

3.2

- an **enlargement**
- scale factor **3**
- centre **$(0, 0)$** – each image coordinate is three times the object

3.3 A shape is enlarged from the origin by scale factor $\frac{1}{2}$. The point $(6, 4)$ is on the shape. Find its image. [2]

3.3

- multiply by $\frac{1}{2}$
- **$(3, 2)$**

Exercise 3.4 (a)

3.4 Triangle P has vertices $(2, 1)$, $(6, 1)$ and $(2, 3)$.

(a) Triangle P is reflected in the line $x = 1$ to give triangle Q . Write the coordinates of the vertices of Q . [3]

Exercise 3.4 (a)

3.4 Triangle P has vertices $(2, 1)$, $(6, 1)$ and $(2, 3)$.

(a) Triangle P is reflected in the line $x = 1$ to give triangle Q . Write the coordinates of the vertices of Q . [3]

(a)

- reflecting in $x = 1$ sends (x, y) to $(2 - x, y)$
- $(\mathbf{0}, \mathbf{1})$, $(\mathbf{-4}, \mathbf{1})$ and $(\mathbf{0}, \mathbf{3})$

Exercise 3.4 (b)

3.4 Triangle P has vertices $(2, 1)$, $(6, 1)$ and $(2, 3)$.

(b) Triangle P is enlarged with centre $(0, 0)$ and scale factor $\frac{3}{2}$ to give triangle R . Write the coordinates of the vertices of R . [3]

Exercise 3.4 (b)

3.4 Triangle P has vertices $(2, 1)$, $(6, 1)$ and $(2, 3)$.

(b) Triangle P is enlarged with centre $(0, 0)$ and scale factor $\frac{3}{2}$ to give triangle R . Write the coordinates of the vertices of R . [3]

(b)

- multiply each coordinate by $\frac{3}{2}$
- **$(3, 1.5)$, $(9, 1.5)$ and $(3, 4.5)$**

Exercise 3.4 (c)

3.4 Triangle P has vertices $(2, 1)$, $(6, 1)$ and $(2, 3)$.

(c) Triangle P is rotated 90° clockwise about the origin to give triangle S . Write the coordinates of the vertices of S . [3]

Exercise 3.4 (c)

3.4 Triangle P has vertices $(2, 1)$, $(6, 1)$ and $(2, 3)$.

(c) Triangle P is rotated 90° clockwise about the origin to give triangle S . Write the coordinates of the vertices of S . [3]

(c)

- 90° clockwise about the origin: $(x, y) \rightarrow (y, -x)$
- $(1, -2)$, $(1, -6)$ and $(3, -2)$

Exercise 3.4 (d)

3.4 Triangle P has vertices $(2, 1)$, $(6, 1)$ and $(2, 3)$.

(d) Triangle T has vertices $(-2, -1)$, $(-6, -1)$ and $(-2, -3)$. Describe fully the single transformation that maps P onto T . [3]

Exercise 3.4 (d)

3.4 Triangle P has vertices $(2, 1)$, $(6, 1)$ and $(2, 3)$.

(d) Triangle T has vertices $(-2, -1)$, $(-6, -1)$ and $(-2, -3)$. Describe fully the single transformation that maps P onto T . [3]

(d)

- a **rotation** of **180°** about the origin **$(0, 0)$**
- (an enlargement of scale factor -1 , centre the origin, is also accepted)