Data representation

IGCSE Computer Science

Why computers use binary

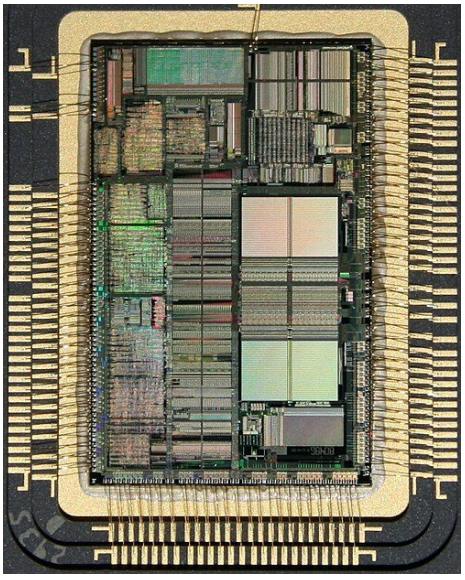
Digital computers represent data using two logical states, written as 1 and 0. A system that uses only two digits is called **binary** (base 2).



Computers represent all data – numbers, text, sound and images – as binary strings of 0s and 1s

Every kind of data – numbers, text, sound and images – must be changed into binary before a computer can use it. The computer processes this binary using **logic gates**, and stores it in **registers** (small, fast stores inside the processor).

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A microprocessor holds millions of tiny transistors, each a switch that is on (1) or off (0) – the physical basis of binary

Number systems

A **number system** is a way of writing numbers using a fixed set of digits. You need three of them.

System	Base	Digits used
Denary	10	0–9
Binary	2	0 and 1
Hexadecimal	16	0–9 then A–F

- **denary** is the normal counting system (also called decimal).
- **binary** uses only 0 and 1.
- **hexadecimal** (hex) uses sixteen digits: 0–9, then A, B, C, D, E, F stand for 10, 11, 12, 13, 14, 15.

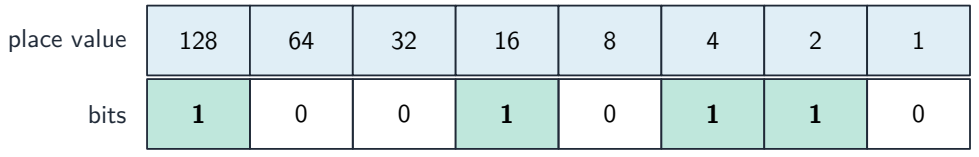
The **base** tells you how many different digits a system uses.

Place value

Each column in a number has a **place value** . In binary the place values double from right to left. For an 8-bit number they are:

```
128  64  32  16  8  4  2  1
```


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An 8-bit place-value chart: the 1s sit under the values that add up to 150

One **bit** is a single 0 or 1. Eight bits make one **byte**. Four bits (half a byte) is a **nibble**.

■ Converting between number systems

Denary → binary. Write the place values. Put a 1 under each value you need so they add up to your number; put 0 under the rest.

Example: change denary 150 to binary. $150 = 128 + 16 + 4 + 2$.

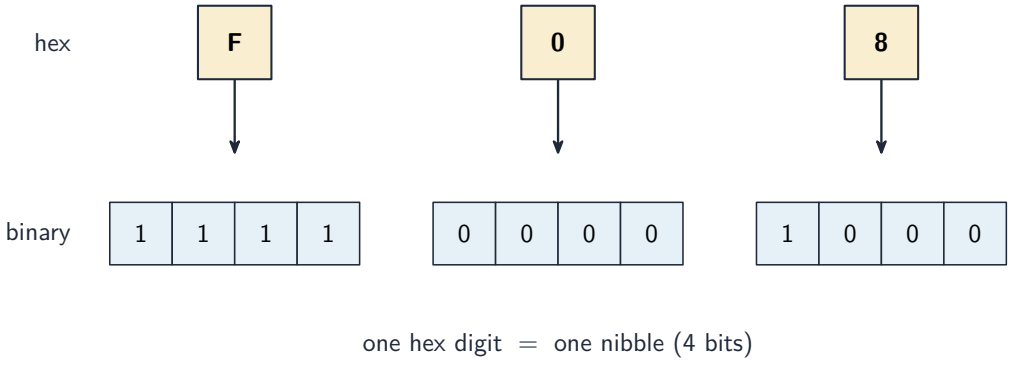
```
128 64 32 16 8 4 2 1
  1  0  0  1 0 1 1 0
```

So 150 = `10010110`.

