3.2 Work energy and power (full A.3)

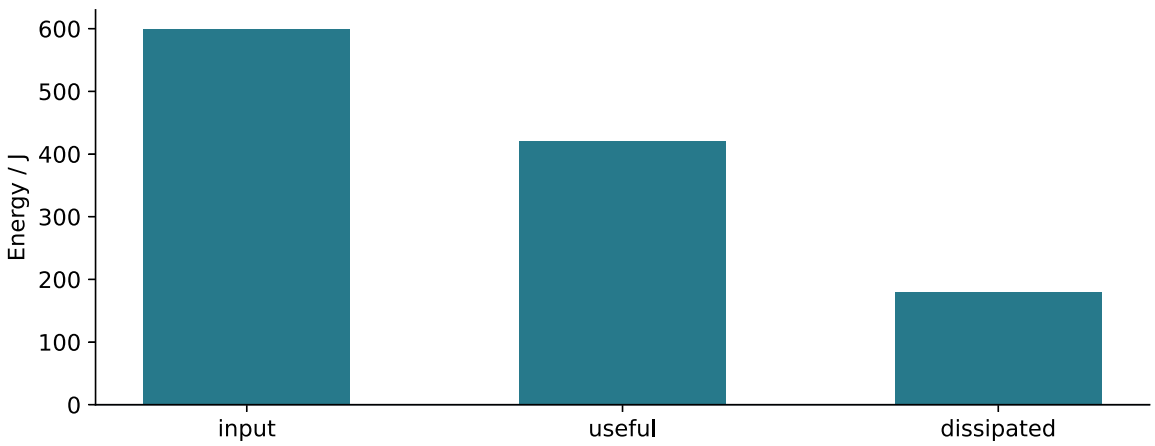
Name: _____ Class: _____ Date: _____ **Total: 27 marks**

KEY WORDS mechanical energy • energy density • non-conservative force

Objective

Practise the full guide A.3 toolkit beyond the focus lesson: work done by a force at an angle (W = Fs cos θ), the three mechanical-energy stores with the momentum form of kinetic energy, conservation of mechanical energy, power as P = ΔW/Δt = Fv, efficiency, and energy density of fuels. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ **The A.3 equation set**

$$W = Fs\cos\theta \qquad E_k = \tfrac{1}{2}mv^2 = \frac{p^2}{2m} \qquad \Delta E_p = mg\Delta h \qquad E_H = \tfrac{1}{2}k(\Delta x)^2$$

$$P = \frac{\Delta W}{\Delta t} = Fv \qquad \eta = \frac{E_{\text{out}}}{E_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{in}}}$$

Mechanical energy is conserved when no frictional or resistive force acts; any change equals the work done by non-conservative forces.

2 Practice

2.1 A 30 N force pulls a sled 8.0 m along a rope at 40° to the horizontal. Calculate the work done. [2]

2.2 A 4.0 kg ball moves at 6.0 m/s. Calculate its kinetic energy, and its momentum. [3]

2.3 A spring (k = 250 N/m) is compressed 0.12 m. Calculate the elastic energy stored. [2]

2.4 A crane lifts a 250 kg beam 12 m at a steady 0.50 m/s. Calculate the useful power ($g = 9.8\ m/s^2$). [3]

2.5 A motor takes 1500 W and delivers 1050 W. Calculate the efficiency as a percentage. [2]

3 Exam-style questions

3.1 A 0.20 kg ball is dropped from 15 m onto a spring (k = 400 N/m) and compresses it with no energy loss. Calculate the ball's speed at 5.0 m above the spring, the maximum compression, and state the energy assumption. [6]

3.2 A 900 kg car accelerates from 12 m/s to 28 m/s in 9.0 s against 500 N of resistance. Calculate the change in kinetic energy, the work done against resistance (take 180 m travelled), and the minimum average engine power. [6]

3.3 Fuel A releases 45 MJ/kg and Fuel B releases 50 MJ/kg but costs 20 % more per kilogram. Explain, using energy density, which fuel gives more energy per unit cost and what else a buyer should consider. [3]

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 $W = 30 \times 8.0 \times \cos 40° = 183.8 \approx 180$ J.

2.2 $E_k = \frac{1}{2} \times 4.0 \times 36 = 72$ J; $p = 4.0 \times 6.0 = 24$ kg · m/s. Check: $p^2/2m = 576/8 = 72$ J ✓.

2.3 $E_H = \frac{1}{2} \times 250 \times 0.0144 = 1.8$ J.

2.4 Force $= mg = 2450$ N; $P = Fv = 2450 \times 0.50 = 1225 \approx 1.2$ kW.

2.5 $\eta = 1050/1500 = 0.70 = 70\%$.

3.1 From 15 m to 5.0 m: $\Delta E_p = 0.20 \times 9.8 \times 10.0 = 19.6$ J lost $\rightarrow E_k = 19.6$ J; $v = \sqrt{2 \times 19.6/0.20} = 14$ m/s. At maximum compression all $mg(15 + x)$ goes into $\frac{1}{2}kx^2$: $1.96(15 + x) = 200x^2 \rightarrow x = 0.39$ m. Assumption: no air resistance – mechanical energy conserved.

3.2 $\Delta E_k = \frac{1}{2} \times 900 \times (784 - 144) = 2.88 \times 10^5$ J; work against resistance $= 500 \times 180 = 9.0 \times 10^4$ J; total energy $= 3.78 \times 10^5$ J over 9.0 s $\rightarrow P = 4.2 \times 10^4$ W $= 42$ kW minimum.

3.3 Per unit cost, B gives $50/1.2 \approx 41.7$ MJ against A's 45 MJ – A delivers more energy per unit cost; buyers must also weigh storage safety, availability and engine compatibility, not energy density alone.