Forces and momentum

A.2 · SL and HL

Why does a passenger lurch forward?

> Forces do not keep things moving; forces change how things move.

- Nothing pushes the passenger forwards when a bus brakes.
- A seat belt supplies the force that changes their motion.
- This unit: Newton's laws $\rightarrow$ forces $\rightarrow$ momentum $\rightarrow$ collisions $\rightarrow$ circles.

Newton's three laws

Law	Statement
First	No **resultant force**, no change in velocity (**inertia**).
Second	Fixed collection: $\overline{F} = \Delta p / \Delta t$; instantaneously $F = ma$ at constant mass.
Third	A on B is equal and opposite to B on A.

Third-law pairs

Earth pulls book

Partner: book pulls Earth up.

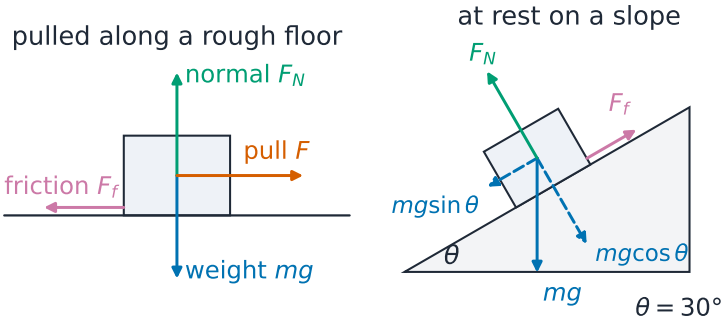
Gravitational.

Table pushes book

Partner: book pushes table down.

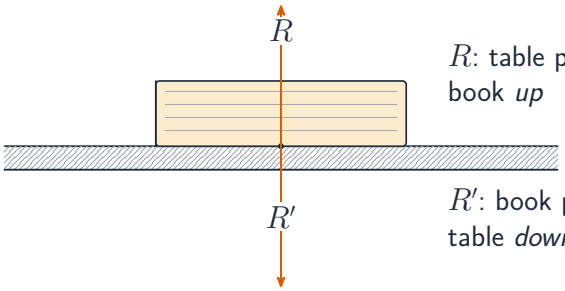
Contact.

Free-body diagrams



- One body, only the forces on it, each labelled.
- Zero resultant means **translational equilibrium**.

A third-law pair acts on two objects



R: table pushes the book *up*

R': book pushes the table *down*

Equal, opposite, the same type, on different objects

Contact forces

Force	Equation
static friction	$F_f \leq \mu_s F_N$
dynamic friction	$F_f = \mu_d F_N$
Hooke's law	$F_H = -kx$ in the linear range
viscous drag	$F_d = 6\pi\eta rv$
buoyancy	$F_b = \rho V g$

Worked example: will it slide?

1. **Given.** A 5.0 kg box on a level floor; horizontal forces, $\mu_s = .50$, $\mu_d = .40$, $g = 9.8$ m/s^2, so $F_N = 49$ N.
2. **Limit.** Largest **static friction** $= 0.50 \times 49 = 24.5$ N.
3. **20 N push.** Below the limit, so friction is 20 N and the box stays still.
4. **30 N pull.** Slides; friction $= 0.40 \times 49 = 19.6$ N.

Decision

$a = (30 - 19.6)/5.0 = 2.1$ m s^{-2}

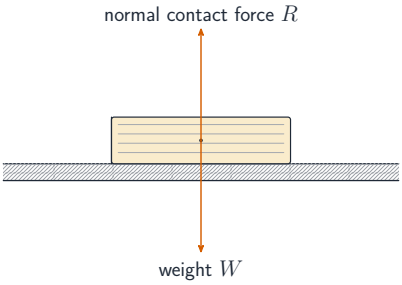
A sphere falling through glycerol

1. **Given.** A sphere has radius 1.0 mm and density 7800 kg/m^3; the fluid has density 1260 kg/m^3 and **viscosity** 1.4 Pa s. Use g = 9.8 m/s^2. Find terminal speed in the Stokes model.
2. **Weight.** 3.20×10^{-4} N.
3. **Buoyancy.** $\rho V g = 0.52 \times 10^{-4}$ N.
4. **Balance.** weight = **buoyancy** + $6\pi\eta r v$.