Binary → denary. Add the place values where there is a 1. `10010110` $= 128 + 16 + 4 + 2 = 150$.

Hexadecimal → binary. Change each hex digit into its own 4-bit group (a nibble).

Example: hex F08. $F = 1111$, $0 = 0000$, $8 = 1000$, so F08 = `1111 0000 1000`.



Each hex digit maps to its own 4-bit nibble – F08 = 1111 0000 1000

Binary → hexadecimal. Group the bits into nibbles of 4, starting from the right. Change each nibble to one hex digit.

Denary → hexadecimal. The easy way is to change to binary first, then binary to hex.

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This table helps with the hex letters:

Denary	Binary	Hex
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Cambridge questions use binary numbers up to 16 bits long.

> **Worked example.** Convert denary 100 to 8-bit binary, then to hexadecimal.

$100 = 64 + 32 + 4$, so the binary is `01100100`. Splitting into nibbles, `0110 0100` = 6 and 4, so the hexadecimal is `64`.

> **Worked example (hexadecimal → denary).** Convert hex `2C` to denary. The hex place values are 16 and 1. `2` is worth $2 \times 16 = 32$ and `C` is 12, so `2C` $= 32 + 12 = 44$. Check through binary: `0010 1100` $= 32 + 8 + 4 = 44$.

■ Why hexadecimal is used

Hex is shorter than binary and easier for people to read and write. One hex digit replaces 4 binary digits, so you make fewer mistakes. The value does not change – hex is just a shorter way to show the same binary.

Computer scientists use hex for:

- MAC addresses and IPv6 addresses
- colour codes in HTML (for example `#FF0000` is red)
- **memory addresses** and error codes
- showing the contents of memory (a "memory dump")

Binary addition

You can add two 8-bit binary numbers, column by column from the right, just like denary. The rules for one column are:

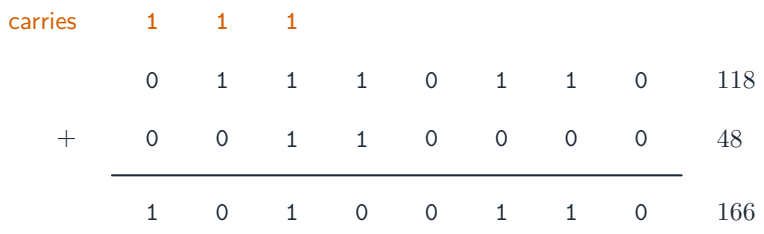
A	B	Result bit	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

When a carry also comes in, $1 + 1 + 1 = 1$ with a carry of 1.

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Example: add `01110110` (118) and `00110000` (48).

```
  0 1 1 1 0 1 1 0    (118)
+ 0 0 1 1 0 0 0 0    (48)
--------------------
  1 0 1 0 0 1 1 0    (166)
```

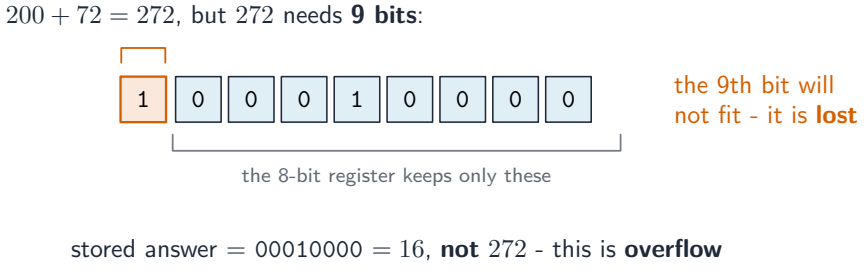


Adding column by column; the carries ripple to the left. 118 + 48 = 166

Overflow

When interpreted as an **unsigned** integer, an 8-bit register holds values from 0 to 255. If an addition gives a result above 255, the answer needs a 9th bit. The register cannot hold this extra bit, so it is lost. This is called **overflow** (an overflow error). It happens when a value goes outside the limit the register can store.

Example: `11001000` (200) + `01001000` (72) = 272. In binary that is `1 00010000`, which needs 9 bits. The leading 1 will not fit in 8 bits, so the stored answer is wrong.



Adding 200 and 72 needs 9 bits, but an 8-bit register drops the ninth, so the answer is wrong

Logical binary shift

A **logical binary shift** moves all the bits left or right by a number of places.

