Fusion and stars

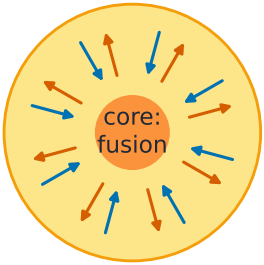
E.5 · SL

How can a hot star be faint?

> Total power depends on emitting area as well as temperature.

- A hot **white dwarf** has a small area; a cooler giant can be much more luminous.
- **Apparent brightness** also depends on distance and extinction.
- Combine nuclear physics with measurements of distance, spectrum and brightness.

A star in equilibrium



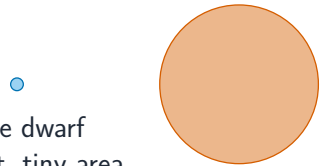
outward: pressure-gradient force (gas + radiation)

inward: gravity

balanced: the star neither expands nor collapses

- The outward pressure-gradient force balances inward gravity.
- Thermal regulation links fusion rate, expansion/contraction and core temperature.

Hot but faint, cool but bright



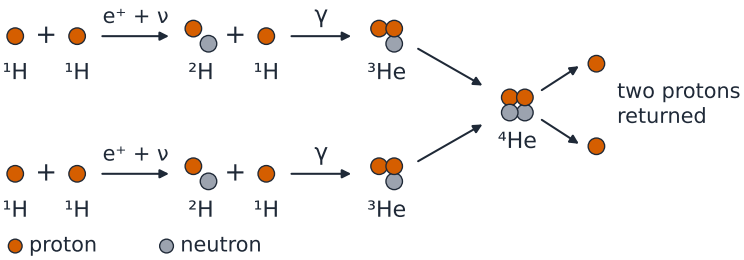
white dwarf
very hot, tiny area
low L

red giant
cool, enormous area
high L

$L = \sigma(4\pi R^2)T^4$:
temperature alone cannot
tell you the power

$L = \sigma 4\pi R^2 T^4$: size matters as well as temperature

The proton–proton chain



net: $4\ {}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 2e^+ + 2\nu$

26.7 MeV including annihilation; neutrinos carry energy away

- The two drawn branches consume four protons net and return two.
- The first step includes a weak interaction, not isolated-proton beta decay.

Energy from hydrogen fusion

1. **Known.** Atomic masses H-1 1.007825 u and He-4 4.002603 u; 1 u $c^2 = 931.5$ MeV.
2. **Initial and final.** $m_i = 4m_H = 4.031300$ u; $m_f = m_{He} = 4.002603$ u.
3. **Energy.** $Q = (m_i - m_f)c^2 = 0.028697 \times 931.5 = 26.73$ MeV.

Decision

Complete net accounting includes **positron** annihilation; neutrinos carry some energy away.

How long will the Sun shine?

- Given photon L $= 3.83 \times 10^{26}$ W and c $= 3.00 \times 10^{8}$ m/s: L/c^2 $= 4.26 \times 10^{9}$ kg/s.
- Assume available fuel $0.10 M_{\odot}$, $M_{\odot} = 1.99 \times 10^{30}$ kg, conversion fraction 0.0071 and constant L: total lifetime is near 10^{10} years.

$$\frac{\Delta m}{\Delta t} = \frac{L_{\odot}}{c^2}$$

Photon mass-equivalent is not hydrogen consumed; total model lifetime is not time remaining.

Fission and fusion

Fission

A heavy nucleus splits into fragments and neutrons.

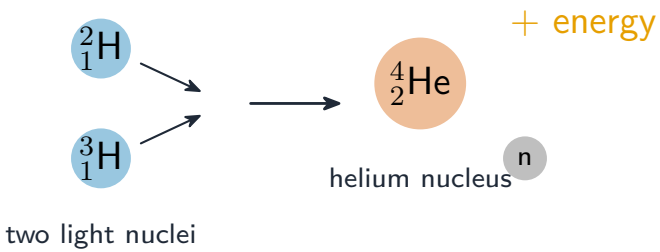
Neutron management depends on the chosen reactor.

Fusion

Suitable light nuclei combine; energy depends on the reaction.

