Radioactive decay

E.3 · SL and AHL

Can random events make a clock?

> An invented sample has 52.5% of its assumed initial carbon-14.

- The fraction is invented, not a measurement of Ötzi.
- A real archaeological date also needs calibration and contamination checks.
- A population trend can be predictable while individual decays remain **random**.

Isotopes and the strong force

Isotopes

Same proton number; different neutron numbers.

Carbon-12 is stable.

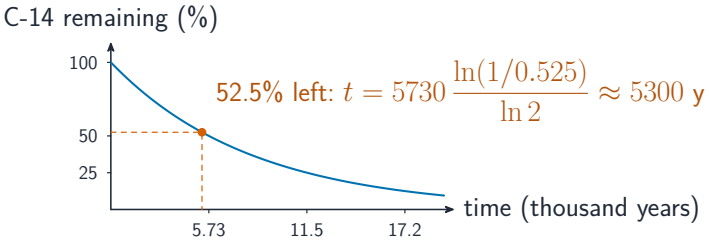
Carbon-14 is radioactive.

Strong nuclear force

Attraction at suitable nuclear separations.

Short-range binding; repulsion at sufficiently small separation.

Carbon dating

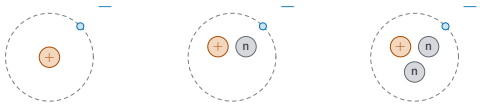


invented example; real dates need calibration

Read the age off the decay curve

The three isotopes of hydrogen

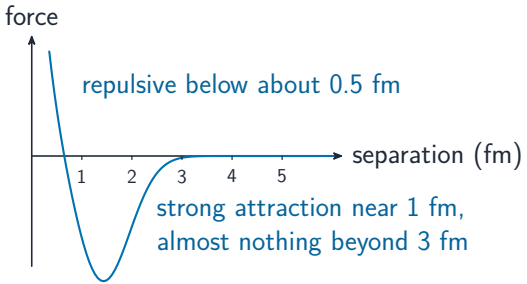
Hydrogen isotopes (not to scale)



hydrogen-1	hydrogen-2	hydrogen-3
$^{1}_{1}\mathrm{H}$	$^{2}_{1}\mathrm{H}$	$^{3}_{1}\mathrm{H}$
1 proton	1 proton	1 proton
0 neutrons	1 neutron	2 neutrons
1 electron	1 electron	1 electron

The same element: one proton, one electron

The strong force against separation



A short-range force holds nucleons together

Mass defect and binding energy

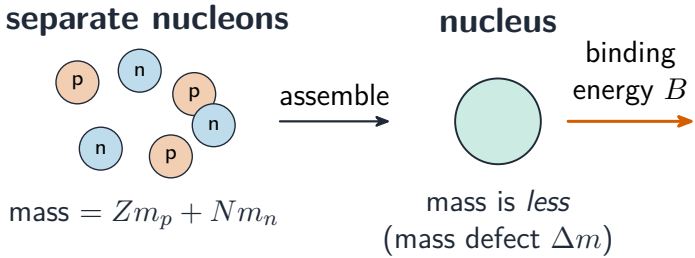
1. **Known.** Helium-4 nuclear mass 4.001506 u; proton 1.007276 u, neutron 1.008665 u; 1 u $c^2 = 931.5$ MeV.
2. **Separate nucleons.** $m_{\text{free}} = 2m_p + 2m_n = 2(1.007276 + 1.008665) = 4.031882$ u.
3. **Mass defect.** $\Delta m = m_{\text{free}} - m_{\text{nucleus}} = 4.031882 - 4.001506 = 0.030376$ u.
4. **Energy.** $E_B = \Delta m c^2 = 0.030376 \times 931.5 = 28.3$ MeV; divide by four for 7.07 MeV per nucleon.

Decision

Use a consistent nuclear-mass convention.