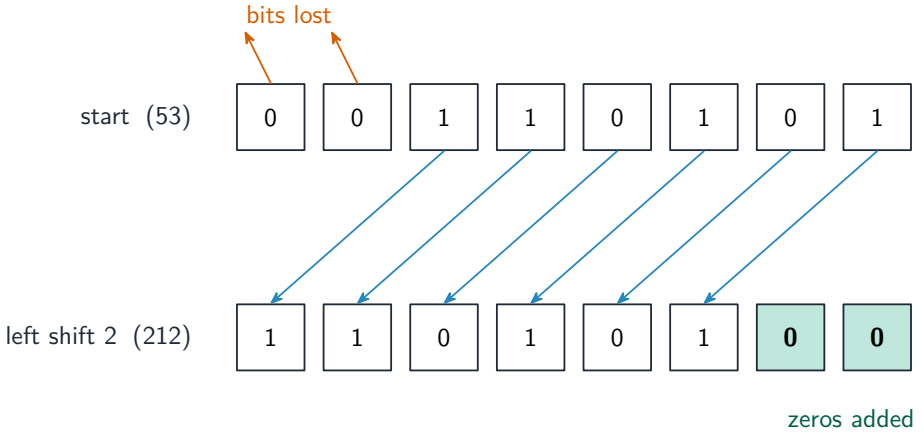
- Bits that move off the end of the register are **lost**.
- **Zeros** are added at the empty end.

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For an unsigned integer, a **left shift** multiplies by 2 for each place moved **if no non-zero bit is lost**. A logical **right shift** gives the whole-number result of division by 2 for each place; the right-most bits (the **least significant bit(s)**) are lost.

Example: left shift `00110101` (53) by 2 places.

```
start:          0 0 1 1 0 1 0 1
left shift 2:   1 1 0 1 0 1 0 0
```



A left shift of 2: every bit moves 2 places left, the top bits are lost and zeros fill the right

The result is `11010100` (212), which is 53×4. The two left-most bits were lost and two zeros came in on the right. If a 1 is pushed off the end, that information is gone for good.

Worked example (right shift). Shift `10110100` (180) **right** by 3 places. Give the result in binary and denary, and say what effect the shift has.

```
start:           1 0 1 1 0 1 0 0
right shift 3:   0 0 0 1 0 1 1 0
```

The three right-most bits (`100`) are lost and three zeros enter on the left: `00010110` $= 16 + 4 + 2 = 22$. Shifting right 3 places divides by $2^3 = 8$: $180 \div 8 = 22.5$, and the shift keeps only the whole-number part, 22. The 0.5 was carried by the lost 1 bit.

> **Worked example (a left shift that goes wrong).** Shift `01100100` (100) left by 2 places. The result is `10010000` = 144, not 400: a 1 was pushed off the left end, and 400 is too large for 8 bits anyway. A left shift multiplies correctly only while no 1 bits are lost.

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Two's complement

So far the numbers were positive. **Two's complement** lets an 8-bit register hold negative numbers too.

In two's complement, the left-most bit (the **most significant bit**, or MSB) has a *negative* place value:

```
-128  64  32  16  8  4  2  1
```

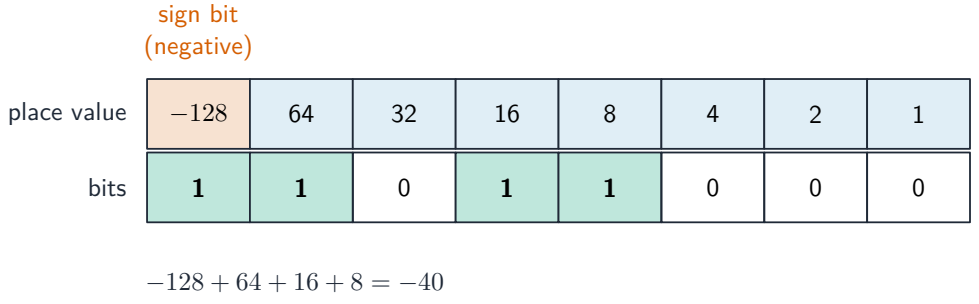
- If the MSB is 0, the number is zero or positive.
- If the MSB is 1, the number is negative.

To make a positive number negative: write the positive binary, flip every bit ($0 \leftrightarrow 1$), then add 1.

Example: make -40.

- $+40 =$ `00101000`
- flip the bits $=$ `11010111`
- add $1 =$ `11011000`

So $-40 =$ `11011000`. Check by adding the place values: $-128 + 64 + 16 + 8 = -40$.



