Course Companion

International Baccalaureate · IB Diploma · Physics · HL

Every topic handout in one volume

全部专题讲义合集

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Kinematics

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches A.1 Kinematics in full, for the course first assessed in 2025. There is no additional higher level content in A.1, so SL and HL students study the same material.

Prerequisites: rearranging equations, gradients of straight lines, areas of rectangles and triangles, and sine and cosine in a right-angled triangle. You will use $g = 9.8\ \text{m s}^{-2}$ near the Earth's surface.

Describing motion

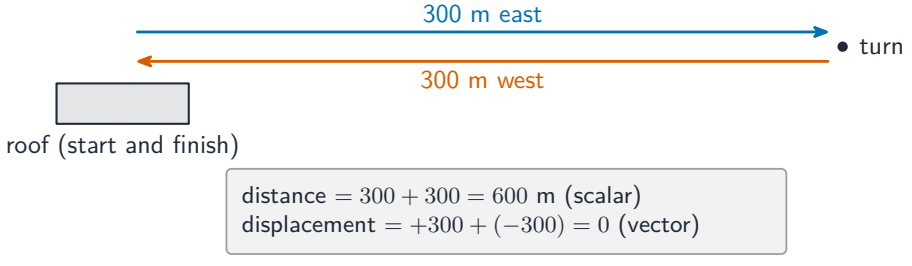
Where is it, and where is it going?

A delivery drone leaves a roof, flies 300 m east, then 300 m back west to land on the same roof. Its trip meter shows 600 m, but it ends exactly where it started. Physics needs two different quantities for these two answers. **Kinematics** 运动学 describes motion with numbers – position, velocity and acceleration – without asking what causes it. Causes (forces) come in A.2.

Distance and displacement

- **Distance** 路程 is the total length of the path travelled. It is a **scalar** 标量: it has size only.
- **Displacement** 位移 is the change in position: how far, and in which direction, from start to finish. It is a **vector** 矢量: it has size and direction.

For the drone, distance = 600 m and displacement = 0. In one dimension, choose a positive direction (for example "east is +") and write the direction as a sign: 300 m west is −300 m.



Distance adds the path lengths; displacement adds them with their signs

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Speed, velocity and acceleration

- **Speed** 速率 is the magnitude of instantaneous velocity (a scalar). Average speed over an interval is total distance divided by elapsed time.
- **Velocity** 速度 is the rate of change of displacement (a vector).
- **Acceleration** 加速度 is the rate of change of velocity, in m s^{-2}. Over a finite interval, $\bar{a} = \dfrac{\Delta v}{\Delta t}$ is the average acceleration. Instantaneous acceleration is the tangent gradient on a velocity–time graph; the two agree throughout an interval of constant acceleration.

An object accelerates whenever its velocity changes: when it speeds up, slows down **or changes direction**. A car going round a bend at a steady 15 m s^{-1} is accelerating.

Average and instantaneous values

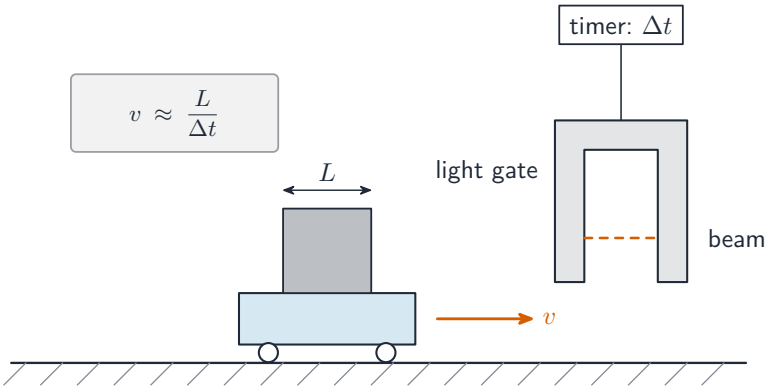
- An **average** 平均 value uses a whole interval: average velocity = total displacement ÷ total time.
- An **instantaneous** 瞬时 value is the value at one moment: the average over an interval so short that the value has no time to change.

A speedometer shows instantaneous speed. A 120 km journey that takes 2 hours has an average speed of 60 km/h, even if the car stopped at traffic lights and reached 100 km/h on the motorway. For the drone, the average speed is $600 \div 120 = 5$ m s^{-1} if the trip took 120 s, but the average velocity is zero.



A speedometer shows speed: the distance travelled per unit time.