Decision

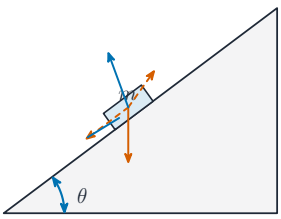
Terminal speed $v = 0.010$ m s^{-1}

Two forces on one book



These two are not a third-law pair

A block on a slope with friction



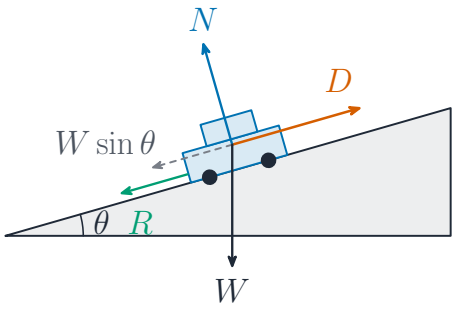
- mg (weight)
- $mg\sin\theta$, $mg\cos\theta$
- R (normal reaction)
- $F = \mu R$ (friction)

$\tan\theta = \frac{3}{4}$, so
$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

resolve both ways: $R = \frac{4}{5}mg$ and $a = g(\sin\theta + \mu\cos\theta) = \frac{4}{5}g$ when $\mu = \frac{1}{4}$

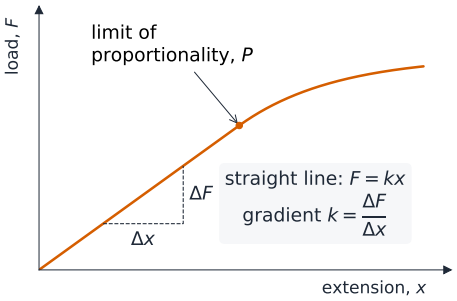
Compare $mg\sin\theta$ with $\mu mg\cos\theta$

A car driving up a slope



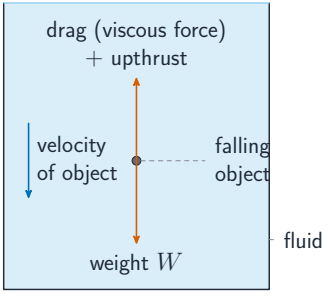
Steady speed: $D = R + W\sin\theta$

Hooke's law and its limit



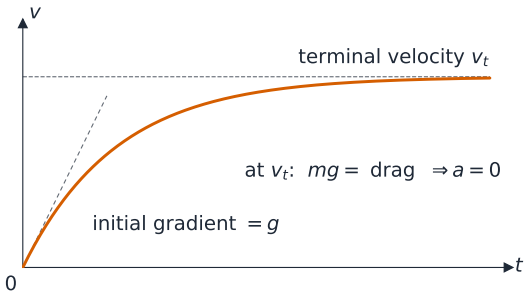
Beyond P the line curves: the limit of proportionality

Forces on an object falling through a fluid



Drag grows with speed until the forces balance

Reaching terminal velocity



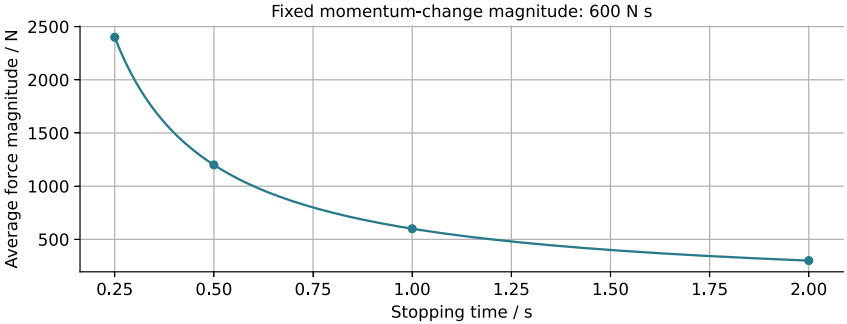
The acceleration falls as drag grows

Momentum

- A 1200 kg car at 15 m s^{-1} has $p = 18\,000$ kg m s^{-1}.
- Total momentum is constant unless a resultant external force acts.

