Exercise walk-through

E4.6
Angles

IGCSE Mathematics

You must be able to

- use corresponding, alternate and co-interior angles
- use the angle sums of triangles and quadrilaterals
- find interior and exterior angles of a regular polygon

1 The method

Method — Angle facts

- Angles on a line $= 180°$; at a point $= 360°$; **vertically opposite** equal.
- **Corresponding** (F) and **alternate** (Z) angles equal; **co-interior** (C) add to $180°$.
- Polygon: sum of interior angles $= (n-2) \times 180°$; exterior angles sum to $360°$.
- **Worked:** regular hexagon — exterior $= \frac{360}{6} = 60°$, interior $= 120°$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

Exercises 2.1·2.3

2.1 Three angles on a straight line are x, 55° and 65°. Find x. [2]

2.2 A triangle has angles x, $2x$ and 60°. Find x. [2]

2.3 Find the exterior angle of a regular pentagon. [2]

Exercises 2.1·2.3

2.1 Three angles on a straight line are x, $55°$ and $65°$. Find x. [2]

2.1

- angles on a straight line add to $180°$
- $x + 55 + 65 = 180$ so $\mathbf{x = 60°}$

2.2 A triangle has angles x, $2x$ and $60°$. Find x. [2]

2.2

- angles in a triangle add to $180°$
- $3x = 120$ so $\mathbf{x = 40°}$

2.3 Find the exterior angle of a regular pentagon. [2]

2.3

$$\frac{360°}{5} = \mathbf{72°}$$

Exercises 3.1–3.2

3.1 A line crosses two parallel lines. One angle is 72°. Find the co-interior angle, giving a reason. [2]

3.2 Find the size of each interior angle of a regular octagon. [3]

Exercises 3.1–3.2

3.1 A line crosses two parallel lines. One angle is $72°$. Find the co-interior angle, giving a reason. [2]

3.1

- co-interior angles add to $180°$
- $180 - 72 = \mathbf{108}°$

3.2 Find the size of each interior angle of a regular octagon. [3]

3.2

- exterior $= \dfrac{360}{8} = 45°$
- interior $= 180 - 45 = \mathbf{135}°$

Exercise 3.3

3.3 The interior angle of a regular polygon is 156°. Find the number of sides. [3]

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3.3

- exterior $= 180 - 156 = 24°$

$$\frac{360}{24} = \mathbf{15}$$

sides

Exercise 3.4 (a)

3.4 In the diagram, PQ is parallel to RS. A straight line crosses PQ at A and RS at B. The angle between the line and PQ on the upper right at A is $(3x + 10)°$, and the angle between the line and RS on the lower left at B is $(5x - 30)°$.

(a) Explain why these two angles are equal, naming the angle fact you use, and find x. [4]

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3.4 In the diagram, PQ is parallel to RS. A straight line crosses PQ at A and RS at B. The angle between the line and PQ on the upper right at A is $(3x + 10)°$, and the angle between the line and RS on the lower left at B is $(5x - 30)°$.

(a) Explain why these two angles are equal, naming the angle fact you use, and find x. [4]

(a)

- they are **alternate angles** between parallel lines PQ and RS
- $3x + 10 = 5x - 30$
- $40 = 2x$ so $\mathbf{x = 20}$

Exercise 3.4 (b)

3.4 In the diagram, PQ is parallel to RS. A straight line crosses PQ at A and RS at B. The angle between the line and PQ on the upper right at A is $(3x + 10)°$, and the angle between the line and RS on the lower left at B is $(5x - 30)°$.

(b) Find the size of each of the two angles. [2]

Exercise 3.4 (b)

3.4 In the diagram, PQ is parallel to RS. A straight line crosses PQ at A and RS at B. The angle between the line and PQ on the upper right at A is $(3x + 10)°$, and the angle between the line and RS on the lower left at B is $(5x - 30)°$.

(b) Find the size of each of the two angles. [2]

(b)

- $3(20) + 10 = \mathbf{70}°$
- $5(20) - 30 = \mathbf{70}°$ – the same, as expected

Exercise 3.4 (c)

3.4 In the diagram, PQ is parallel to RS. A straight line crosses PQ at A and RS at B. The angle between the line and PQ on the upper right at A is $(3x + 10)°$, and the angle between the line and RS on the lower left at B is $(5x - 30)°$.

(c) A regular polygon has an exterior angle of $24°$. Find the number of sides and the sum of its interior angles. [4]

Exercise 3.4 (c)

3.4 In the diagram, PQ is parallel to RS. A straight line crosses PQ at A and RS at B. The angle between the line and PQ on the upper right at A is $(3x + 10)°$, and the angle between the line and RS on the lower left at B is $(5x - 30)°$.

(c) A regular polygon has an exterior angle of 24°. Find the number of sides and the sum of its interior angles. [4]

(c)

- exterior angles of any polygon add to 360°
- $n = \dfrac{360}{24} = \mathbf{15}$ sides
- interior angle sum $= (n - 2) \times 180 = 13 \times 180 = \mathbf{2340}°$

Exercise 3.4 (d)

3.4 In the diagram, PQ is parallel to RS. A straight line crosses PQ at A and RS at B. The angle between the line and PQ on the upper right at A is $(3x + 10)°$, and the angle between the line and RS on the lower left at B is $(5x - 30)°$.

(d) Explain why a regular polygon cannot have an interior angle of $160.5°$. [2]

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(d) Explain why a regular polygon cannot have an interior angle of $160.5°$. [2]

(d)

- exterior $= 180 - 160.5 = 19.5°$
- $\dfrac{360}{19.5} = 18.46\ldots$, not a whole number, so no such regular polygon exists