Gravitational fields

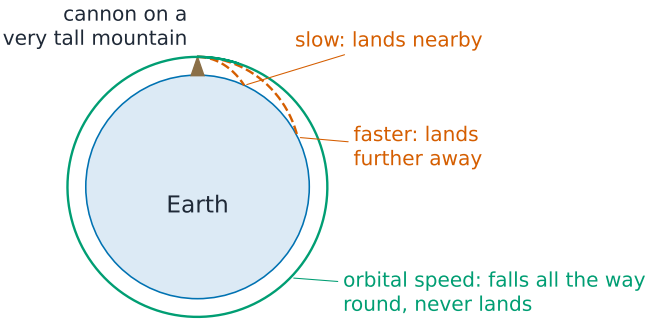
D.1 · SL and AHL

Why doesn't the space station fall down?

> An orbiting satellite is falling all the time, but the ground curves away beneath it.

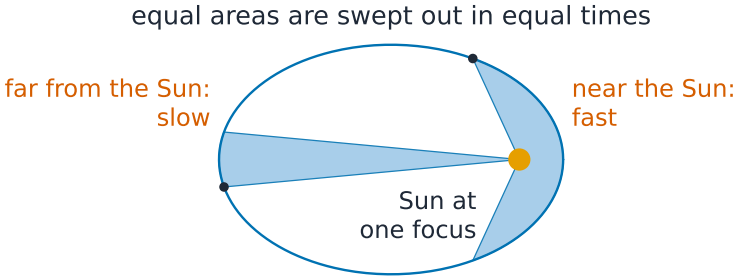
- At 400 km, gravity is still 89% as strong as on the ground.
- It moves sideways at 7.7 km every second.
- This unit: Kepler $\to$ Newton $\to$ fields $\to$ orbits $\to$ energy.

Newton's cannon



- Faster shots land further round the Earth.
- At **orbital speed** the ball falls all the way round.

Kepler's laws



- **Ellipse** with the Sun at a **focus**; equal areas in equal times; $T^2 \propto a^3$ for semi-major axis a.
- Mars at 1.88 years orbits at $1.88^{2/3} = 1.52$ AU.

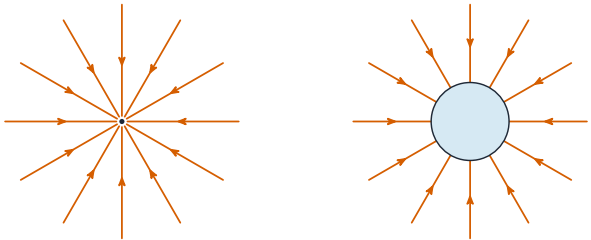
Newton's law of gravitation

- Measure r between centres; spheres act as point masses.
- Earth and Moon attract with 2.0×10^{20} N.

$$F = \frac{Gm_1 m_2}{r^2}$$

Double the distance and the force falls to one-quarter.

A sphere acts like a point mass



point mass

uniform sphere

Use $g = GM/r^2$ from the centre

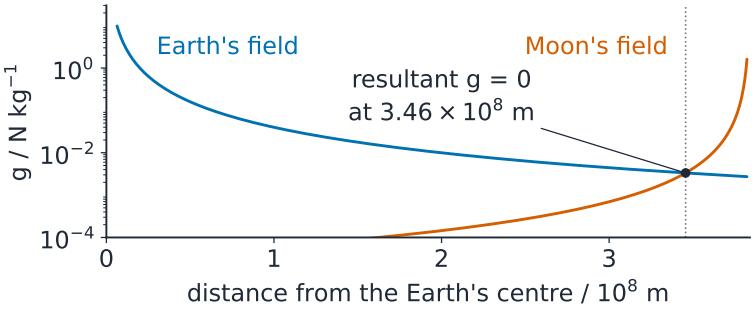
Gravitational field strength

- Force per unit mass on a small test mass.
- 9.81 N kg^{-1} at the surface; 8.69 N kg^{-1} at 400 km.