Momentum mass multiplied by velocity, $p = mv$, a vector in kg m s^{-1}

Impulse is the change in momentum



- Each point is a different stop: fixed $|\Delta p| = 600\ \mathrm{N\,s}$, so $|F_{\mathrm{avg}}| = 600/\Delta t$.
- This curve compares stopping durations; its area is not one collision's **impulse**.

Separate a stopping comparison from one force history

1. **Given.** Different stopping events each have momentum-change magnitude 600 N s.
2. **Model.** $|F_{\text{avg}}| = |\Delta p|/\Delta t$: an inverse curve, not straight segments.
3. **At 0.75 s.** $|F_{\text{avg}}| = 600/0.75 = 800$ N.
4. **Distinguish.** The horizontal axis compares total stopping durations for different events. This is not the force history of one collision; its area is not that event's **impulse**.

Worked example: a rebound

1. **Given.** A 0.058 kg ball approaches at 20 m/s and rebounds at 15 m/s; contact time 0.010 s. Rebound is positive.
2. **Change.** $\Delta p = 0.058(15 - (-20)) = 2.0$ kg m s^{-1}.
3. **Force.** $F = 2.0/0.010 = 200$ N over 0.010 s.

Decision

Ignoring the sign change gives only 0.29 kg m s^{-1}

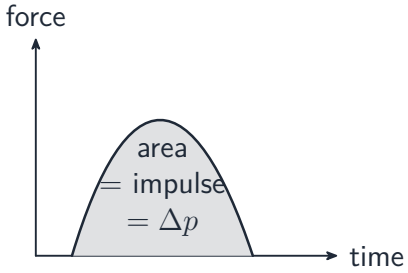
When the mass changes

- Hose: 2.0 kg of water per second at 15 m s^{-1} gives 30 N.
- Rocket: 50 kg s^{-1} at 2000 m s^{-1} relative to the rocket gives 1.0×10^5 N thrust.

$$F = \frac{\Delta m}{\Delta t} v$$

Use the appropriate relative flow velocity; neglect exhaust-pressure thrust for this rocket model.

Impulse is an area



A longer contact time means a smaller peak force

Elastic or inelastic?

Elastic

Kinetic energy conserved.

1.0 kg at 4.0 m s^{-1} stops; the other leaves at 4.0.

Inelastic

Kinetic energy falls.

Sticking together loses the most.

Worked example: trolleys stick

1. **Given.** A 2.0 kg trolley at 3.0 m/s hits a stationary 1.0 kg trolley and sticks. Neglect external **impulse**.
2. **Momentum.** $2.0 \times 3.0 = 3.0v$, so $v = 2.0$ m s^{-1}.
3. **KE before.** $\frac{1}{2}(2.0)(3.0^2) = 9.0$ J.
4. **KE after.** $\frac{1}{2}(3.0)(2.0^2) = 6.0$ J.

Decision

3.0 J transferred, so inelastic

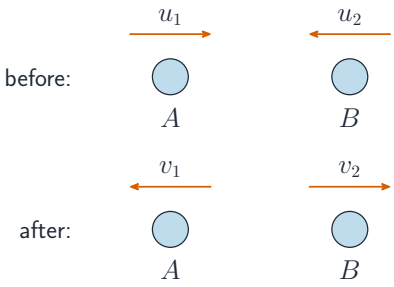
Check: a rifle recoils

Your turn

A 4.0 kg rifle fires a 0.010 kg bullet at 400 m s^{-1}. What is the recoil speed?

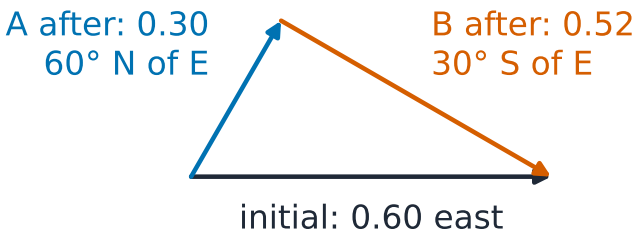
A. 0.010 m s^{-1}
B. 1.0 m s^{-1}
C. 4.0 m s^{-1}
D. 400 m s^{-1}

