

Exercise walk-through

E4.4

Similarity

IGCSE Mathematics

You must be able to

- find a missing length in similar shapes
- use the area ratio k^2 for similar shapes
- use the volume ratio k^3 for similar solids

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The method

Method — Similar shapes and solids

- **Similar** : same angles, all sides $\times$ the **scale factor** k .
- $\frac{\text{area } A}{\text{area } B} = k^2$; $\frac{\text{volume } A}{\text{volume } B} = k^3$.
- **Worked:** lengths $2 : 3$, smaller volume $40 \text{ cm}^3 \rightarrow$ ratio $8 : 27 \rightarrow$ larger
 $= 40 \times \frac{27}{8} = 135 \text{ cm}^3$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

Exercises 2.1·2.3

2.1 Two similar triangles have a scale factor of 3. A side of the small triangle is 4 cm. Find the matching side of the large triangle. [1]

2.2 Two similar shapes have lengths in the ratio 2 : 5. Write down the ratio of their areas. [1]

2.3 Two similar solids have lengths in the ratio 1 : 2. Write down the ratio of their volumes. [1]

Exercises 2.1·2.3

2.1 Two similar triangles have a scale factor of 3. A side of the small triangle is 4 cm. Find the matching side of the large triangle. [1]

2.1

- $4 \times 3 = \mathbf{12}$ cm

2.2 Two similar shapes have lengths in the ratio 2 : 5. Write down the ratio of their areas. [1]

2.2

- $2^2 : 5^2 = \mathbf{4 : 25}$

2.3 Two similar solids have lengths in the ratio 1 : 2. Write down the ratio of their volumes. [1]

2.3

- $1^3 : 2^3 = \mathbf{1 : 8}$

Exercises 3.1·3.2

3.1 Two similar shapes have lengths in the ratio 3 : 4. The area of the smaller is 18 cm^2 . Find the area of the larger. [3]

3.2 Two similar bottles have heights 10 cm and 15 cm. The smaller holds 400 ml. Find the volume of the larger. [3]

Exercises 3.1·3.2

3.1 Two similar shapes have lengths in the ratio 3 : 4. The area of the smaller is 18 cm^2 . Find the area of the larger. [3]

3.1

- area ratio $= 3^2 : 4^2 = 9 : 16$

$$18 \times \frac{16}{9} = \mathbf{32} \text{ cm}^2$$

3.2 Two similar bottles have heights 10 cm and 15 cm. The smaller holds 400 ml. Find the volume of the larger. [3]

3.2

- length ratio $10 : 15 = 2 : 3$; volume ratio $2^3 : 3^3 = 8 : 27$

$$400 \times \frac{27}{8} = \mathbf{1350} \text{ ml}$$

Exercise 3.3 (a)

3.3 Triangle ABC has $AB = 6$ cm, $BC = 9$ cm and area 20 cm^2 . Triangle PQR is similar to ABC , with PQ corresponding to AB and $PQ = 15$ cm.

(a) Write down the scale factor from ABC to PQR , and find QR . [3]

Exercise 3.3 (a)

3.3 Triangle ABC has $AB = 6$ cm, $BC = 9$ cm and area 20 cm^2 . Triangle PQR is similar to ABC , with PQ corresponding to AB and $PQ = 15$ cm.

(a) Write down the scale factor from ABC to PQR , and find QR . [3]

(a)

$$\text{scale factor} = \frac{15}{6} = \mathbf{2.5}$$

- $QR = 9 \times 2.5 = \mathbf{22.5}$ cm

Exercise 3.3 (b)

3.3 Triangle ABC has $AB = 6$ cm, $BC = 9$ cm and area 20 cm^2 . Triangle PQR is similar to ABC , with PQ corresponding to AB and $PQ = 15$ cm.

(b) Find the area of triangle PQR .

[3]

Exercise 3.3 (b)

3.3 Triangle ABC has $AB = 6$ cm, $BC = 9$ cm and area 20 cm^2 . Triangle PQR is similar to ABC , with PQ corresponding to AB and $PQ = 15$ cm.

(b) Find the area of triangle PQR .

[3]

(b)

- areas scale by the **square** of the length scale factor
- $2.5^2 = 6.25$
- $\text{area} = 20 \times 6.25 = \mathbf{125} \text{ cm}^2$

Exercise 3.3 (c)

3.3 Triangle ABC has $AB = 6$ cm, $BC = 9$ cm and area 20 cm^2 . Triangle PQR is similar to ABC , with PQ corresponding to AB and $PQ = 15$ cm.

(c) A solid cone has a volume of 54 cm^3 . A mathematically similar cone has a volume of 128 cm^3 . Find the ratio of their heights. [3]

Exercise 3.3 (c)

3.3 Triangle ABC has $AB = 6$ cm, $BC = 9$ cm and area 20 cm². Triangle PQR is similar to ABC , with PQ corresponding to AB and $PQ = 15$ cm.

(c) A solid cone has a volume of 54 cm³. A mathematically similar cone has a volume of 128 cm³. Find the ratio of their heights. [3]

(c)

- volumes scale by the cube of the length scale factor

$$\left(\frac{h_2}{h_1}\right)^3 = \frac{128}{54} = \frac{64}{27}$$

- $\frac{h_2}{h_1} = \frac{4}{3}$, so the ratio is **3 : 4**

Exercise 3.3 (d)

3.3 Triangle ABC has $AB = 6$ cm, $BC = 9$ cm and area 20 cm^2 . Triangle PQR is similar to ABC , with PQ corresponding to AB and $PQ = 15$ cm.

(d) Explain why two shapes with equal angles must be similar, but two shapes with equal areas need not be. [3]

Exercise 3.3 (d)

3.3 Triangle ABC has $AB = 6$ cm, $BC = 9$ cm and area 20 cm^2 . Triangle PQR is similar to ABC , with PQ corresponding to AB and $PQ = 15$ cm.

(d) Explain why two shapes with equal angles must be similar, but two shapes with equal areas need not be. [3]

(d)

- equal angles fix the **shape**, so one figure is an enlargement of the other
- equal areas fix only the space enclosed – a 2×8 rectangle and a 4×4 square both have area 16 but are not similar