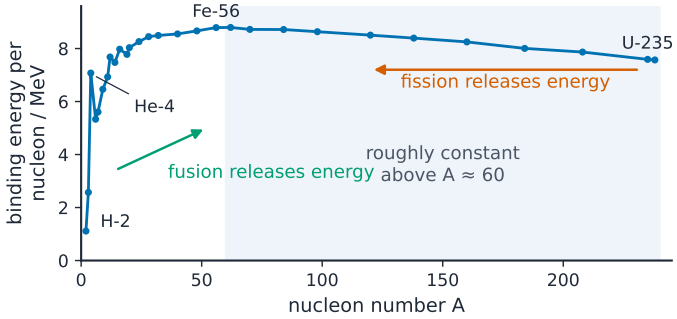
> **Binding energy** is the energy needed to separate the nucleons.

Where the missing mass goes



$B = \Delta m\, c^2$

Binding energy per nucleon



- The tightly bound region is near iron-56 and nickel-62.
- A favourable reaction increases total product **binding energy**; compare every product.

Random and spontaneous

> Half-life predicts an expected population fraction, not an individual deadline.

- Single nuclear decay times are unpredictable.
- The ordinary constant-rate model is approximately insensitive to temperature and chemistry.

Decay equations

- α: $A-4$, $Z-2$. β^-: $Z+1$. β^+: $Z-1$. γ: no change.
- Beta decay also emits a **neutrino** or **antineutrino**.

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e} + \overline{\nu}$$

Nucleon numbers and proton numbers must balance.

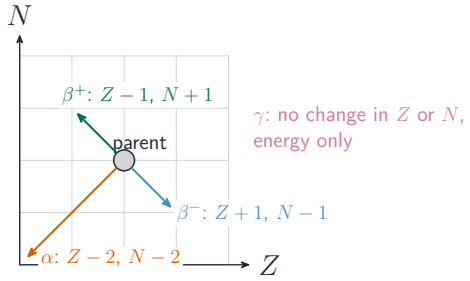
Check: a beta-minus decay

Your turn

Strontium-90 ($A = 90$, $Z = 38$) emits beta-minus. What are the daughter numbers?

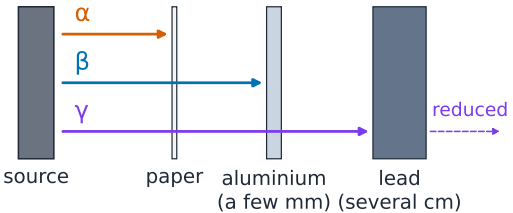
A. $Z = 37$, $A = 90$
B. $Z = 39$, $A = 90$
C. $Z = 36$, $A = 86$
D. $Z = 38$, $A = 89$

Decay moves on an N–Z chart



Gamma changes neither

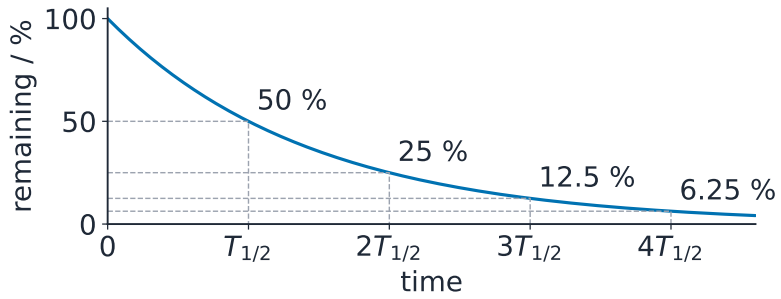
Penetration and ionisation



- Typical classroom energies: alpha is readily stopped; beta penetrates further.
- Gamma is progressively attenuated; ranges depend on energy and material.

Irradiation means radiation exposure; contamination means radioactive material on or inside an object. Under school rules, reduce time, increase distance and use appropriate shielding.

Half-life



- After n half-lives, $(1/2)^n$ remains.
- 800 Bq with $T_{1/2} = 6.0$ h becomes 50 Bq after 24 h.