The most significant bit is worth −128, so 11011000 = −128 + 64 + 16 + 8 = −40

To read a negative two's complement number, just add the place values (the MSB counts as -128). The range of an 8-bit two's complement number is -128 to $+127$.

> **Worked example.** Convert the two's complement number `10011010` to denary. The MSB is 1, so the number is negative: $-128 + 16 + 8 + 2 = -102$. Check by the other method: flip the bits (`01100101`) and add 1 (`01100110` $= 102$), so the number is -102.

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Representing text

Computers store text by giving every character a number, then storing that number in binary. The set of characters a computer can use, together with their numbers, is a **character set** .

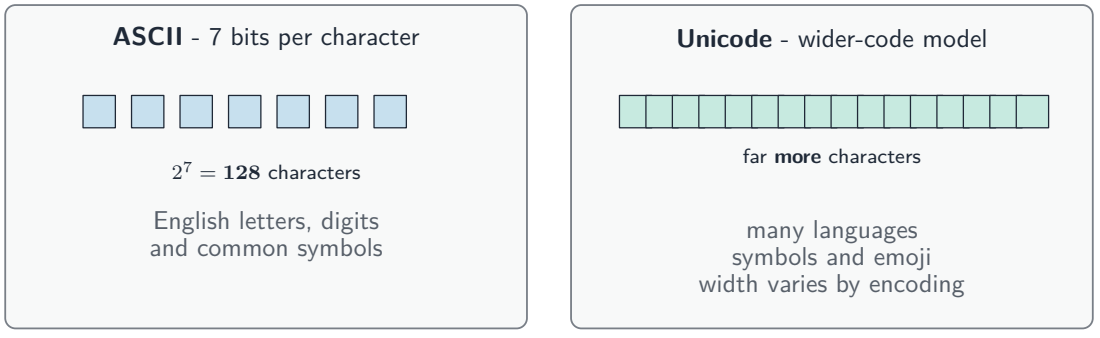
ASCII uses 7-bit character codes, giving $N = 2^b = 2^7 = 128$ different codes. It includes English letters, digits and common symbols; not all 128 codes are letters. **Unicode** represents a much greater range of characters, including different languages and **emoji** .

Try it – encode a short message. This extract from ASCII gives A = 65, B = 66, C = 67 and space = 32. Write the four 7-bit groups for `CAB` , including its final space. Then decode the groups `1000010 1000001 1000011`.

Worked answer. Look up each character, then convert its code to binary:

Character	Code	Seven bits
C	67	1000011
A	65	1000001
B	66	1000010
space	32	0100000

So `CAB` is `1000011 1000001 1000010 0100000`. The second message decodes to `BAC`. A space has a code too; it is not an absence of stored data. The sender and receiver must use the same character-code mapping to interpret the message correctly.



ASCII has 128 codes. Unicode supports a much greater range; the wider row illustrates the syllabus model.

Why a larger range matters. ASCII cannot directly represent a Chinese name. Unicode includes Chinese characters as well as English characters and many other languages. For the syllabus comparison, explain that a greater range needs more distinct binary codes, requiring more bits per character than the 7-bit ASCII model.

Try it – compare stated storage models. Store the four characters `CAB` using (A) 7 bits per character and (B) 16 bits per character. Assume the bits are packed with no padding or extra file information. Find the text data size in each model. The

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16-bit value is an assumption for this example, not a rule that all Unicode characters take 16 bits.

Worked answer. With the same number of characters, multiply by the stated bits per character:

