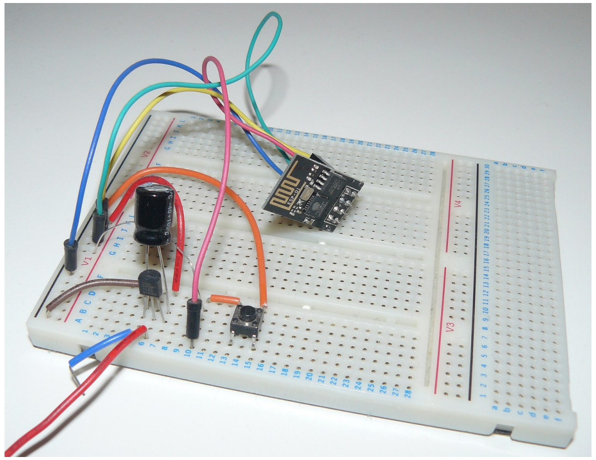
Boolean logic

IGCSE Computer Science

What is Boolean logic?

Boolean logic 布尔逻辑 works with values that are either **true** or **false**. In electronics these are shown as **1** (true) and **0** (false). A **logic gate** 逻辑门 takes one or more of these inputs and gives one output, following a fixed rule.

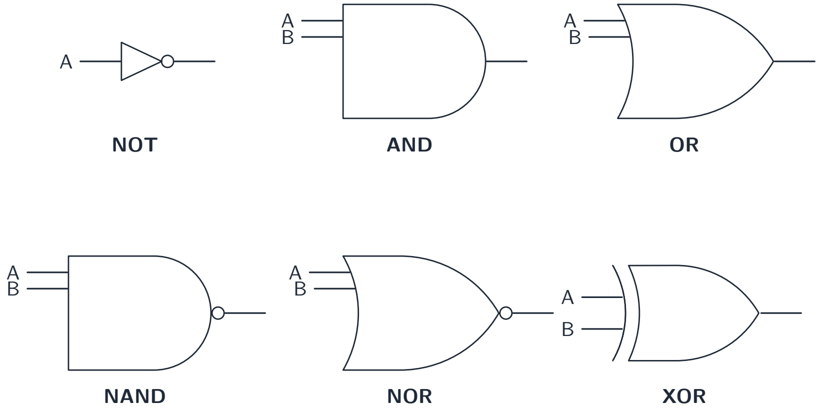
A **truth table** 真值表 lists every possible set of inputs and the output for each. You build it by writing out all the input combinations.



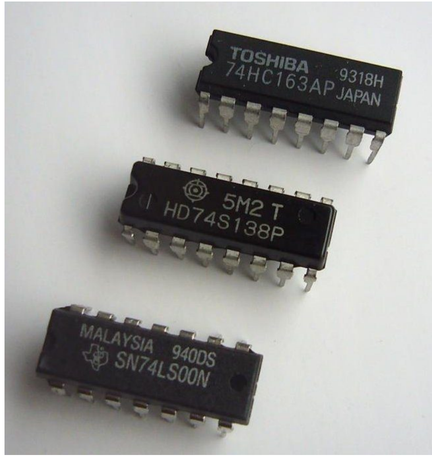
Logic gates are built from electronic circuits like this one, where each gate switches on 1s and 0s

The six logic gates

You must know six gates. `NOT` has one input; all the others have two inputs (A and B).



The six logic gates. A small circle on the output means the result is inverted (NOT, NAND, NOR)



A real logic chip: inside are logic gates like the ones on this page

NOT gate

The **NOT gate** 非门 reverses the input. Output is 1 when the input is 0.

A	Output
0	1
1	0

AND gate

The **AND gate** 与门 gives output 1 **only when both** inputs are 1.

A	B	A · B
0	0	0
0	1	0
1	0	0
1	1	1

AND

AND outputs 1 only when both inputs are 1

A	B	Output
0	0	0
0	1	0
1	0	0
1	1	1

OR gate

The **OR gate** 或门 gives output 1 when **at least one** input is 1.

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

OR

OR outputs 1 when either input is 1

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1

NAND gate

The **NAND gate** 与非门 is AND followed by NOT. The output is the **opposite** of AND.

A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0

NOR gate

The **NOR gate** 或非门 is OR followed by NOT. The output is the **opposite** of OR.

A	B	Output
0	0	1
0	1	0
1	0	0
1	1	0

XOR gate

The **XOR gate** 异或门 (exclusive OR) gives output 1 when the inputs are **different**.