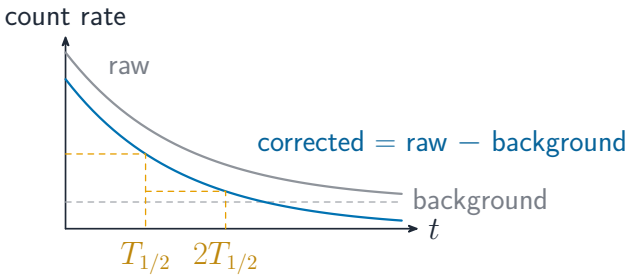
Background and measuring half-life

1. **Readings.** Invented detector rates 100 and 55 counts/min; constant background 10 counts/min; fixed detector geometry.
2. **Correct.** 90 and 45 counts per minute from the source.

Decision

Corrected ratio $= 45/90 = 1/2$: consistent with one half-life.

Subtract the background first



Read the half-life from the corrected curve

Choosing an isotope

Medical tracer (Tc-99m)

Gamma photons can escape to the detector.

A 6 h physical half-life provides a measurement window; dose also depends on **activity** and biology.

Paper thickness

Choose partial attenuation over the operating thickness range.

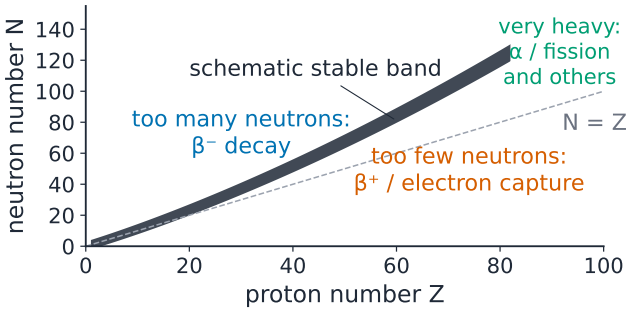
A long-lived source reduces drift; keep geometry fixed and calibrate.

Evidence for the strong force

Electric repulsion and gravity cannot explain why protons bind in a nucleus.

- With $k = 8.99 \times 10^9$ N m^2/C^2, $e = 1.60 \times 10^{-19}$ C and $r = 2.0$ fm, $ke^2/r^2 = 58$ N repulsion.
- Short-range nuclear attraction and saturation explain binding and broad density trends.

Neutrons and stability



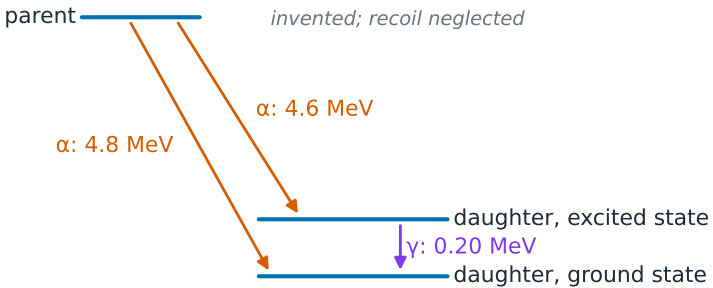
- Neutron-rich: beta-minus can move towards the band.
- Proton-rich: beta-plus or electron capture when allowed; heavy nuclei have several possible modes.

A flat curve above A = 60

Saturation gives approximately similar **binding energy per nucleon**.

- Short range: most binding comes from nearby nucleons.
- Surface, shell and electrical terms affect the detailed curve.

Nuclear energy levels



- Invented branches illustrate distinct daughter energy levels.
- The alpha-energy difference estimates the gamma gap only when daughter recoil is neglected.