In the lab: measure instantaneous speed with a light gate: a card of known length L cuts the beam for a time Δt, so $v \approx L/\Delta t$. A shorter card reduces the averaging interval, but card-length uncertainty and timer resolution still limit the estimate; an arbitrarily short card does not guarantee a better measurement. Video analysis with a known scale and frame interval also works. Avoid hand timing over short intervals: human reaction time (about 0.2 s) is then a large fraction of the measurement.

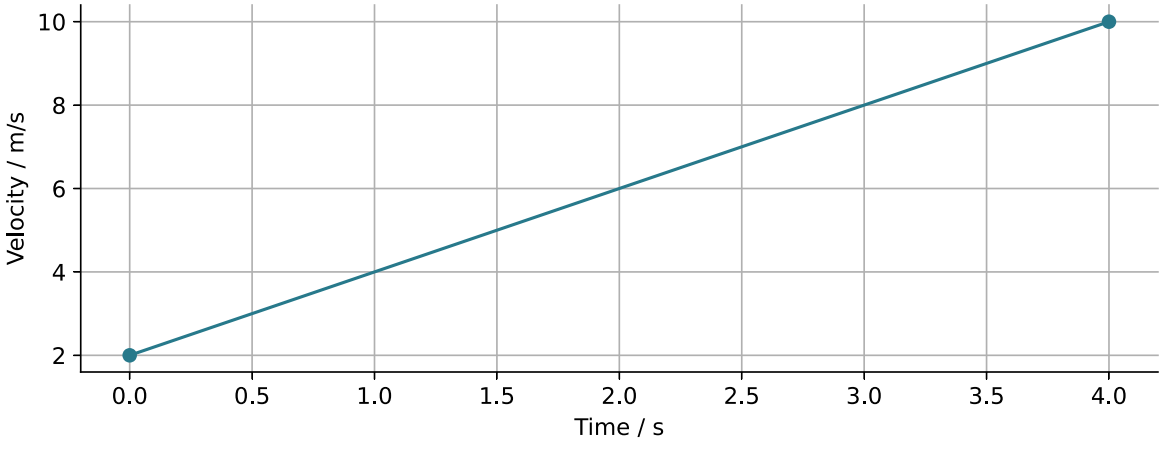


A light gate times how long the card blocks the beam: $v \approx L/\Delta t$

Motion graphs

■ Three graphs, linked by gradients and areas

Graph	Gradient gives	Area under it gives
displacement–time (s–t)	velocity	(not used)
velocity–time (v–t)	acceleration	displacement
acceleration–time (a–t)	(not used)	change in velocity



A straight v–t line means constant acceleration: $a = (10 - 2) \div 4 = 2.0\ m/s^2$

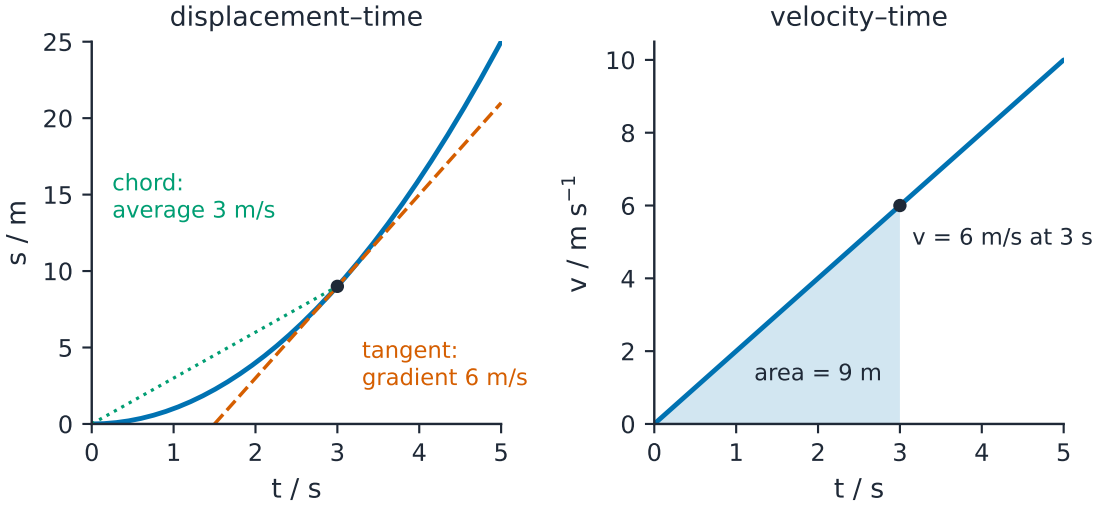
Worked example: velocity rises uniformly from 2 m s^{-1} to 10 m s^{-1} in 4 s.

