Standing waves and resonance

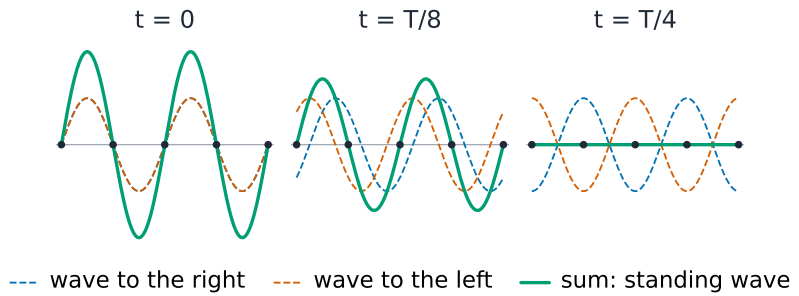
C.4 · SL

Why does a guitar string play only certain notes?

Only wavelengths that fit between the ends can form a standing wave.

- Touching the midpoint suppresses modes that move there.
- In the ideal string, the lowest surviving mode is the second harmonic.
- Uniform strings and pipes select modes through their end conditions.

Forming a standing wave



- Two identical waves travel in opposite directions and superpose.
- Usually one is the reflection of the other.

Nodes and antinodes

Amplitude

Zero at nodes.

Greatest at antinodes.

Phase

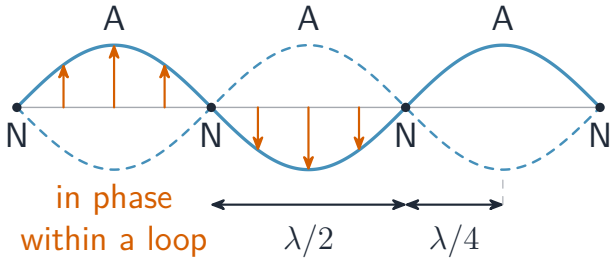
In phase within one loop.

Antiphase across a **node**.

Standing and travelling waves

	Travelling	**Standing**
net energy transfer	along propagation	zero in ideal pattern
amplitude	uniform in ideal plane wave	varies with position
shape	moves along	stays in place

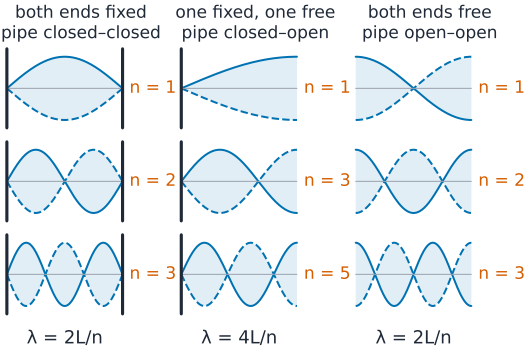
Where the nodes sit



neighbouring loops:
antiphase

Neighbouring loops move in antiphase

Harmonics on a string



- A fixed end is a **node**; a free end is an **antinode**.
- One fixed and one free end allows odd harmonics only.

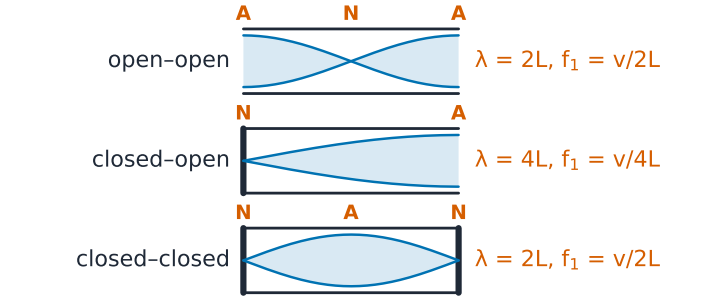
Harmonic frequencies

- Both ends fixed or both free; for one of each, $f_n = nv/4L$ with odd n.
- A 0.65 m guitar string at 110 Hz has $v = 143$ m s^{-1}.

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{nv}{2L}$$

The lowest mode is the **first harmonic**.

Standing waves in pipes



N = displacement node at a closed end; A = antinode at an open end

- A closed end is a displacement **node**; an open end is an **antinode**.
- At 0.34 m, open–open gives 500 Hz; closed–open gives 250 Hz.

Worked example: the speed of sound

1. **Known.** Invented 512 Hz fork, closed–open air column 0.166 m long; neglect end correction.
2. **Wavelength.** $\lambda = 4 \times 0.166 = 0.664$ m.
3. **Speed.** $v = 512 \times 0.664 = 340$ m s^{-1}.