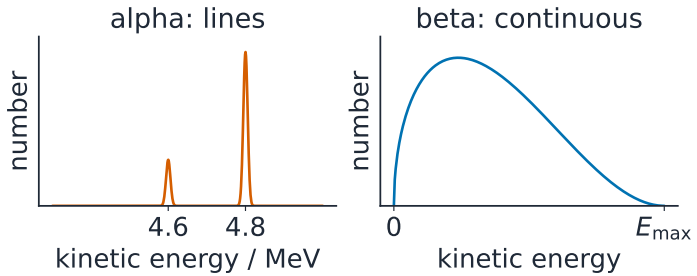
Two photons from one excited nucleus

1. **Given.** Invented levels 0.50, 0.20 and 0 MeV; follow $0.50 \to 0.20 \to 0$. Neglect daughter recoil.
2. **First photon.** $E_{\gamma 1} = E_i - E_f = 0.50 - 0.20 = 0.30$ MeV.
3. **Second photon.** $E_{\gamma 2} = 0.20 - 0 = 0.20$ MeV.
4. **Check.** $E_{\gamma 1} + E_{\gamma 2} = 0.50$ MeV. Proton and nucleon numbers are unchanged.

Decision

Each photon carries one level difference, which need not be the whole initial excitation energy.

The beta spectrum



- For one beta branch, energy is shared continuously.
- Include the (anti)**neutrino** and daughter recoil in energy and momentum conservation.

The decay law

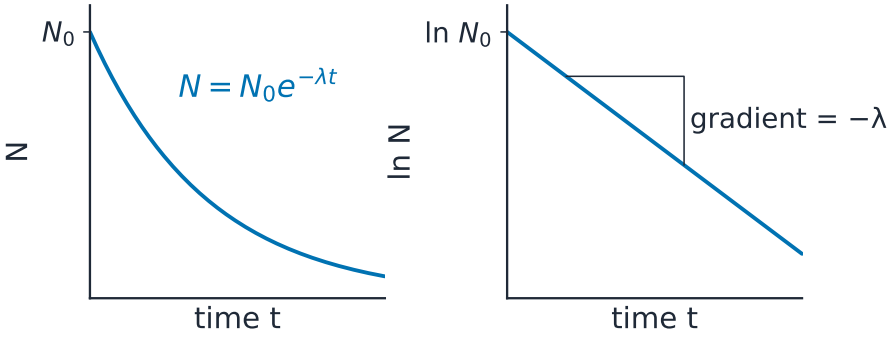
- Valid for any time, not only whole half-lives.
- A graph of $\ln A$ against t has gradient $-\lambda$.

$$N = N_0 e^{-\lambda t}, \quad A = \lambda N$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

Independent parent nuclei, constant λ, no replenishment; use matching time units.

Fit the decay constant



One corrected exponential gives a straight log-rate trend with gradient $-\lambda$.

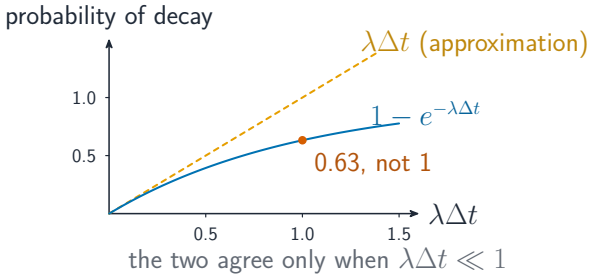
Check: iodine-131

Your turn

Iodine-131 has half-life 8.0 days. In the single-parent model, what fraction remains after 20 days?

A. 0.063
B. 0.18
C. 0.25
D. 0.40

When $\lambda \Delta t$ is not small



$\lambda \Delta t$ is only valid for small intervals

Exam tips

> Compare total energies and correct background before using a decay fraction.

- Balance A and Z and include the (anti)**neutrino** in beta decay.
- Subtract background before using count rates.
- Explain an **isotope** choice by its radiation and its half-life.

Check yourself

Try these

Use proton mass 1.007276 u, neutron mass 1.008665 u and carbon-12 nuclear mass 11.996706 u. With 1 u $c^2 = 931.5$ MeV, find binding energy per nucleon.

A 640 Bq source has half-life 2.0 h. Find activity after 8.0 h.

AHL: for an 8.0 day half-life and 86400 s/day, find decay constant and activity of 2.0×10^{15} parent nuclei.