Signs in a head-on collision



A velocity to the left enters with a minus sign

HL: Collisions in two dimensions



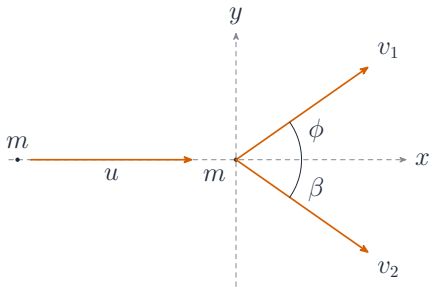
momentum / kg m s^{-1}

x: $0.60 = 0.15 + 0.45$

y: $0 = 0.26 - 0.26$

- Conserve momentum separately in x and in y.
- B moves off at 1.7 m s^{-1}, 30° south of east.

Momentum in two dimensions



Momentum is a vector: resolve before adding

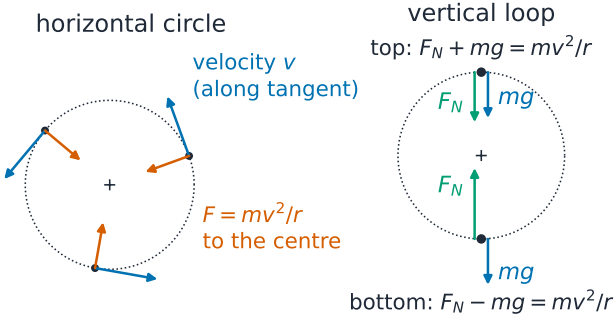
Circular motion

- $\omega = 2\pi/T$ and $v = \omega r$.
- **Centripetal force** is the resultant, supplied by tension, friction or gravity.

$$a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$$

The force is perpendicular to the velocity, so it does no work.

Forces in a circle



- Level, unbanked bend: friction must supply mv^2/r.
- Top of a loop: $v \geq \sqrt{gr}$ to keep contact.

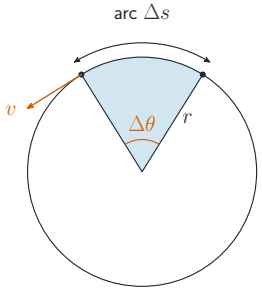
Worked example: a stone on a string

1. **Given.** Radius 0.80 m; two revolutions per second. Find **period**, angular speed, speed and radial acceleration.
2. **Period.** $T = 0.50$ s, so $\omega = 2\pi/0.50 = 12.6$ rad s^{-1}.
3. **Speed.** $v = \omega r = 10$ m s^{-1}.

Decision

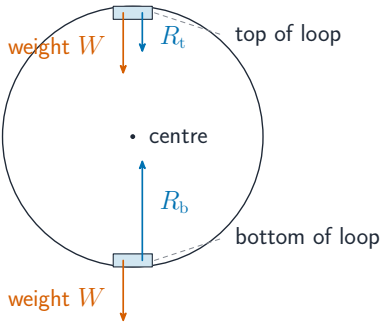
$a = \omega^2 r = 126$ m s^{-2}

Angular speed and the tangent



The velocity changes direction, so the object accelerates

Forces at the top and bottom of a loop



The normal force is largest at the bottom

Exam tips

Centripetal force is the resultant, never an extra force.

- **Free-body diagram**: only forces on the chosen body.
- Give momentum a sign; a rebound changes it.
- A kinetic-energy increase needs energy from another store, as in an **explosion**.
- HL. Write separate x and y momentum equations.

Check yourself

Try these

On a level floor, a 3.0 kg block slides under a horizontal 25 N pull with $\mu_d = .30$. Take $g = 9.8$ m/s^2. Find acceleration.

A 3000 kg truck at 2.0 m/s couples with a stationary 2000 kg truck. Neglect external impulse; find their common velocity.

Find the tension at the top of a vertical circle, 0.20 kg at 4.0 m s^{-1}, $r = 0.50$ m.

HL. A 2.0 kg ball initially moves east at 5.0 m/s; a 3.0 kg ball is stationary. After collision the 2.0 kg ball moves north at 2.0 m/s. Find the other velocity; neglect external impulse.