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

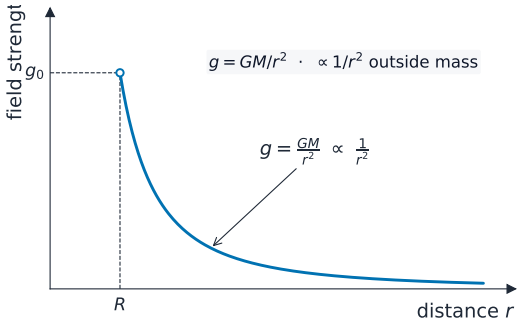
A vector, pointing towards the mass.

The Earth and the Moon together



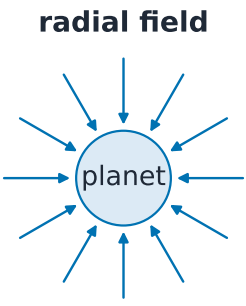
- Add the two fields as vectors along the line.
- They cancel at 3.46×10^8 m, nine-tenths of the way.

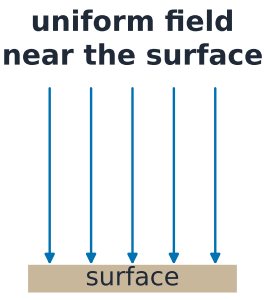
Field strength falls as $1/r^2$



g is largest at the surface

Field lines





- Closer lines mean a stronger field.
- Near the surface the field is almost uniform.

Circular orbits

- Gravity provides the **centripetal force**; the satellite's mass cancels.
- Circular equatorial prograde **geostationary orbit**: $T = 86164$ s and $r \approx 4.215 \times 10^7$ m.

$$T^2 = \frac{4\pi^2}{GM} r^3$$

Apparent weightlessness means falling together, not zero gravity.

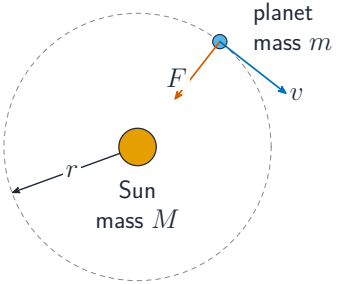
Check: a higher orbit

Your turn

A negligible-mass satellite moves between circular orbits around the same central mass. Orbital radius doubles. By what factor does its period change?

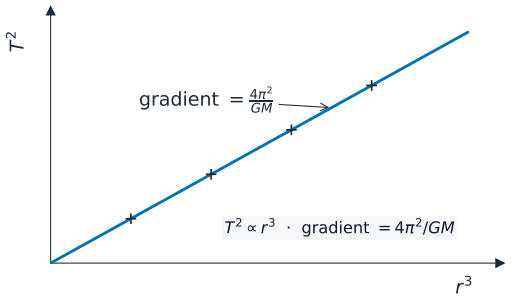
A. 2
B. 2.8
C. 4
D. 8

Gravity provides the centripetal force



A lower orbit is faster

Kepler's third law as a straight line



The gradient gives the central mass

Gravitational potential energy

- Zero at infinity, negative everywhere else.
- 1000 kg on the Earth's surface has -6.25×10^{10} J.

$$E_p = -\frac{Gm_1m_2}{r}$$

Use *mgh* only for small heights.

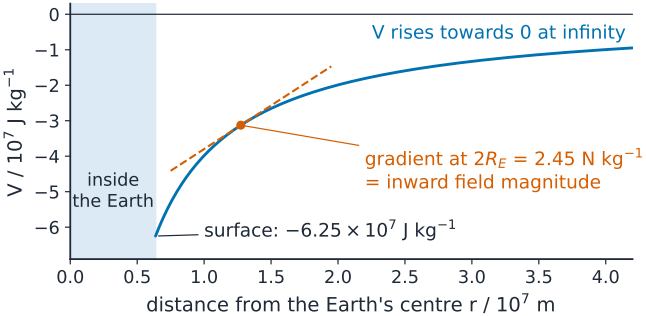
Gravitational potential

- External work per unit mass for a slow move from infinity.
- Lifting 1000 kg to 400 km takes 3.7×10^9 J.

