Work, energy and power

A.3 · SL and HL

Where does the energy go?

> Energy is conserved when all stores and transfers are included.

- A roller coaster climbs, races down, slows and is braked to a stop.
- At every moment the energy is in some store.
- This unit: work $\rightarrow$ stores $\rightarrow$ conservation $\rightarrow$ power $\rightarrow$ **efficiency** $\rightarrow$ fuels.

Work is energy transferred by a force

- Only the component of the force along the displacement does work.
- At 90° no work is done; above 90° the work is negative.

$$W = Fs\cos\theta$$

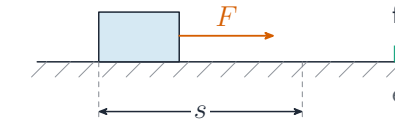
Suitcase: 50 N at 30° for 20 m gives $W = 870$ J.

The work–energy principle

> Work done by the resultant force equals the change in kinetic energy.

- Use the signed area of the force component parallel to displacement.
- Car $10 \to 20$ m s^{-1} (1200 kg): $\Delta E_k = 1.8 \times 10^5$ J.

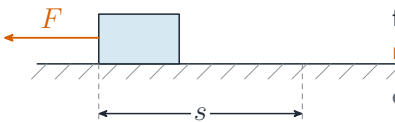
Positive and negative work



force along motion

positive work: $W = +Fs$

energy given **to** the object



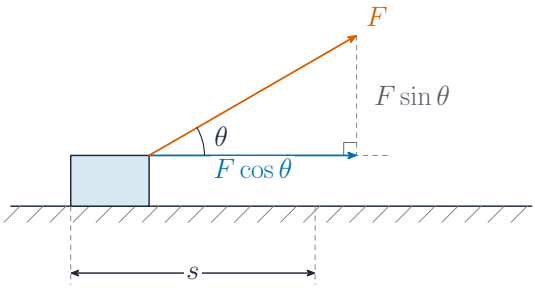
force opposite to motion

negative work: $W = -Fs$

energy taken **from** the object

Negative work takes energy away from the object

Work done by a force at an angle



$$W = Fs\cos\theta$$

Three mechanical energy stores

Store	Equation
kinetic	$E_k = \frac{1}{2}mv^2 = p^2/2m$
gravitational potential	$\Delta E_p = mg\Delta h$
linear elastic spring	$E_H = \frac{1}{2}kx^2$ from natural length

Use the evidence

Use constant g near Earth; spring formula requires the linear elastic range and zero energy at natural length.

Check: a spring launcher

Your turn

A linear elastic spring ($k = 400$ N/m) compressed by 0.10 m transfers all its 2.0 J to a 0.050 kg ball initially at rest. Find the launch speed.

A. 2.0 m s^{-1}

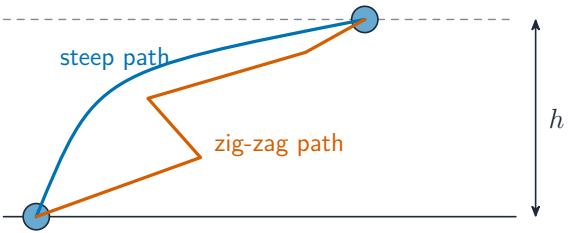
B. 6.3 m s^{-1}

C. 8.9 m s^{-1}

D. 80 m s^{-1}

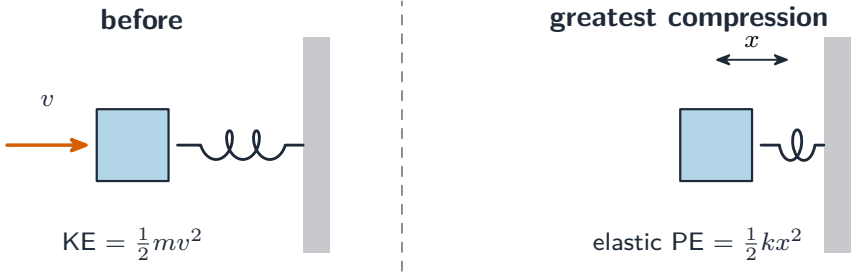
Gravitational energy depends only on height

same $\Delta E_P = mgh$ – only the height h matters



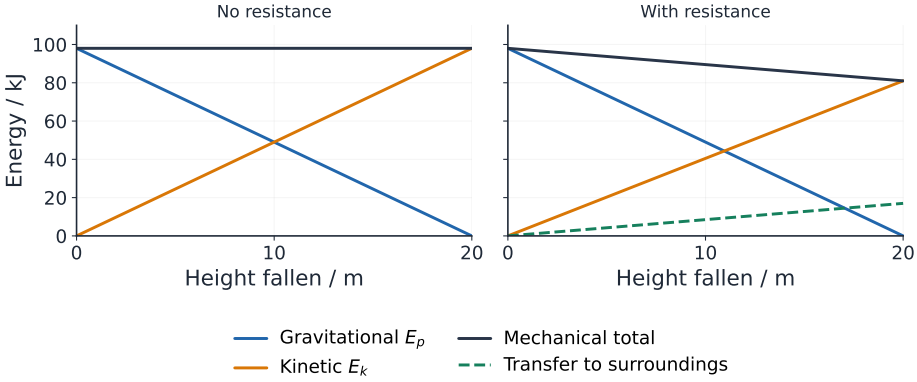
The path does not matter for ΔE_p

Kinetic into elastic energy



The block is momentarily at rest there

Energy moves between stores



Within each panel, potential + kinetic = mechanical total.