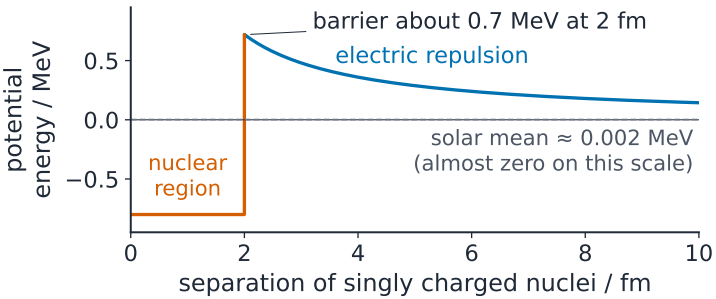
D–T produces neutrons that can activate materials; tritium is radioactive.

Fusion joins light nuclei



The product has more binding energy per nucleon

The Coulomb barrier



- For charge $+e$ and separation 2 fm, electric potential energy is about 0.719 MeV.
- Mean solar-core thermal energy near 1.5×10^7 K is only about 0.00194 MeV.

Why the Sun can still fuse

Energy distributions, tunnelling and reaction physics determine fusion rates.

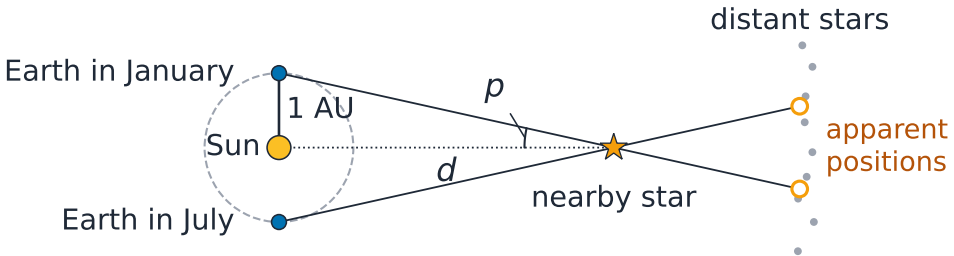
- Tunnelling allows penetration below a classical barrier.
- The proton–proton weak step also limits its rate; sustained stellar encounters need density and confinement.

Distance units

A **parsec** is the distance at which 1 AU makes an angle of one arc-second.

- 1 AU $= 1.50 \times 10^{11}$ m; 1 ly $= 9.46 \times 10^{15}$ m.
- 1 pc $= 3.09 \times 10^{16}$ m $= 3.26$ ly.

Stellar parallax



- The formula uses semi-amplitude p, not the full opposite-position shift 2p.
- Invented example: p = 0.050 arc-seconds gives d = 20 pc.

The parallax equation

- Useful distance range depends on angular precision and the target.
- For small relative error, $\Delta d/d \approx \Delta p/p$.

$$d\,(\mathrm{pc}) = \frac{1}{p\,(\text{arc-second})}$$

Assigned $p = 0.050 \pm 0.002$ arc-seconds gives $d \approx 20 \pm 0.8$ pc, not a universal instrument cutoff.

Luminosity and brightness

- Luminosity is the total power; brightness is power per unit area received.
- Wien's law gives the temperature; absorption lines give the composition.

$$L = 4\pi R^2 \sigma T^4$$

$$b = \frac{L}{4\pi d^2}$$

Use bolometric brightness and state the isotropic, negligible-extinction model.

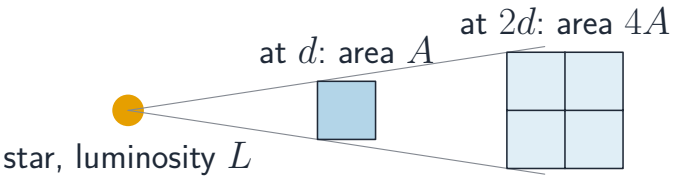
Check: apparent brightness

Your turn

Stars X and Y have equal luminosity; X is three times as far away. Neglect extinction. What is bX/bY?