Decision

The next resonance is at $3\lambda/4 = 0.498$ m

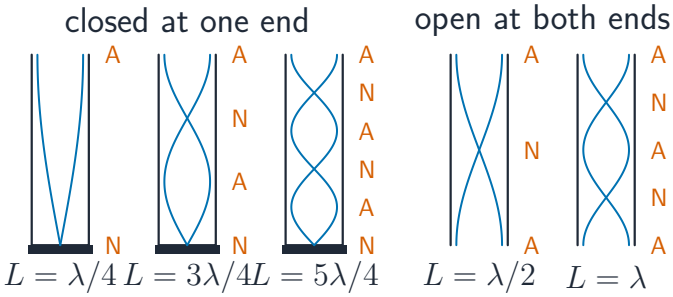
Check: a closed pipe

Your turn

An ideal uniform pipe closed at one end has first-harmonic frequency 200 Hz. Which other frequency is an allowed harmonic?

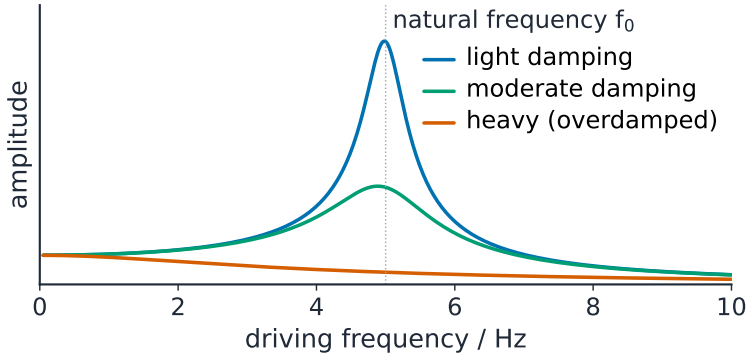
A. 300 Hz
B. 400 Hz
C. 600 Hz
D. 800 Hz

Which modes fit a pipe



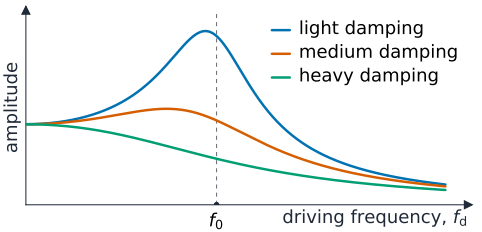
A closed end is a **node**; an open end is an **antinode**

Resonance



Same driving force: **light damping** gives a sharp peak. Overdamping removes the displacement resonance peak.

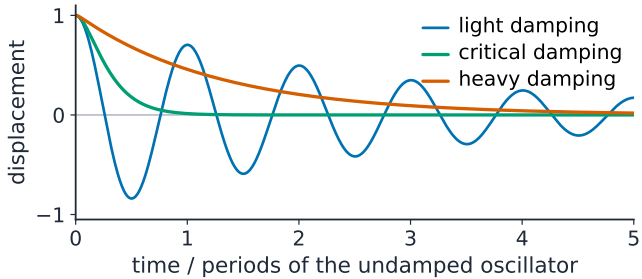
Damping and the resonance peak



resonance: max amplitude near f_0 · lighter damping
⇒ sharper peak

Heavy damping: a low, broad peak

Damping



- **Critical damping** gives the fastest non-oscillatory return after release from rest in this ideal model.
- More **damping** lowers and broadens a driven displacement peak; strong damping can remove it.

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Measuring oscillations and damping

1. **Release.** Use a small pendulum angle and keep its path clear.
2. **Time.** Count several cycles, returning to the same position in the same direction; divide the elapsed time by the cycle count.
3. **Repeat.** Repeat the timing and record the decrease in amplitude.

Decision

Ideal SHM has greatest speed at equilibrium and greatest acceleration magnitude at the extremes.

Uses and dangers of resonance

Useful

Instruments, radio tuning and quartz timing references.

MRI nuclear-spin excitation.

Harmful

Crowd–bridge feedback can amplify lateral motion.

Earthquake response depends on modes, **damping** and strength.

Exam tips

> Neighbouring nodes are half a wavelength apart.

- Draw the boundary first, then fit the pattern.
- Closed–open pipes have odd harmonics only.
- Say **first harmonic**, never fundamental.

Check yourself

Try these

Find the first three harmonics of a 0.80 m string with $v = 240$ m s^{-1}.

Find the two lowest harmonics of a 0.25 m pipe closed at one end.

For an ideal suspension released from rest, compare light, critical and heavy damping; give a limit of the model.