- Acceleration = gradient = $\dfrac{10 - 2}{4} = 2.0$ m s^{-2}.
- Displacement = area of the trapezium = $\dfrac{(2 + 10)}{2} \times 4 = 24$ m.

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Curved graphs: use a tangent

When the gradient changes, the velocity (or acceleration) is changing. The instantaneous value at a point is the gradient of the **tangent** 切线 drawn at that point.



The tangent's gradient at 3 s (6 m/s) is the velocity there; the shaded v–t area (9 m) is the displacement after 3 s

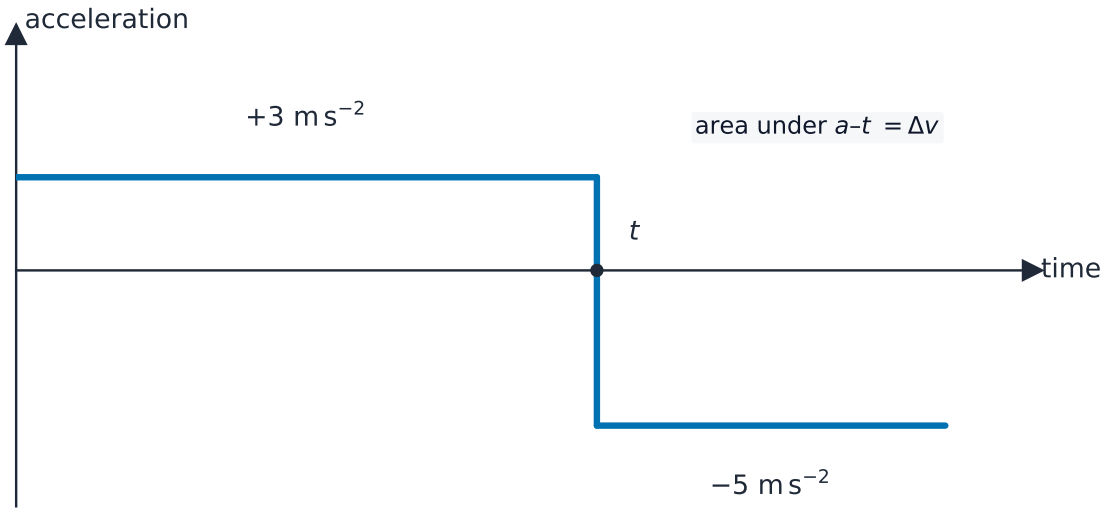
Worked example: a trolley's displacement is $s = t^2$ (s in m, t in s).

- At $t = 3$ s, the tangent passes through (1.5 s, 0 m) and (5 s, 21 m). Gradient $= \dfrac{21 - 0}{5 - 1.5} = 6.0$ m s^{-1}: the instantaneous velocity.
- Over the first 3 s, the average velocity is $\dfrac{9 - 0}{3} = 3.0$ m s^{-1}: the gradient of the **chord** from (0, 0) to (3, 9), not the tangent.

On a v–t graph, area **below** the time axis is displacement in the negative direction. Add areas with their signs for displacement; add their sizes for distance.

Reading the shape of a graph

- s–t: a straight line means constant velocity; a curve getting steeper means speeding up; a flat line means at rest.
- v–t: a straight line means **uniform acceleration** 匀加速; a flat line means constant velocity; a line crossing the time axis means the object reverses.
- a–t: the area between t_1 and t_2 is the change in velocity, even when the acceleration varies.

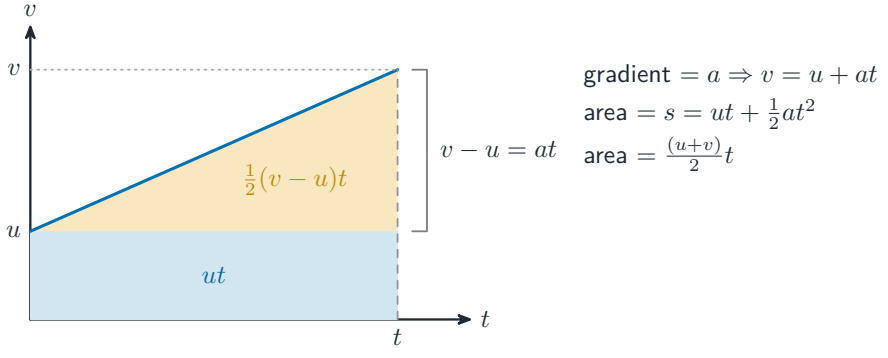


A separate motion: constant positive acceleration followed by constant negative acceleration

Equations of uniformly accelerated motion

■ Where the equations come from

For constant acceleration the v–t graph is a straight line from u (initial velocity) to v (final velocity) over time t.



