Simple harmonic motion

C.1 · SL

Why does a tall building sway at its own rhythm?

SHM approximates small oscillations when the **restoring force** is proportional to displacement.

- Swings, guitar strings, car springs and atoms in crystals all oscillate.
- Each repeats its motion about a fixed **equilibrium position**.
- This unit: conditions $\rightarrow$ a $= -\omega^2$x $\rightarrow$ graphs $\rightarrow$ springs and pendulums $\rightarrow$ energy.

Conditions for SHM

SHM

Ideal linear spring: resultant $F_{\text{net}} = -kx$.

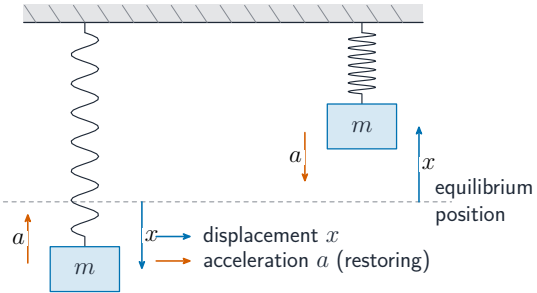
Pendulum at small angles.

Not SHM

A bouncing ball.

In flight its acceleration is g, independent of height.

Acceleration points back to equilibrium



$$a = -\omega^2 x$$

The defining equation

- Acceleration is proportional to displacement.
- The minus sign means acceleration always points back to equilibrium.

$$a = -\omega^2 x$$

$\omega = 12.6$ rad s^{-1} at $x = +0.020$ m gives $a = -3.2$ m s^{-2}.

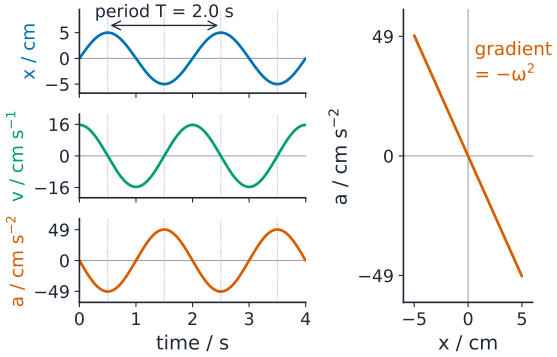
Period, frequency and angular frequency

- **Amplitude** x_0 is measured from equilibrium, not end to end.
- A 440 Hz string has $T = 2.3$ ms and $\omega = 2.8 \times 10^3$ rad s^{-1}.

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

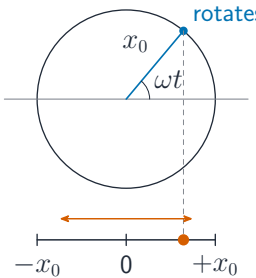
The period of SHM does not depend on the amplitude.

Graphs of SHM



Speed peaks at equilibrium; acceleration magnitude peaks at the ends.

SHM is the shadow of circular motion



One turn of 2π is one **oscillation**

The mass–spring system

- 0.40 kg on 40 N m^{-1} gives $T = 0.63$ s.
- A four times larger mass doubles the period.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Ideal constant-k model; x is from equilibrium. Gravity shifts that equilibrium.

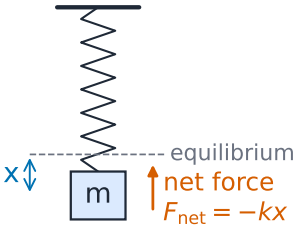
Worked example: a hanging spring

1. **Given.** Mass 0.40 kg; stiffness 40 N/m; g = 9.8 m/s^2; x = +0.025 m below equilibrium. Down is positive.
2. **Equilibrium.** $s_{\text{eq}} = mg/k = 0.098$ m; actual extension $s = 0.123$ m.
3. **Forces.** Weight +3.92 N; spring force −4.92 N; resultant −1.00 N.
4. **Acceleration.** $a = -1.00/0.40 = -2.5$ m/s^2, upward.

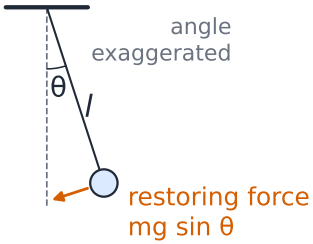
Decision

Check: $-kx = -40(0.025) = -1.00$ N uses x from equilibrium.

Springs and pendulums



$$T = 2\pi\sqrt{\frac{m}{k}}$$



$$T = 2\pi\sqrt{\frac{l}{g}}$$

Spring: net **restoring force**. Pendulum: tangential weight.

The simple pendulum

- A 1.0 m pendulum has $T = 2.0$ s on Earth and 5.0 s on the Moon.
- Plot T^2 against l; the gradient is $4\pi^2/g$.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Small-angle tangential motion; the bob also has radial acceleration.

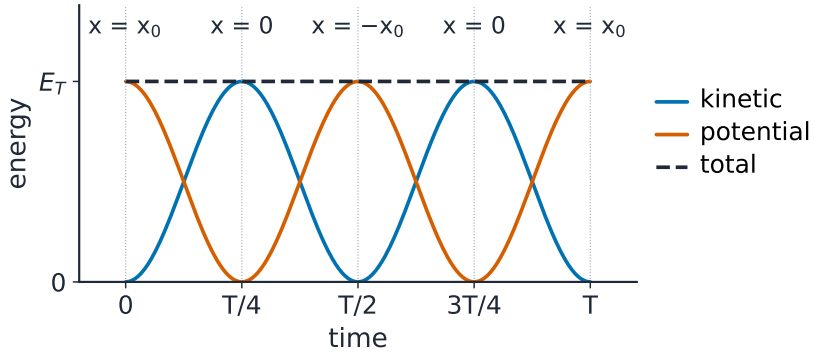
Check: a pendulum

Your turn

An ideal small-angle pendulum has period 2.0 s. Its length is multiplied by four at unchanged g. What is the new period?

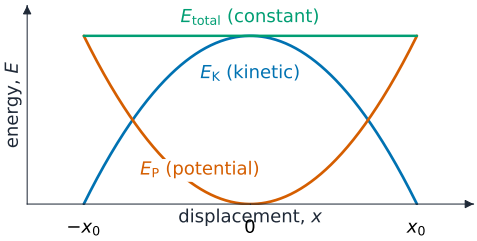
A. 1.0 s
B. 2.0 s
C. 4.0 s
D. 8.0 s

Energy in SHM



Undriven, no dissipation: kinetic + potential energy stays constant.

Energy in SHM against displacement



SHM: $E_k + E_p =$ const · max E_k at centre

Their sum is constant

Exam tips

> To show SHM, show that $a \propto -x$.

- Measure **amplitude** from equilibrium.
- Ideal linear spring or small-angle pendulum: period is independent of amplitude.
- Speed peaks at equilibrium; acceleration magnitude peaks at the ends.

Check yourself

Try these

Find the period of a 0.50 kg mass on a spring of 200 N m^{-1}.

Find the length of a pendulum with a period of 1.0 s.

Describe the energy changes of a pendulum during one oscillation.