Worked example: a 20 m drop

1. **Given.** Released from rest; 20 m drop; $g = 9.8$ m/s^2. Neglect resistance and rotation.
2. **Equate.** $mgh = \frac{1}{2}mv^2$; the mass cancels.
3. **Solve.** $v = \sqrt{2(9.8)(20)} = 20$ m s^{-1}.

Decision

Same speed for either mass under these assumptions

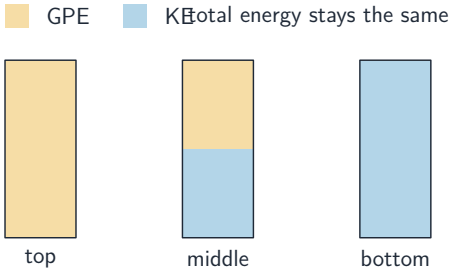
When friction acts

1. **Top.** 500 kg, released from rest through 20 m: $mgh = 98\,000$ J.
2. **Bottom.** $\frac{1}{2}(500)(18^2) = 81\,000$ J.
3. **Difference.** 17 000 J of work against friction over 60 m.

Decision

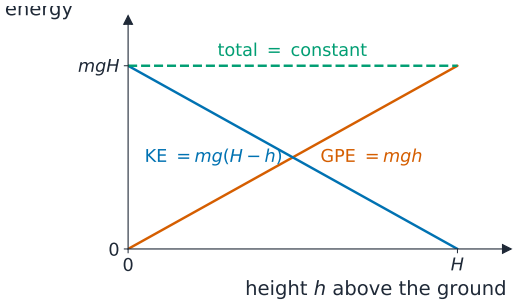
Average resistive force $= 17\,000/60 = 280$ N

Energy bars down a frictionless track



The total bar stays the same height

Energy against height for a falling ball



The two lines cross at half the release height

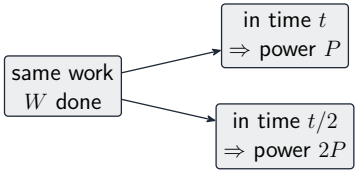
Power

- The force's power can be calculated while the object accelerates.
- Steady driving at 25 m/s against 600 N needs 15 kW.

$$P_{\text{instantaneous}} = Fv\cos\theta$$

Average power: $P_{\text{av}} = \Delta W/\Delta t$. Use instantaneous force and velocity above.

The same work in half the time



$$P = \frac{W}{t}$$ — less time, more power

> Power is the rate of doing work

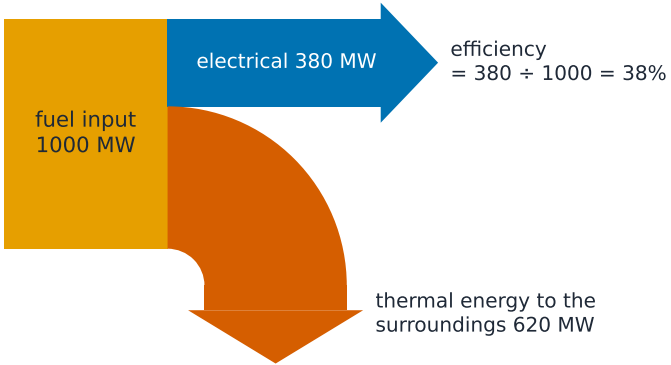
Efficiency

1. **Useful.** Crane lifts 500 kg by 12 m in 20 s, $mgh/t = 2940$ W.
2. **Input.** 4.2 kW of electrical power.

Decision

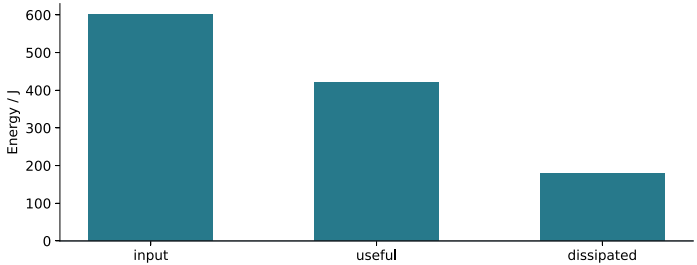
$\eta = 2940/4200 = 70\%$

Sankey diagrams



1000 MW input = 380 MW useful + 620 MW other transfers.

600 J in, 420 J useful



The input bar equals useful plus wasted

Energy density of fuels

Fuel or store	**MJ kg^{-1}**
coal	about 30
petrol	46
hydrogen	142
lithium-ion battery	about 0.7
uranium-235	about 8×10^7

Use the evidence

Rounded examples, not exact specifications. Hydrogen value includes water condensation; compare like-for-like system masses.

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Worked example: coal per second

1. **Given.** 500 MW electrical output, **efficiency** 0.38; coal releases 30 MJ/kg.
2. **Input.** $500/0.38 = 1316$ MW.
3. **Mass.** $\dot{m} = \dfrac{1316 \times 10^6}{30 \times 10^6} = 44$ kg/s.

Decision

About 3800 tonnes of coal per day

Exam tips

> Energy is dissipated, never lost.

- θ is between force and displacement, not the slope angle.
- Include external driving work and dissipation in the energy balance.
- 1 kW h $=$ 3.6 MJ.

Check yourself

Try these

A 70 kg skier starts from rest and drops 40 m, dissipating 6000 J. Find the final speed ($g = 9.8$ m/s^2).

At steady speed 10 m/s, a cyclist supplies 300 W mechanically. Find the resistive force.

A 25%-efficient engine delivers 15 kW for one hour. Petrol releases 46 MJ/kg. Find the mass burned.