Gas laws

B.3 · SL and HL

Why does a tyre feel harder on a hot day?

Pressure, volume and temperature are linked by molecules hitting the walls.

- No air is added, yet the pressure rises.
- A crisp bag swells on a mountain; a closed bottle is crushed coming down.
- This unit: pressure $\rightarrow$ gas laws $\rightarrow$ $PV = nRT$ $\rightarrow$ **kinetic theory** $\rightarrow$ real gases.

Pressure and amount of substance

- A 600 N person on 0.030 m^2 of shoe exerts 2.0×10^4 Pa.
- 32 g of O_2 is 1.0 mol, 6.02×10^{23} molecules.

$$P = \frac{F}{A} \qquad n = \frac{N}{N_A}$$

Atmospheric pressure is about 1.0×10^5 Pa.

The empirical gas laws

Law	Constant	Relationship
Boyle's law	T	PV constant
Charles's law	P	V/T constant
pressure law	V	P/T constant

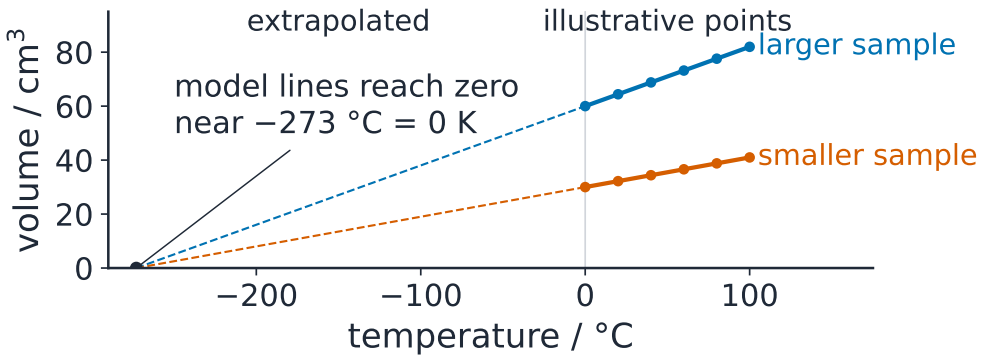
Use the evidence

Fixed amount; ideal-gas approximation. Use absolute pressure and kelvin.

Investigating the pressure law

1. **Set up.** Use teacher-approved constant-volume apparatus in a water bath. The teacher sets temperature and pressure limits.
2. **Measure.** At each bath temperature, wait for thermal equilibrium; record the gas's absolute pressure and temperature in kelvin.
3. **Test.** Plot pressure against kelvin temperature; repeat readings to check consistency. Never heat an improvised sealed vessel.

Absolute zero



Model lines extrapolate to zero; a **real gas** departs from this model.

Combining the laws

- 50 cm^3, 100 kPa, 300 K $\to$ 20 cm^3 at 360 K.
- $P_2 = 100 \times \frac{50}{20} \times \frac{360}{300} = 300$ kPa.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Any units for P and V if both sides match; T always in kelvin.

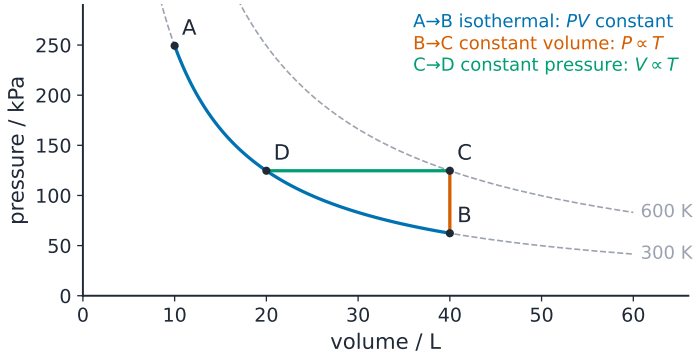
Worked example: a gauge reading

1. **Given.** Rigid sealed vessel: 150 kPa gauge at 300 K; outside 100 kPa; final 360 K.
2. **Absolute.** $P_1 = 150 + 100 = 250$ kPa.
3. **Scale.** $P_2 = 250(360/300) = 300$ kPa absolute.
4. **Gauge.** $300 - 100 = 200$ kPa.

Decision

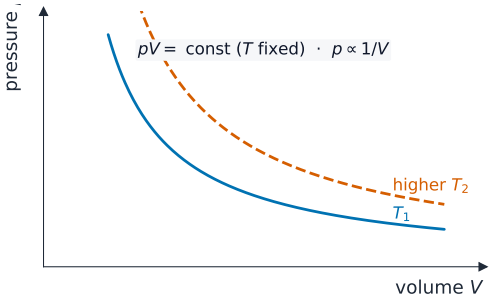
Scaling the gauge value directly gives the wrong 180 kPa.

Pressure–volume diagrams



Follow the path: constant volume is vertical; constant pressure horizontal.