$$V_g = -\frac{GM}{r} \qquad W = m\,\Delta V_g$$

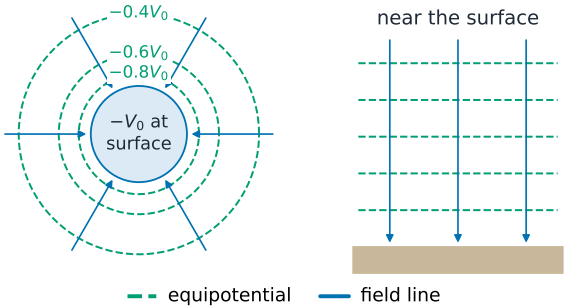
Here W is external work at unchanged kinetic energy; gravitational work has the opposite sign.

Potential gradient



- With outward r positive, $g_r = -dV_g/dr$; field magnitude is the positive slope here.
- The field points towards lower potential.

Equipotential surfaces



- Field lines cross equipotentials at right angles.
- Gravity does no work along an equipotential.

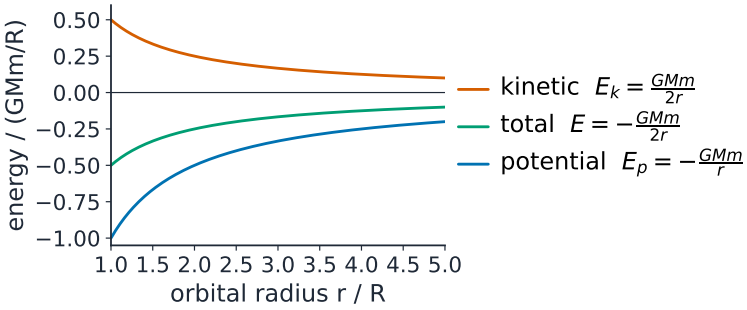
Orbital speed and escape speed

- The space station moves at 7.67 km s^{-1}.
- Escape from the Earth's surface needs 11.2 km s^{-1}.

$$v_{\text{orbital}} = \sqrt{\frac{GM}{r}} \qquad v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

Ideal energy threshold: negligible test mass, no atmosphere, no collision or other gravitating body.

Energy in orbit



- $E_k = -E_{\text{total}}$ and $E_p = 2E_{\text{total}}$.
- A higher orbit has more total energy but is slower.

Worked example: changing orbit

1. **Known.** Circular radii 7.0×10^6 m and 1.4×10^7 m; $GMm = 3.98 \times 10^{17}$ J m; $E = -GMm/(2r)$.
2. **Before.** $E_i = -\frac{3.98\times10^{17}}{2\times7.0\times10^{6}} = -2.84 \times 10^{10}$ J.
3. **After.** $E_f = -\frac{3.98\times10^{17}}{2\times1.4\times10^{7}} = -1.42 \times 10^{10}$ J.

Decision

Mechanical energy rises by 1.42×10^{10} J; this is not total fuel energy.

Atmospheric drag

Decreases

Across slowly decaying near-circular orbits: radius and total energy.

Potential energy falls by twice the net energy loss.

Increases

Later lower-orbit speed and kinetic energy.

Drag itself initially opposes motion; gravity accelerates descent.

Exam tips

> Measure r from the centre of the planet, not from its surface.

- Inverse square: double r, one-quarter F and g.
- Never add an outward force to an orbit.
- Potentials are negative and zero at infinity.

Check yourself

Try these

Use $G = 6.67 \times 10^{-11}$, $M_E = 5.97 \times 10^{24}$ kg and $R_E = 6.37 \times 10^{6}$ m. Find g at altitude 400 km.

With the same Earth constants and T = 86164 s, calculate geostationary radius and altitude.

HL. Find escape speed from a spherical Moon of mass 7.35×10^{22} kg and radius 1.74×10^{6} m, using the stated G.