Standing waves and resonance

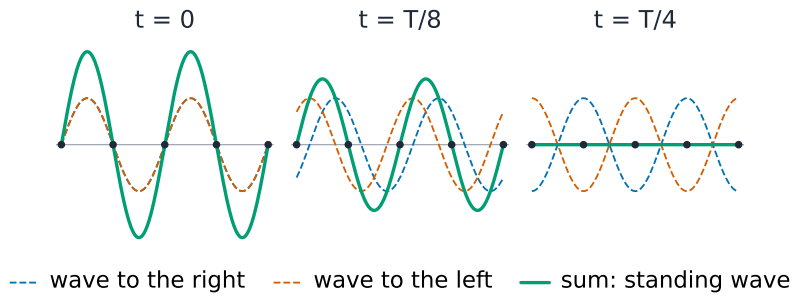
C.4 · SL

Why does a guitar string play only certain notes?

> Only wavelengths that fit between the ends can form a standing wave.

- Touching the midpoint suppresses modes that move there.
- In the ideal string, the lowest surviving mode is the second **harmonic**.
- Uniform strings and pipes select modes through their end conditions.

Forming a standing wave



- Two identical waves travel in opposite directions and superpose.
- Usually one is the reflection of the other.

Nodes and antinodes

Amplitude

Zero at nodes.

Greatest at antinodes.

Phase

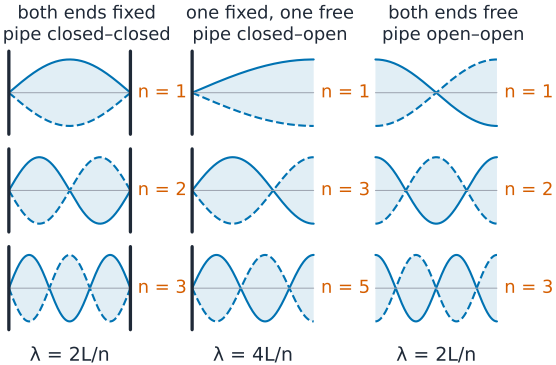
In phase within one loop.

Antiphase across a **node**.

Standing and travelling waves

	Travelling	**Standing**
net energy transfer	along propagation	zero in ideal pattern
amplitude	uniform in ideal plane wave	varies with position
shape	moves along	stays in place

Harmonics on a string



- A fixed end is a **node**; a free end is an **antinode**.
- One fixed and one free end allows odd harmonics only.

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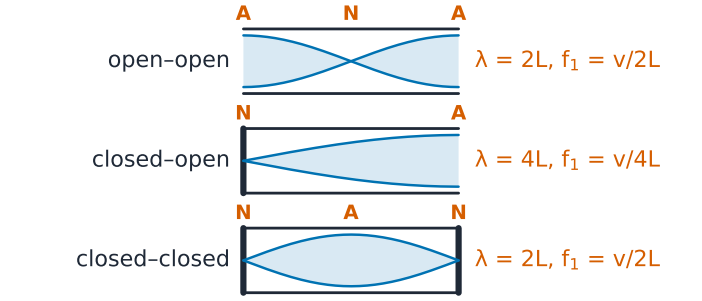
Harmonic frequencies

- Both ends fixed or both free; for one of each, $f_n = nv/4L$ with odd n.
- A 0.65 m guitar string at 110 Hz has $v = 143$ m s^{-1}.

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{nv}{2L}$$

The lowest mode is the **first harmonic**.

Standing waves in pipes



N = displacement node at a closed end; A = antinode at an open end

- A closed end is a displacement **node**; an open end is an **antinode**.
- At 0.34 m, open–open gives 500 Hz; closed–open gives 250 Hz.

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Worked example: the speed of sound

1. **Known.** Invented 512 Hz fork, closed–open air column 0.166 m long; neglect end correction.
2. **Wavelength.** $\lambda = 4 \times 0.166 = 0.664$ m.
3. **Speed.** $v = 512 \times 0.664 = 340$ m s^{-1}.

Decision

The next resonance is at $3\lambda/4 = 0.498$ m

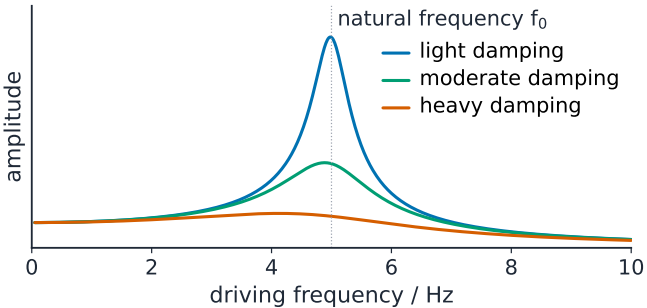
Check: a closed pipe

Your turn

An ideal uniform pipe closed at one end has first-harmonic frequency 200 Hz. Which other frequency is an allowed harmonic?

A. 300 Hz
B. 400 Hz
C. 600 Hz
D. 800 Hz

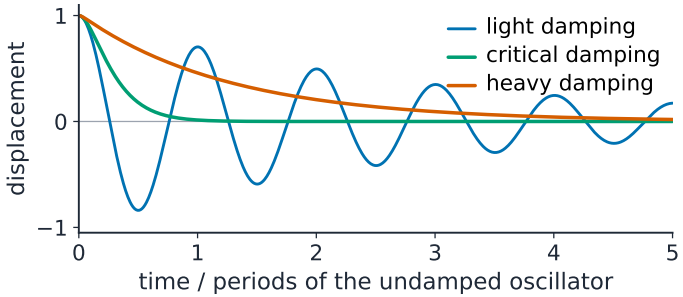
Resonance



- Enhanced response near a **natural frequency**; **light damping** gives a tall, narrow displacement peak.
- **Damping** can shift the displacement peak below the undamped natural frequency.

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Damping



- **Critical damping** gives the fastest non-oscillatory return after release from rest in this ideal model.
- More **damping** lowers and broadens a driven displacement peak; strong damping can remove it.

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Uses and dangers of resonance

Useful

Instruments, radio tuning and quartz timing references.

MRI nuclear-spin excitation.

Harmful

Crowd–bridge feedback can amplify lateral motion.

Earthquake response depends on modes, **damping** and strength.

Exam tips

> Neighbouring nodes are half a wavelength apart.

- Draw the boundary first, then fit the pattern.
- Closed–open pipes have odd harmonics only.
- Say **first harmonic**, never fundamental.

Check yourself

Try these

Find the first three harmonics of a 0.80 m string with $v = 240$ m s^{-1}.

Find the two lowest harmonics of a 0.25 m pipe closed at one end.

For an ideal suspension released from rest, compare light, critical and heavy damping; give a limit of the model.