Boyle's law curves



Plot p against $1/V$ for a straight line

The ideal gas law

- 1.0 m^3 at 100 kPa and 300 K holds 40 mol, 2.4×10^{25} molecules.
- One mole at 273 K and 101 kPa occupies 22.5 L.

$$PV = nRT = Nk_BT$$

$R = N_A k_B = 8.31$ J K^{-1} mol^{-1}; V in m^3.

Check: an oxygen cylinder

Your turn

Use an ideal-gas estimate for oxygen in a 0.020 m^3 cylinder at absolute pressure 1.5×10^7 Pa and 290 K. Molar mass is 32 g mol^{-1} and $R = 8.31$ J mol^{-1} K^{-1}. What mass is in the cylinder?

A. 0.12 kg

B. 1.2 kg

C. 4.0 kg

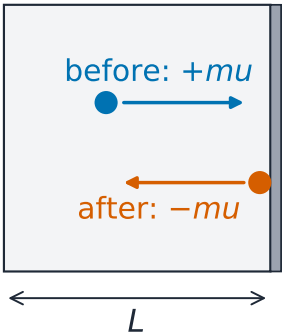
D. 124 kg

The kinetic model

> An ideal gas is a model that explains the gas laws through molecular motion.

- Very many identical molecules in random motion.
- Negligible molecular volume; no forces except in elastic collisions.

Pressure from collisions



each collision: $\Delta p = -2mu$

time between hits on this wall: $2L/u$

force on the wall $= \dfrac{2mu}{2L/u} = \dfrac{mu^2}{L}$

add all N molecules, average over 3 directions:

$P = \dfrac{1}{3}\rho\overline{v^2}$

Normal impulse on the molecule is negative; the wall receives the opposite impulse.

Explaining the gas laws

Halve the volume

At fixed temperature, number density doubles; momentum transfer per area per second doubles.

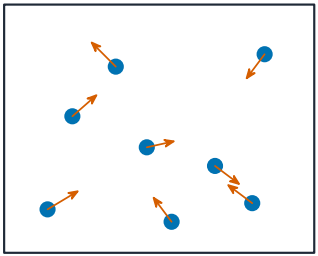
Pressure doubles (Boyle).

Heat at constant volume

Faster molecules hit more often and harder.

Pressure rises (pressure law).

The kinetic-theory assumptions

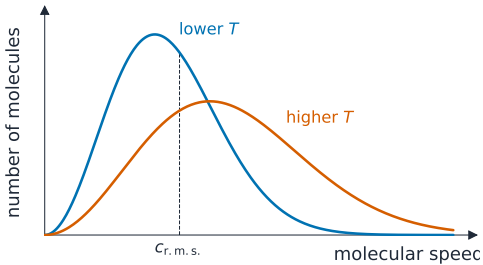


1. many identical molecules, in continuous **random motion**
2. their own volume is **tiny** next to the container
3. straight lines between hits - intermolecular forces **ignored**
4. collisions are **elastic**: no kinetic energy is lost

poor at very high pressure (volume matters) or very low temperature (forces matter)

> An ideal gas obeys $pV = nRT$ exactly

Speeds spread out when the gas is hotter



higher T: peak → higher speeds, flatter curve

The area under each curve is the same number of molecules

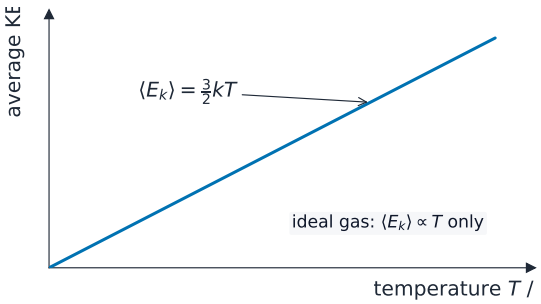
Internal energy of an ideal gas

- Neglect intermolecular interaction energy; polyatomic internal modes can remain.
- 2.0 mol of helium at 400 K has $U = 1.0 \times 10^4$ J.

$$U = \frac{3}{2} N k_B T = \frac{3}{2} n R T$$

Classical **monatomic** ideal gas; fixed amount and composition.

Average kinetic energy is proportional to T



Double the kelvin temperature, double the average KE

Real gases

A **real gas** is nearly ideal at low pressure, low density and high temperature.

- At high pressure, molecular volume matters.
- Attractions matter near condensation; phase behaviour also depends on pressure.

Exam tips

> Decide which quantity is constant before choosing a gas law.

- Kelvin for T; m^3 for V in $PV = nRT$.
- n goes with R; N goes with k_B.
- Explain pressure with change of momentum and rate of collisions.

Check yourself

Try these

A rigid, sealed ideal-gas sample at 20°C and 150 kPa absolute is heated to 120°C. Find the pressure.

Find the volume of 0.25 mol of gas at 300 K and 120 kPa.

Find v_{rms} for a gas of density 0.18 kg m^{-3} at 100 kPa.