8.2 Simple harmonic motion (full C.1, SL)

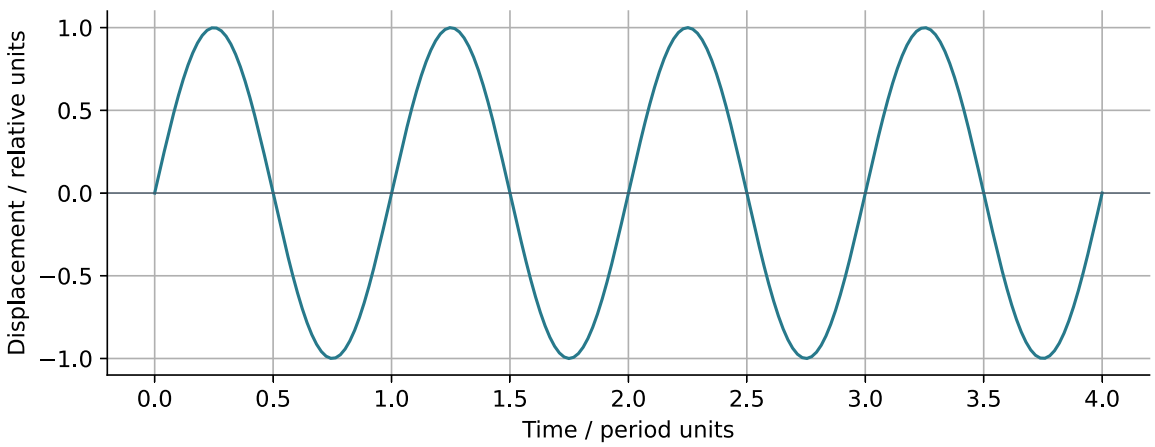
Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS simple harmonic motion • restoring force • amplitude • equilibrium position

Objective

Practise the full guide C.1 toolkit at standard level: the defining condition $a = -\omega^2 x$ and the meaning of its minus sign, $T = 1/f = 2\pi/\omega$, the mass–spring period $T = 2\pi\sqrt{m/k}$, the pendulum period $T = 2\pi\sqrt{l/g}$, and qualitative energy exchange through a cycle. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ The C.1 equation set (SL)

$$a = -\omega^2 x \qquad T = \frac{1}{f} = \frac{2\pi}{\omega} \qquad T = 2\pi\sqrt{\frac{m}{k}} \qquad T = 2\pi\sqrt{\frac{l}{g}}$$

Restoring condition: acceleration proportional to displacement and directed back toward equilibrium. Energy: total constant, kinetic greatest at equilibrium, potential greatest at the extremes.

■ **Worked example: rating a spring–mass oscillator**

A 0.50 kg mass hangs on a spring of stiffness 20 N/m and is pulled down and released. Period $T = 2\pi\sqrt{0.50/20} = 0.99$ s, so $f \approx 1.0$ Hz and $\omega = 6.3$ rad/s. With amplitude 0.050 m the maximum acceleration is $\omega^2 x_0 = 158 \times 0.050 = 7.9$ m/s^2, at the extremes where the mass momentarily stops. Two habits: check that the restoring condition holds before using these equations, and place maximum speed at equilibrium, never at the extremes.

2 Practice

2.1 State the defining equation of simple harmonic motion and explain the significance of its minus sign. [2]

2.2 An oscillator has frequency 2.5 Hz. Calculate its period and angular frequency. [2]

2.3 A 0.20 kg mass on a spring of stiffness 5.0 N/m oscillates. Calculate the period. [2]

2.4 Describe the energy changes over one complete cycle of a simple pendulum, from release at maximum displacement. [3]

3 Exam-style questions

3.1 A simple pendulum has length 2.5 m (g = 9.8 m/s^2). Calculate its period, and calculate the length that would double this period. [4]

3.2 A 0.30 kg mass on a spring oscillates with amplitude 0.040 m and period 0.25 s. Calculate the angular frequency and the maximum acceleration, and state where in the cycle the speed is greatest. [4]

3.3 A student times 20 oscillations of a mass–spring system as 8.8 s, with mass 0.40 kg. Calculate the period and the spring stiffness, and state one assumption the calculation makes about the system. [5]

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 $a = -\omega^2 x$; the minus sign shows acceleration is always directed opposite to displacement – toward equilibrium, the restoring condition.

2.2 $T = 1/f = 1/2.5 = 0.40$ s; $\omega = 2\pi f = 2\pi \times 2.5 = 16$ rad/s.

2.3 $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.20/5.0} = 2\pi(0.200) = 1.3$ s.

2.4 Starts all potential, zero kinetic; potential falls to zero as it passes equilibrium, where kinetic energy and speed are greatest; the exchange reverses on the far side and the cycle repeats, the total staying constant throughout.

3.1 $T = 2\pi\sqrt{2.5/9.8} = 2\pi(0.505) = 3.2$ s; doubling T needs four times the length since $T \propto \sqrt{l}$: $l = 4 \times 2.5 = 10$ m.

3.2 $\omega = 2\pi/T = 2\pi/0.25 = 25$ rad/s; $a_{max} = \omega^2 x_0 = 632 \times 0.040 = 25$ m/s^2; speed greatest as it passes equilibrium.

3.3 $T = 8.8/20 = 0.44$ s; $k = 4\pi^2 m/T^2 = 4\pi^2(0.40)/(0.44^2) = 82$ N/m; assumes ideal (massless) spring and no damping – amplitude constant.