Two suvat equations read off one graph

- Gradient: $a = \dfrac{v-u}{t}$, so $v = u + at$.
- Area of the trapezium: $s = \dfrac{(u+v)}{2}\,t$.

Substituting one into the other gives two more:

$$s = ut + \tfrac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

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Each equation leaves out one of the five quantities s, u, v, a, t. Pick the equation that leaves out the quantity you neither know nor need.

Leaves out	Equation
s	$v = u + at$
a	$s = \frac{(u+v)}{2}t$
v	$s = ut + \frac{1}{2}at^2$
t	$v^2 = u^2 + 2as$

These equations are valid **only** when the acceleration is constant. Check this before you use them; for changing acceleration use graphs instead.

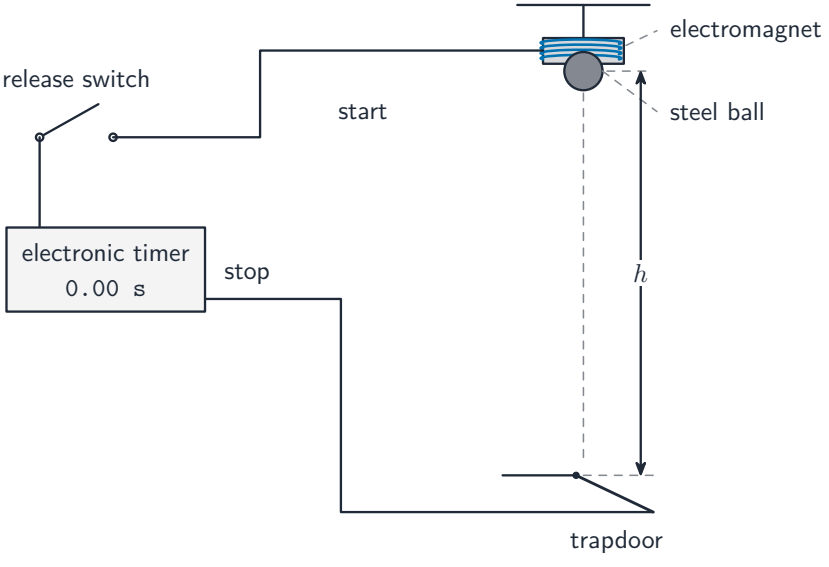
■ Worked example: braking

A car at 25 m s^{-1} brakes uniformly and stops in 50 m. Take the direction of travel as positive.

- Known: $u = 25$, $v = 0$, $s = 50$. Unknown: a. Not needed: t, so use $v^2 = u^2 + 2as$.
- $0 = 625 + 2a(50)$, so $a = -6.25$ m s^{-2}. The negative sign means the acceleration points against the motion.
- Time: $s = \frac{(u+v)}{2}t$ gives $50 = 12.5t$, so $t = 4.0$ s.

■ Free fall

An object moving only under gravity is in **free fall** 自由落体: its acceleration is $g = 9.8$ m s^{-2} downwards, whether it is rising, at the top, or falling. At the top of a purely vertical throw the velocity is zero, but the acceleration is still 9.8 m s^{-2} down.



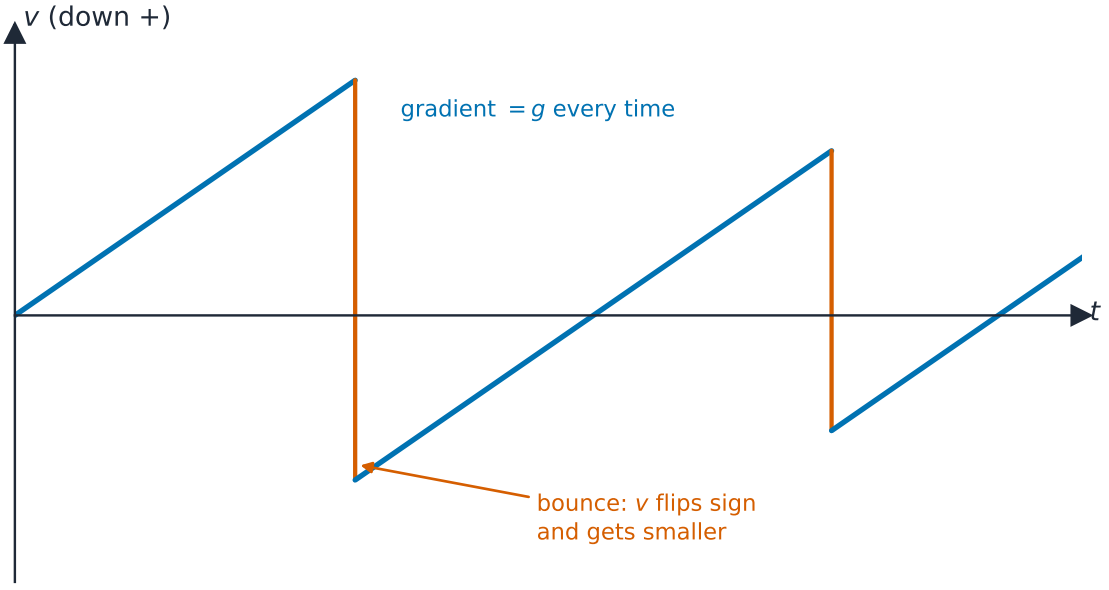
Measuring g: the electromagnet releases the ball and starts the timer; the trapdoor stops it. Then $h = \frac{1}{2}gt^2$

