Galilean and special relativity

A.5 · AHL

Who is moving?

> Motion is only defined relative to a **reference frame**.

- A train pulls away beside yours, and for a moment you think you are moving.
- No experiment inside a smoothly moving train can tell.
- The whole unit is additional higher level.

Galilean relativity

Quantity	**Galilean transformation**
position	$x' = x - vt$
time	$t' = t$
velocity	$u' = u - v$

The two postulates

Postulate 1

The laws of physics are the same in all inertial frames.

Postulate 2

Light in a vacuum travels at c for all inertial observers.

The Lorentz transformations

- $\gamma = 1.25$ at $0.60c$, 1.67 at $0.80c$, 7.1 at $0.99c$.
- At everyday speeds $\gamma \approx 1$ and they become Galilean.

$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

The same forms apply to intervals Δx and Δt.

Worked example: transform an event

1. **Given.** Coincident origins at $t = t' = 0$; S′ moves at $+0.60c$. **Event**: $x = 3.0 \times 10^{8}$ m, $t = 2.0$ s; $c = 3.0 \times 10^{8}$ m s^{-1}.
2. **Position.** $x' = 1.25(3.0 \times 10^{8} - 3.6 \times 10^{8}) = -7.5 \times 10^{7}$ m.
3. **Time.** $t' = 1.25(2.0 - 0.60) = 1.75$ s.
4. **Check.** Both frames give $(\Delta s)^2 = 2.7 \times 10^{17}$ m^2.

Decision

$x' = -7.5 \times 10^{7}$ m, $t' = 1.75$ s

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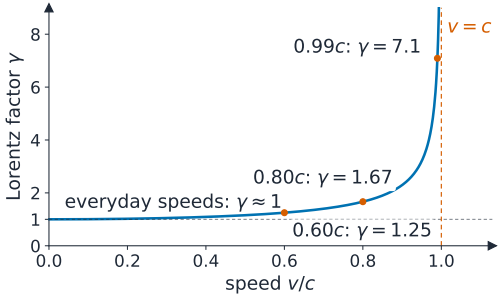
Relativistic velocity addition

- Two craft approach, each at $0.80c$ relative to the Earth.
- In A's frame, B moves at $-1.60c/1.64 = -0.98c$, not $1.6c$.

$$u' = \frac{u - v}{1 - uv/c^2}$$

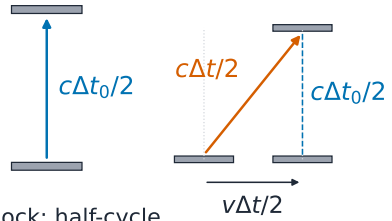
If $u = c$ then $u' = c$ in every frame.

How γ grows with speed



γ rises without limit as $v \to c$

Time dilation



rest clock: half-cycle
full tick Δt_0

moving at v: each leg is half a tick; light travels at c, giving
$(c\Delta t)^2 = (c\Delta t_0)^2 + (v\Delta t)^2$
$\Rightarrow \Delta t = \gamma \Delta t_0$

- Proper time Δt_0 is measured where both events happen at one place.
- At $0.60c$: 4.0 μs on board is 5.0 μs for Earth observers.

Length contraction

- **Proper length** L_0 is measured where the object is at rest.
- Only lengths along the motion contract.

$$\Delta t = \gamma \Delta t_0 \qquad L = \frac{L_0}{\gamma}$$

A 100 m craft at $0.80c$ is 60 m long in the Earth's frame.

Check: which is proper?

Your turn

A rod of proper length 2.0 m moves along its length at $0.60c$. What length do you measure?

A. 1.2 m

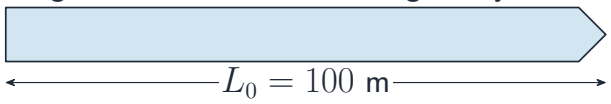
B. 1.6 m

C. 2.0 m

D. 2.5 m

A spacecraft at 0.80c

height and width do not change: only the length along the motion



Only the length along the motion shrinks

The invariant interval

- 900 m and 5.0 μs apart in S give $\Delta s = 1200$ m.
- In the frame where both are at one place, $c\Delta t' = 1200$ m, so 4.0 μs.