A. 1/9
B. 1/3
C. 3
D. 9

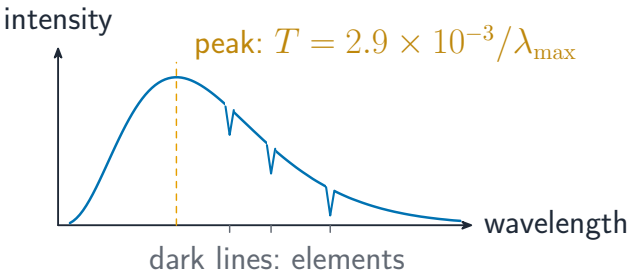
Brightness falls as $1/d^2$



$b = \frac{L}{4\pi d^2}$: twice as far, a quarter as bright

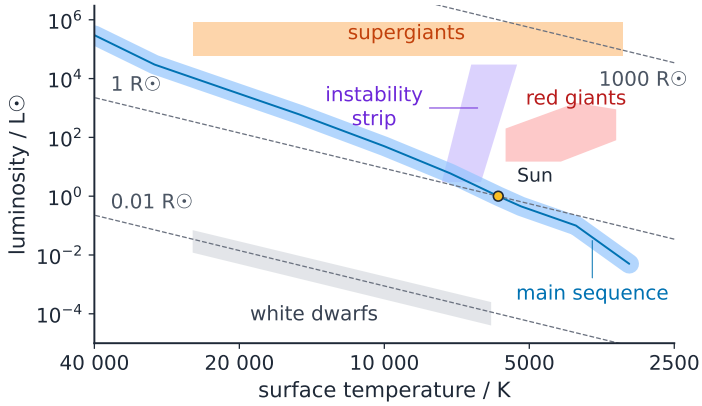
Measure b and L to find the distance

What a star's spectrum tells you



Wien: $\lambda_{\max} T = 2.9 \times 10^{-3}$ m K

The HR diagram



Hotter to the left; the plot shows star populations, not one age track.

What defines the main sequence?

> A main-sequence star has sustained hydrogen burning in its core.

- The population band is not a route every star traverses from end to end.
- The **instability strip** includes many pulsating variables; schematic regions are not exact boundaries.

Regions of the HR diagram

Giants and supergiants

Red giants: cool and luminous, so large.

Supergiants: very high luminosities over a range of temperatures.

White dwarfs

Hot but dim, so very small.

A high temperature does not imply a large total luminosity.

Stellar radii

- Assigned red-supergiant model: L $= 1.2 \times 10^5\ L_\odot$, T $= 3500$ K and $T_\odot = 5800$ K give R $\approx 951\ R_\odot$.
- Use both luminosity and surface temperature.

$$\frac{R}{R_\odot} = \sqrt{\frac{L}{L_\odot}} \left(\frac{T_\odot}{T} \right)^2$$

Assigned hot-white-dwarf model: $0.026\ L_\odot$, 25000 K and $T_\odot = 5800$ K give $0.00868\ R_\odot$; these are illustrative data.

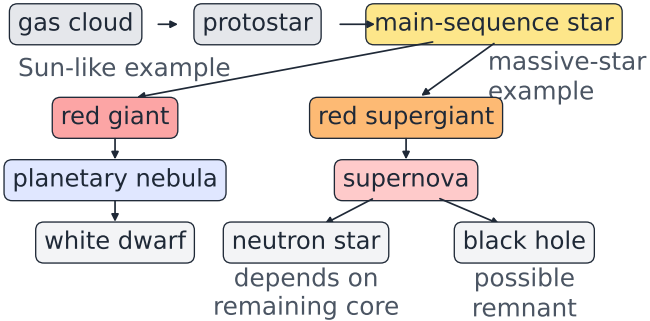
Check: equal luminosity

Your turn

A and B have equal luminosity and TA = 2TB. What is RA/RB?

A. 1/4
B. 1/2
C. 2
D. 4

Mass decides a star's life



selected paths; alternatives and direct collapse omitted

- Initial mass is not remaining-core mass; other pathways exist.

Making the elements

Element origins involve several processes and sites.

- The early universe supplied hydrogen, much of the helium and some lithium.
- Stellar burning makes many heavier nuclei; slow capture occurs in evolved stars.
- Rapid capture in extreme neutron-rich events, including neutron-star mergers, makes other heavy nuclei.

Exam tips

Explain a selected evolutionary path and distinguish initial from remnant mass.

- Use p in arc-seconds for d in parsecs.
- Use $b = L/(4\pi d^2)$ and $L = 4\pi R^2 \sigma T^4$ with consistent units.
- Label HR axes with temperature increasing to the left.

Check yourself

Try these

An assigned parallax is 0.050 arc-seconds. With 1 pc = 3.26 ly, find distance in light years.

Use L = 3.0×10^{28} W, T = 4000 K and $\sigma = 5.67 \times 10^{-8}$ W/(m^2 K^4). Find black-body radius in metres.

Describe a Sun-like single-star path after the main sequence and identify the pressure supporting the remnant.