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Worked example: a ball is thrown straight up at 12 m s^{-1}. Take up as positive, so $a = -9.8$ m s^{-2}.

- Maximum height ($v = 0$): $0 = 144 - 19.6s$, so $s = 7.3$ m.
- Time to the top: $0 = 12 - 9.8t$, so $t = 1.2$ s. By symmetry it returns to the same level after 2.4 s, moving at 12 m s^{-1} downwards.

A negative velocity does not always mean slowing down. The object slows down only when acceleration and velocity have opposite signs. Falling back down, the ball has negative velocity and negative acceleration: it speeds up.



A bouncing ball: every sloping line has gradient g; the velocity flips sign and shrinks at each bounce

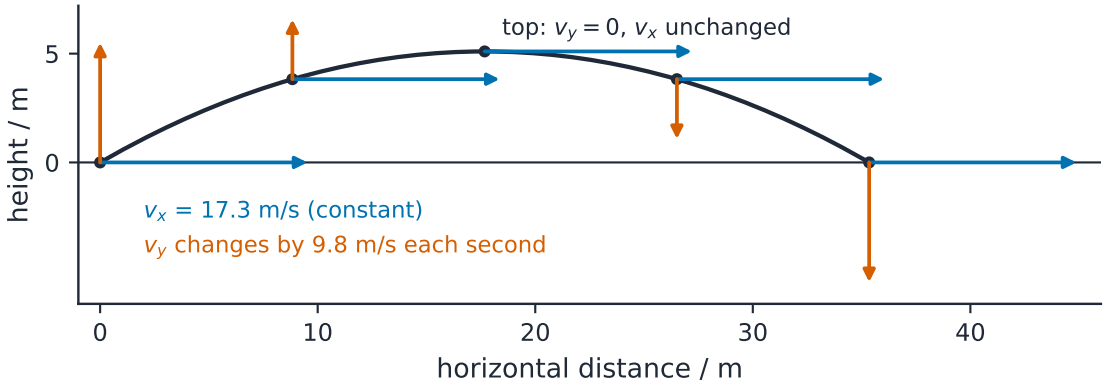
Projectile motion

Two independent motions

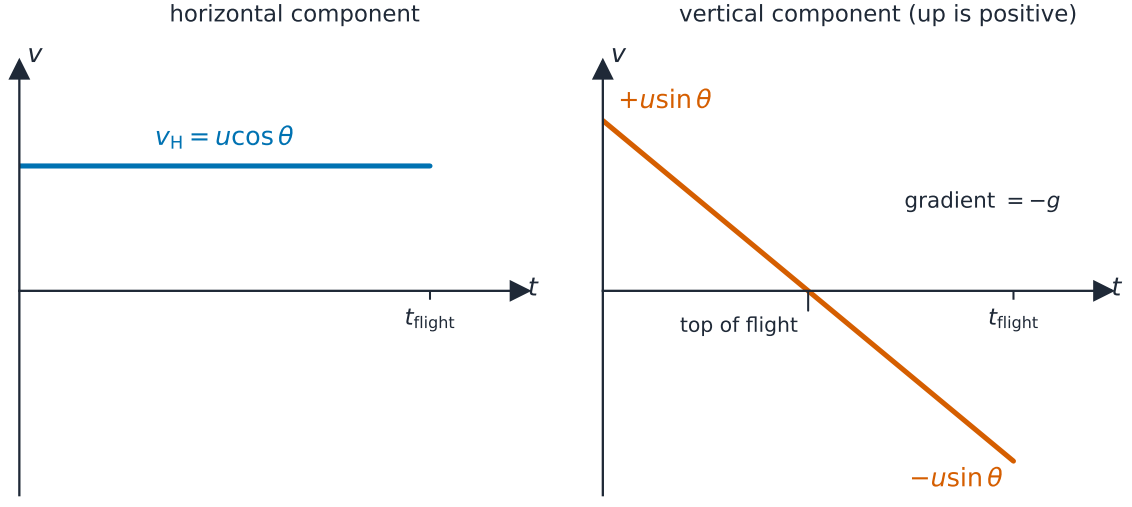
A **projectile** 抛体 is an object that moves through the air after launch, acted on only by gravity (when fluid resistance can be neglected). Split its velocity into horizontal and vertical **components** 分量:

