Gravitational fields

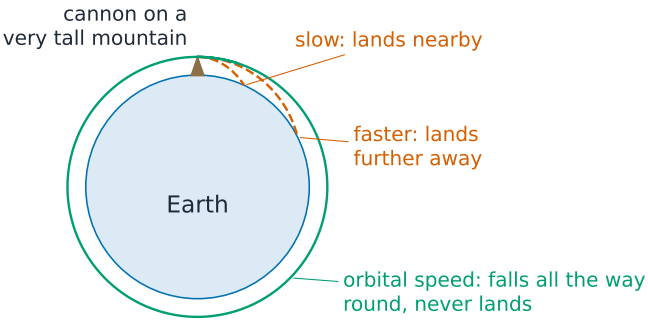
D.1 · SL

Why doesn't the space station fall down?

> An orbiting satellite is falling all the time, but the ground curves away beneath it.

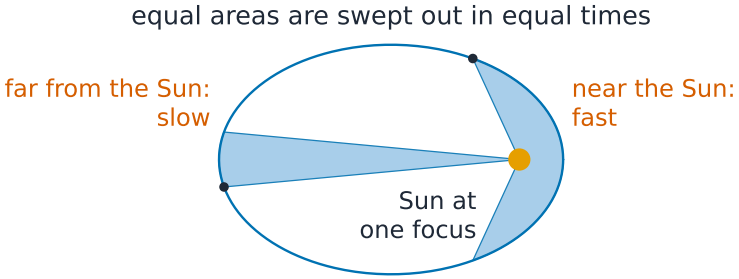
- At 400 km, gravity is still 89% as strong as on the ground.
- It moves sideways at 7.7 km every second.
- This unit: Kepler $\rightarrow$ Newton $\rightarrow$ fields $\rightarrow$ circular orbits.

Newton's cannon



- Faster shots land further round the Earth.
- At orbital speed the ball falls all the way round.

Kepler's laws



- **Ellipse** with the Sun at a **focus**; equal areas in equal times; $T^2 \propto a^3$ for semi-major axis a.
- Mars at 1.88 years orbits at $1.88^{2/3} = 1.52$ AU.

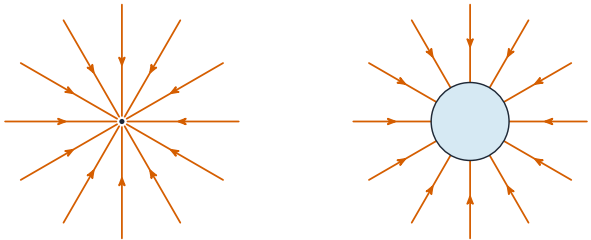
Newton's law of gravitation

- Measure r between centres; spheres act as point masses.
- Earth and Moon attract with 2.0×10^{20} N.

$$F = \frac{Gm_1 m_2}{r^2}$$

Double the distance and the force falls to one-quarter.

A sphere acts like a point mass



point mass

uniform sphere

Use $g = GM/r^2$ from the centre

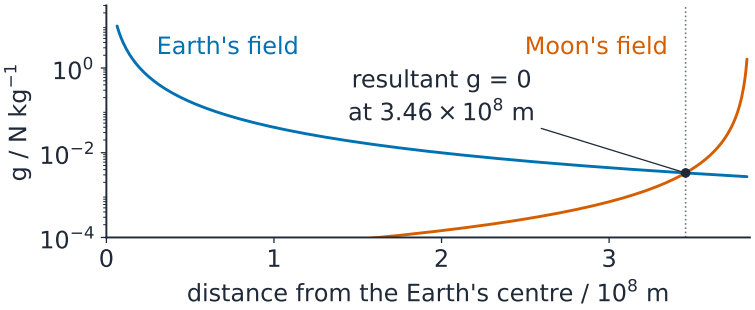
Gravitational field strength

- Force per unit mass on a small test mass.
- 9.81 N kg^{-1} at the surface; 8.69 N kg^{-1} at 400 km.

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

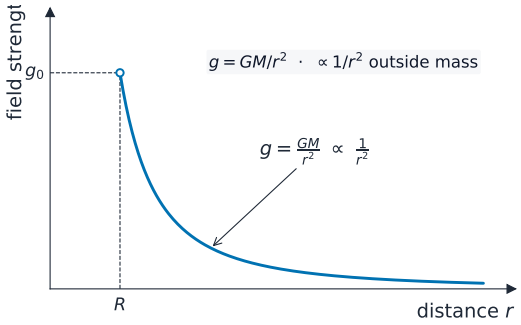
A vector, pointing towards the mass.

The Earth and the Moon together



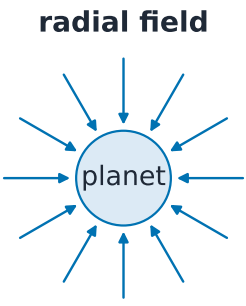
- Add the two fields as vectors along the line.
- They cancel at 3.46×10^8 m, nine-tenths of the way.

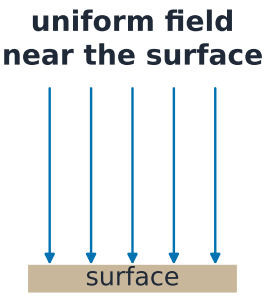
Field strength falls as $1/r^2$



g is largest at the surface

Field lines





- Closer lines mean a stronger field.
- Near the surface the field is almost uniform.

Circular orbits

- Gravity provides the **centripetal force**; the satellite's mass cancels.
- Circular equatorial prograde **geostationary orbit**: $T = 86164$ s and $r \approx 4.215 \times 10^7$ m.

$$T^2 = \frac{4\pi^2}{GM} r^3$$

Apparent weightlessness means falling together, not zero gravity.

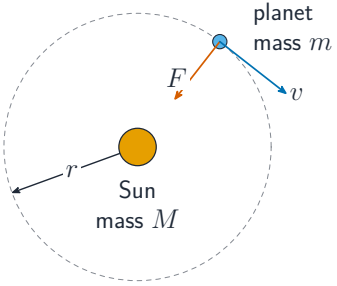
Check: a higher orbit

Your turn

A negligible-mass satellite moves between circular orbits around the same central mass. Orbital radius doubles. By what factor does its period change?

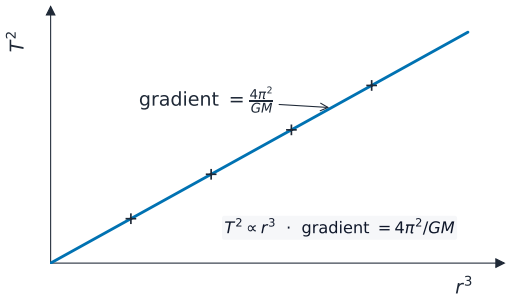
A. 2
B. 2.8
C. 4
D. 8

Gravity provides the centripetal force



A lower **orbit** is faster

Kepler's third law as a straight line



The gradient gives the central mass

Exam tips

> Measure r from the centre of the planet, not from its surface.

- Inverse square: double r, one-quarter F and g.
- Never add an outward force to an **orbit**.
- Compare periods using the same central mass and centre distances.

Check yourself

Try these

Use $G = 6.67 \times 10^{-11}$, $M_E = 5.97 \times 10^{24}$ kg and $R_E = 6.37 \times 10^{6}$ m. Find g at altitude 400 km.

With the same Earth constants and T = 86164 s, calculate geostationary radius and altitude.