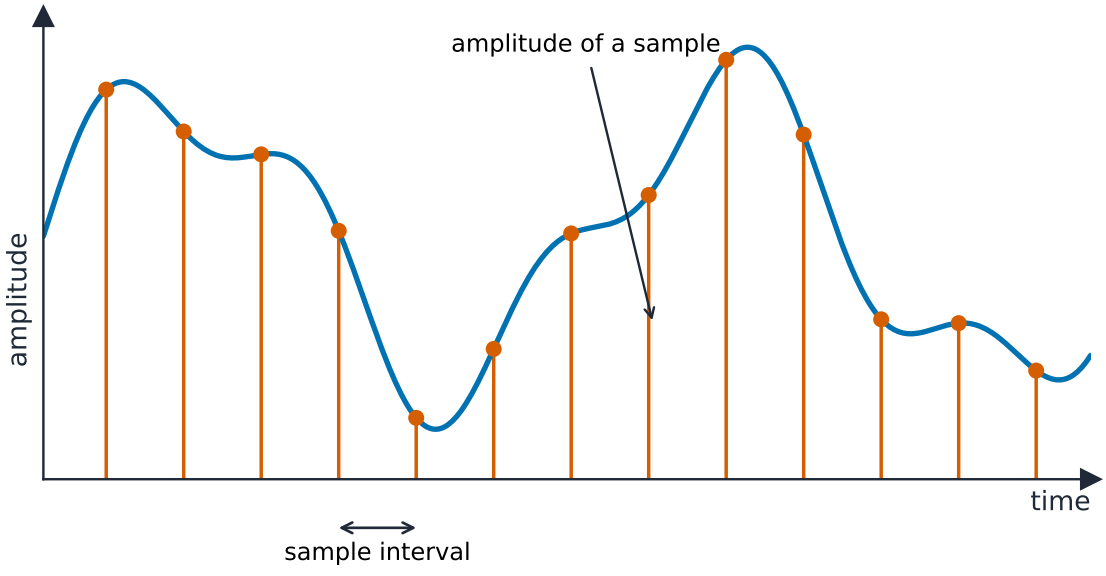
$$S = nb = 4 \times 7 \text{ bits} = 28 \text{ bits}$$

$$S = nb = 4 \times 16 \text{ bits} = 64 \text{ bits}$$

Model B uses more **storage** for this example because its stated code width is larger. Use the width given in an exam question. In real files, the encoding also matters: UTF-8 stores ASCII characters in the same bytes as an ASCII file, so Unicode text does not always occupy more bytes. You do not need the detailed UTF-8 rules for this syllabus comparison.

Representing sound

A **sound wave** is smooth and always changing. To store it, the computer measures the height of the wave at regular moments. This is called **sampling**, and each measurement is a sample.



Sampling records the wave's height (amplitude) at regular moments

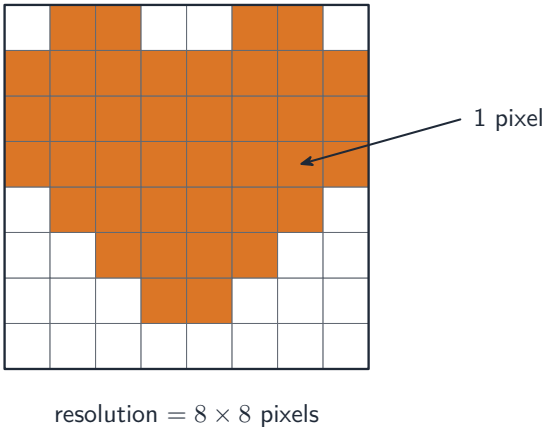
- **sample rate** is the number of samples taken each second (measured in Hz).
- **sample resolution** is the number of bits used for each sample. The height of the wave at a sample point is its **amplitude**.

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A higher sample rate and a higher sample resolution give a more accurate recording, but a larger file.

Representing images

A computer image is made of a grid of small dots called **pixels** .



A bitmap is a grid of pixels; the resolution is how many pixels it has

- **resolution** is the number of pixels in the image (for example 1920×1080).
- **colour depth** is the number of bits used to store the colour of each pixel.

A higher resolution and a higher colour depth give a better-quality image, but a larger file.

The colour depth decides how many different colours each pixel can be: with n bits there are 2^n colours. A colour depth of 1 bit gives 2 colours (black and white), 8 bits give $2^8 = 256$ colours, and 24 bits give over 16 million.

> **Worked example.** An image uses 64 different colours. What is the smallest colour depth it can have? Find the smallest n with $2^n \geqslant 64$: $2^6 = 64$, so **6 bits** per pixel. If it used 65 colours it would need 7 bits, because 6 bits can only make 64 different patterns.

Measuring data storage

Data storage is measured in the units below. A nibble is 4 bits and a byte is 8 bits; from the kibibyte upward, each unit is 1024 times the one before it (because $1024 = 2^{10}$, which fits binary).

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Unit	Equals
bit	a single 0 or 1
nibble	4 bits
byte	8 bits
kibibyte (KiB)	1024 bytes
mebibyte (MiB)	1024 KiB
gibibyte (GiB)	1024 MiB
tebibyte (TiB)	1024 GiB
pebibyte (PiB)	1024 TiB
exbibyte (EiB)	1024 PiB



Hard-disk platters: storage is measured in bytes – knowing file size needs width × height × colour depth for images

Calculating file size

Image file size (in bits) = resolution × colour depth = width × height × colour depth.