- **Horizontal:** no force, so no acceleration. u_x stays constant: $x = u_x t$.
- **Vertical:** acceleration g downwards, so use the equations of motion with u_y and $a = -9.8$ m s^{-2} (up positive).
- **Time** t is the same for both, and links them.

For a launch at speed u and angle θ above the horizontal: $u_x = u\cos\theta$ and $u_y = u\sin\theta$. Without fluid resistance the **trajectory** 轨迹 is a parabola.



The horizontal component never changes; the vertical component changes by 9.8 m/s every second



Velocity–time sketches for a projectile: the horizontal component stays constant; the vertical component falls in a straight line through zero at the top

■ Launched horizontally

A ball rolls off a 1.25 m high table at 3.0 m s^{-1}.

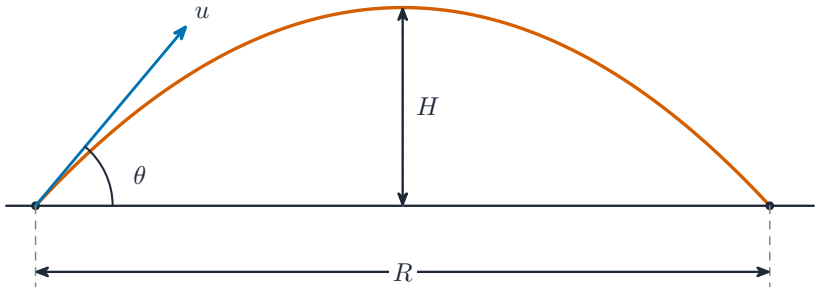
- Vertical: $u_y = 0$, $s_y = 1.25$ m down. $1.25 = \frac{1}{2}(9.8)t^2$ gives $t = 0.505$ s.
- Horizontal: $x = 3.0 \times 0.505 = 1.5$ m from the table.
- Landing velocity: $v_y = 9.8 \times 0.505 = 4.9$ m s^{-1} down, $v_x = 3.0$ m s^{-1}. Speed $= \sqrt{3.0^2 + 4.9^2} = 5.8$ m s^{-1} at $\tan^{-1}(4.9/3.0) = 59°$ below the horizontal.

A ball dropped from the same height at the same moment lands at the same time: horizontal motion does not change vertical motion.

Launched at an angle above the horizontal

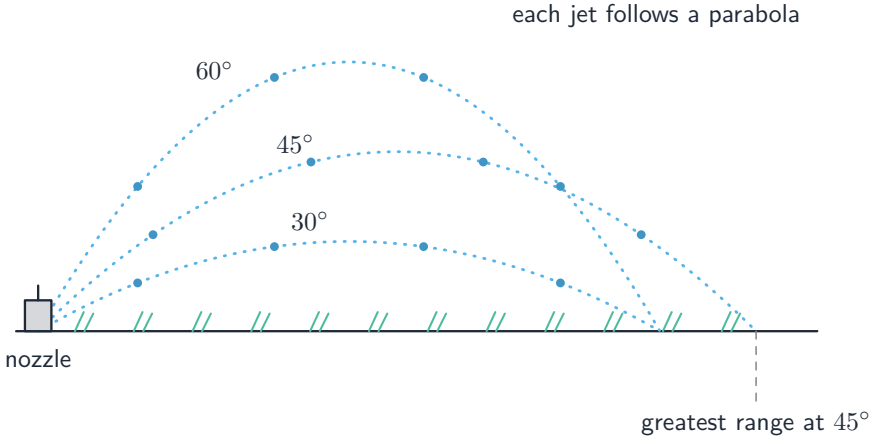
A ball is kicked from flat ground at 20 m s^{-1}, 30° above the horizontal.