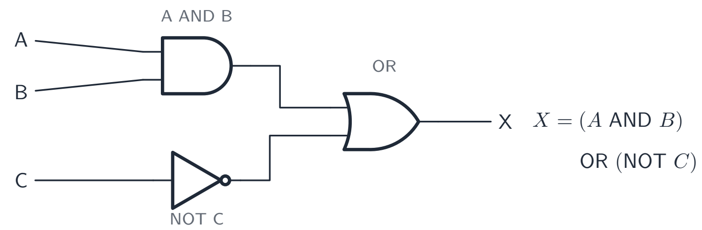
A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0

Logic expressions

A **logic expression** 逻辑表达式 writes a circuit using letters and gate words. The usual way to write the gates:

Gate	In words
NOT A	NOT A
A AND B	A AND B
A OR B	A OR B

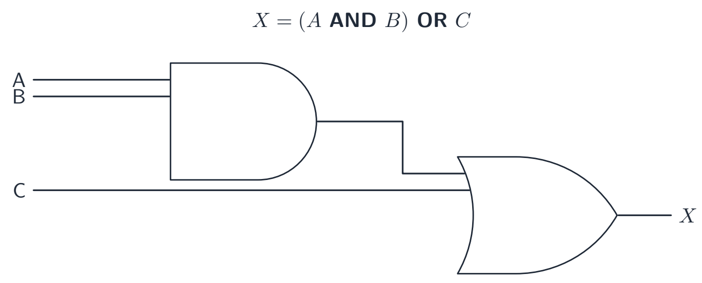
For example, the expression `(A AND B) OR (NOT C)` means: do A AND B, do NOT C, then OR the two results together.



The expression X = (A AND B) OR (NOT C) drawn as a logic circuit

Logic circuits

A **logic circuit** 逻辑电路 joins gates together to carry out a task. The output of one gate can become the input of another. At IGCSE a circuit has up to **three inputs** and **one output**.



Building the circuit for X = (A AND B) OR C — the AND gate's output feeds the OR gate

You must be able to move between four forms:

- a **problem statement** 问题陈述 (a description in words),
- a logic expression,
- a logic circuit,
- a truth table.

From a problem statement to a circuit

Read the statement and pick out the conditions and the logic words (and, or, not). For example:

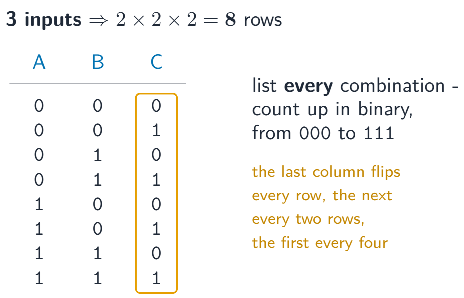
> An alarm (X) sounds when the door is open (A) AND the system is switched on (B).

This is `X = A AND B`, so you draw one AND gate with inputs A and B.

Completing a truth table from a circuit or expression

To fill in a truth table:

1. Write **all** the input combinations. For three inputs there are 8 rows (000 up to 111).
2. Work out each gate's output in order, one column at a time.
3. The last column is the final output.



Three inputs give eight rows: every combination counted in binary

A	B	C	A AND B	(A AND B) OR C
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

Adding a middle "working" column for each gate makes the final output easy to fill in. Always draw the circuit exactly as the statement says, **without simplifying** it.

Worked example. Complete the truth table for `X = (A AND B) OR (NOT C)` for the row A = 1, B = 0, C = 0. Work **outwards from the brackets**, one gate at a time. First `A AND B` = 1 AND 0 = **0**, because AND needs both inputs to be 1. Next `NOT C` = NOT 0 = **1**. Finally OR the two results: 0 OR 1 = **1**. So X = **1**. Give each intermediate gate **its own column** rather than trying to do the whole expression in one step: with three inputs there are $2^3 = 8$ rows, and those intermediate columns are where the method marks live even if the final answer slips.

Exam tips

- Learn all six gates and their truth tables: **NOT**, **AND**, **OR**, **NAND** (NOT AND), **NOR** (NOT OR), **XOR** (output 1 when the inputs are different).
- Build a truth table with *all* input rows (2 inputs → 4 rows, 3 inputs → 8), counting up in binary, and add a working column for each gate.
- Turn a problem statement into a logic expression by picking out the AND / OR / NOT words, then draw it exactly as written — **do not simplify** it.
- NAND and NOR give the *opposite* output to AND and OR; a small circle on a gate's output means the result is inverted.