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

Every inertial observer agrees on Δs.

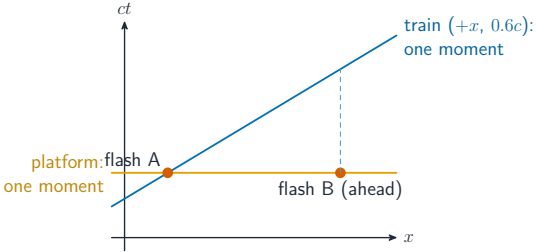
The relativity of simultaneity

- Lamps 3.0 km apart flash together in the platform frame.
- Train coordinates at $0.60c$ assign the forward lamp an emission time 7.5 μs earlier.

$$\Delta t' = -\gamma \frac{v \Delta x}{c^2}$$

Compare assigned **event** times, allowing for light travel; reception times at one passenger are different.

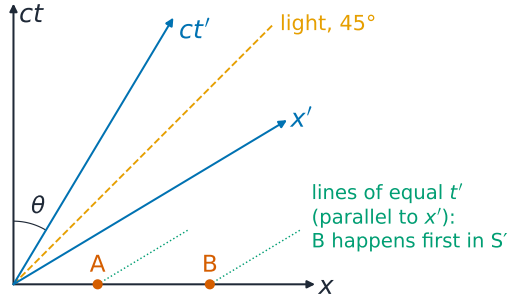
Simultaneous in one frame only



the train's lines of equal time tilt: in its frame flash B happened *first*

In the train frame, the flash ahead happens first

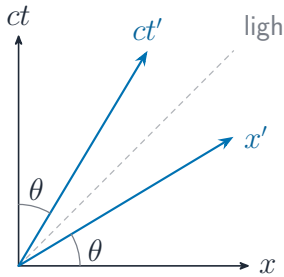
Space–time diagrams



- Light is at 45°; a **world line** at θ from ct has $\tan\theta = v/c$.
- Lines of equal t' are parallel to x'.

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Axes of a moving frame

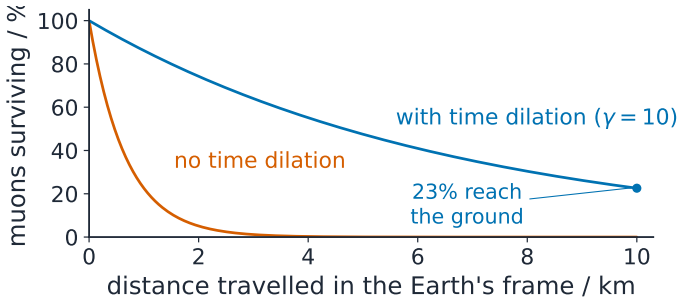


$\tan\theta = v/c = 0.6$

both axes tilt by the same angle, symmetric about the light line; faster frame $\Rightarrow$ closer to $45°$

The light line bisects both pairs of axes

Muons reach the ground



- Model: constant $0.995c$ over 10 km; at-rest half-life 1.56 μs; $\gamma \approx 10$.
- Illustrative survival model: about 3×10^{-7} without dilation; about 23% with dilation.

Two frames, one result

Earth frame

Half-life dilated to 15.6 μs.

Trip of 33.5 μs.

Muon frame

Half-life 1.56 μs.

Atmosphere contracted to 1.0 km, 3.35 μs.

Exam tips

Decide which frame measures the proper quantity before you substitute.

- Different place and different time? Use the Lorentz transformations.
- Check transformed coordinates with the interval.
- Label the time axis ct; $\tan\theta = v/c$ from the ct axis.

Check yourself

Try these

Transform $x = 600$ m, $ct = 900$ m into a frame moving at $0.80c$.

Find the speed of one proton relative to another when each moves at $0.90c$ in opposite directions.

Find the fraction of muons that survive 2.0 km at $0.98c$ in both frames. Use an at-rest half-life of 1.56 μs and $c = 3.0 \times 10^{8}$ m s^{-1}.