- Components: $u_x = 20\cos 30° = 17.3$ m s^{-1}; $u_y = 20\sin 30° = 10.0$ m s^{-1}.
- **Time of flight** 飞行时间: it lands when $s_y = 0$: $0 = 10.0t - 4.9t^2$, so $t = 2.04$ s.
- **Range** 射程: $x = 17.3 \times 2.04 = 35$ m.
- Maximum height: $v_y = 0$ at the top, so $0 = 100 - 19.6h$, $h = 5.1$ m.



Range R of a projectile launched from and landing on level ground

On level ground, without fluid resistance, the greatest range comes from a 45° launch, and complementary angles (30° and 60°) give the same range.



Water jets from a sprinkler trace parabola paths – a real example of projectile motion

Launched at an angle below the horizontal

A stone is thrown at 15 m s^{-1}, 20° below the horizontal, from a 30 m cliff. Take **down** as positive for the vertical motion.

- $u_x = 15\cos 20° = 14.1$ m s^{-1}; $u_y = 15\sin 20° = 5.13$ m s^{-1} downwards.

- $30 = 5.13t + 4.9t^2$. Solve the quadratic: $t = \dfrac{-5.13 + \sqrt{5.13^2 + 4(4.9)(30)}}{2(4.9)} = 2.01$ s (reject the negative root).
- Horizontal distance $= 14.1 \times 2.01 = 28$ m.

Energy as a shortcut

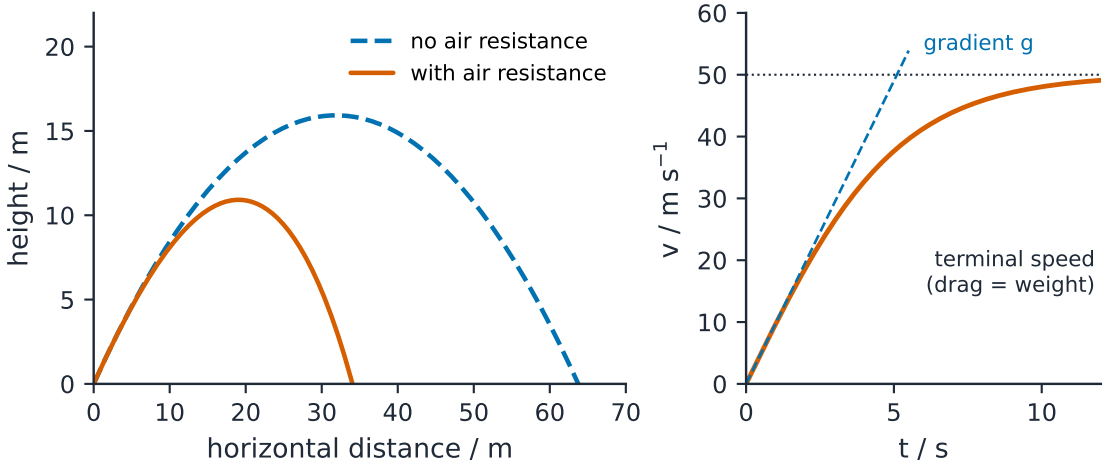
When only the landing speed is asked for, energy conservation (A.3) is quicker: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgh$, so $v = \sqrt{u^2 + 2gh}$. For the cliff stone, $v = \sqrt{15^2 + 2(9.8)(30)} = 29$ m s^{-1}, whatever the launch angle.

Fluid resistance and terminal speed

What air does to a projectile

Fluid resistance 流体阻力 (**drag** 阻力) is the force from a gas or liquid that opposes motion through it. It grows as speed increases. Compared with the same launch in a vacuum, a projectile with fluid resistance has:

- a **shorter range** and a **lower maximum height**;
- a **non-parabolic, asymmetric** trajectory: the way down is steeper than the way up;
- a horizontal velocity that **decreases** throughout the flight;
- a **shorter** time to reach the top, and a lower landing speed;
- an acceleration that is **not** constant and not just g: during ascent its vertical component is more downward than g; during descent the upward drag component opposes weight. The total acceleration magnitude also depends on horizontal drag, so it need not be smaller than g throughout descent.



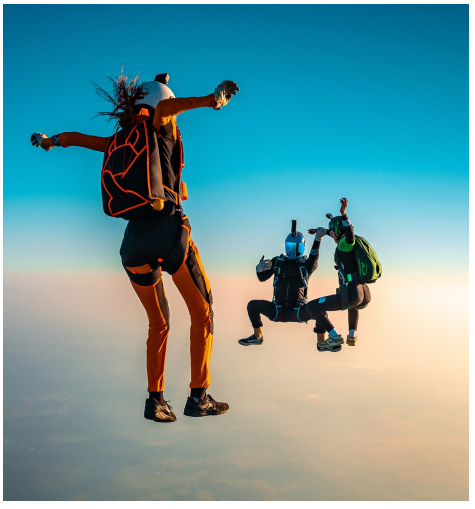
Drag shortens and skews the trajectory; a falling object's speed levels off at terminal speed

Terminal speed

A skydiver steps out of a plane.