Example: an image is 1024 × 1024 pixels with a colour depth of 2 bytes (= 16 bits).

- bits = $1024 \times 1024 \times 16 = 16\,777\,216$ bits
- bytes = $\div 8 = 2\,097\,152$ bytes
- KiB = $\div 1024 = 2048$ KiB
- MiB = $\div 1024 = 2$ MiB

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1024 × 1024 pixels × 16 bits; divide by 1024, not 1000, after the bytes

Bits to bytes is divide by 8; every step after that is divide by 1024

Sound file size (in bits) = sample rate × sample resolution × length in seconds.

These formulas count uncompressed data without file headers. The sound formula is for one channel; multiply by the channel count if a question specifies more channels.

Always divide by 1024 (not 1000) to change to KiB, MiB and so on. Give your answer in the unit the question asks for.

> **Worked example.** A sound is recorded for 30 seconds at a sample rate of 8,000 Hz with a sample resolution of 16 bits. Find the file size in kibibytes (KiB).

- bits = $8\,000 \times 16 \times 30 = 3\,840\,000$ bits
- bytes = $3\,840\,000 \div 8 = 480\,000$ bytes
- KiB = $480\,000 \div 1024 \approx 469$ KiB

Compression

Compression makes a file smaller. A smaller file:

- uses less storage space,
- needs less **bandwidth** (the amount of data a connection can carry),
- takes a shorter time to send (a shorter **transmission** time).

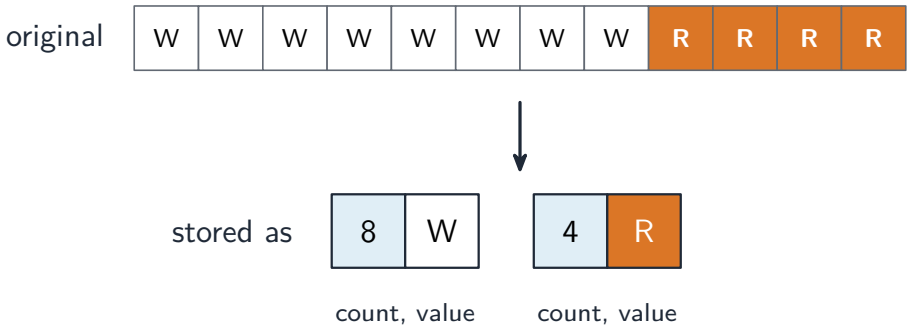
There are two types.

■ Lossless compression

Lossless compression makes the file smaller with **no permanent loss** of data. The original file can be rebuilt exactly.

One method is **run-length encoding** (RLE). It replaces a run of repeated values with one copy of the value and a count of how many times it repeats. For example `WWWWWWWW` (8 whites) is stored as "8 W". This works well when data has many repeats.

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Run-length encoding stores each run once as a count and a value

> **Worked example.** One row of a black-and-white image is `B B B B B W W B B B B B B B B B` (16 pixels). Compress it with RLE, and say whether it saved space if each count and each colour is stored in one byte.

The runs are 5 black, 2 white, 9 black, so the RLE data is `5 B 2 W 9 B`: 6 bytes instead of 16 bytes for the original (one byte per pixel). It saves space because the row has long runs. A row that changes colour at every pixel, `B W B W …`, would become `1 B 1 W 1 B 1 W …`, **twice** as large as the original: RLE only helps when data repeats.

■ Lossy compression

Lossy compression makes the file much smaller by **permanently removing** some data. The removed data cannot be got back. For example:

- reducing the resolution or colour depth of an image,
- reducing the sample rate or sample resolution of a sound.

Use lossless when you must keep every detail (text and program files). Use lossy for photos, music and video, where a small loss of quality is worth a much smaller file.

Exam tips

- Convert denary → binary by subtracting the place values (128, 64, 32 …); binary → denary by adding the place values that hold a 1.
- To convert to hex, group the binary into **nibbles** of 4 bits from the right; each nibble is exactly one hex digit.
- **Overflow** happens when a result is outside the chosen representation's range. Eight-bit unsigned integers hold 0–255; eight-bit two's complement holds −128–127.
- File size in bits: for an image, width × height × colour depth; for sound, sample rate × resolution × seconds. Divide by 8 for bytes, then by 1024 for each larger unit.

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- **Lossless** compression keeps every bit (text; run-length encoding); **lossy** permanently removes data (photos, music) for a much smaller file.

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