1. At first the speed is low, drag is small, and the acceleration is close to g.
2. As speed rises, drag rises, so the resultant force and the acceleration both decrease. The v–t graph gets less steep.
3. When drag equals weight, the resultant force is zero: the skydiver falls at a constant **terminal speed** 终端速度 (about 55 m s^{-1} in a spread position).
4. Opening the parachute suddenly increases drag above the weight, so the skydiver decelerates until drag again equals weight at a much lower terminal speed (about 5 m s^{-1}).

A larger area facing the flow, or a less streamlined shape, means a lower terminal speed. A heavier object of the same shape needs more drag to balance its weight, so its terminal speed is higher.



Why the spread-eagle pose? Spreading out gives the largest area, so the most drag. The bigger the drag, the sooner drag balances weight – and the lower the steady terminal velocity they fall at

Practice

1. A runner completes one lap of a 400 m track in 80 s. Find the distance, the displacement, the average speed and the average velocity.
2. A cyclist's velocity rises from 3.0 m s^{-1} to 15 m s^{-1} in 6.0 s. Find the acceleration.
3. A train starts from rest and accelerates at 0.50 m s^{-2} for 40 s. Find its final velocity and the distance travelled.
4. A velocity–time graph rises uniformly from 0 to 8.0 m s^{-1} in 4.0 s, stays at 8.0 m s^{-1} for 6.0 s, then falls uniformly to 0 in 2.0 s. Find the total displacement and the average speed.

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5. A driver at 30 m s^{-1} sees a hazard. The reaction time is 0.70 s, then the car decelerates uniformly at 6.0 m s^{-2}. Find the total stopping distance.
6. A ball is thrown vertically up at 15 m s^{-1}. Find the maximum height and the time until it returns to the thrower's hand.
7. A dart is thrown horizontally at 12 m s^{-1} towards a board 3.0 m away. How far does it drop before it hits the board?
8. A football is kicked from flat ground at 25 m s^{-1}, 40° above the horizontal. Neglect air resistance. Find the time of flight, the range and the maximum height.
9. A ball is thrown at 8.0 m s^{-1}, 30° below the horizontal, from a window 12 m above the ground. Find how far from the wall it lands.
10. Describe and explain how air resistance changes the range, the maximum height and the shape of the trajectory of the football in question 8.
11. Sketch and explain the velocity–time graph of a skydiver from leaving the plane until landing with an open parachute.

Answers.

1. Distance 400 m; displacement 0; average speed $400/80 = 5.0$ m s^{-1}; average velocity 0.
2. $a = (15 - 3.0)/6.0 = 2.0$ m s^{-2}.
3. $v = 0 + 0.50 \times 40 = 20$ m s^{-1}; $s = \frac{1}{2}(0.50)(40^2) = 400$ m.
4. Area $= \frac{1}{2}(4.0)(8.0) + (6.0)(8.0) + \frac{1}{2}(2.0)(8.0) = 16 + 48 + 8 = 72$ m; average speed $= 72/12 = 6.0$ m s^{-1}.
5. Thinking distance $= 30 \times 0.70 = 21$ m. Braking: $0 = 900 - 12s$, $s = 75$ m. Total $= 96$ m.
6. $0 = 225 - 19.6h$, $h = 11.5$ m. Time $= 2 \times 15/9.8 = 3.1$ s.
7. $t = 3.0/12 = 0.25$ s; drop $= \frac{1}{2}(9.8)(0.25^2) = 0.31$ m.
8. $u_x = 19.2$, $u_y = 16.1$ m s^{-1}. $t = 2 \times 16.1/9.8 = 3.3$ s; range $= 19.2 \times 3.28 = 63$ m; height $= 16.1^2/19.6 = 13$ m.
9. $u_x = 6.93$, $u_y = 4.0$ m s^{-1} down. $12 = 4.0t + 4.9t^2$ gives $t = 1.21$ s; distance $= 6.93 \times 1.21 = 8.4$ m.
10. Drag opposes the velocity throughout, so the horizontal component falls and the range is shorter. On the way up drag adds to weight, so the ball stops rising sooner: lower maximum height. The path is no longer a parabola: the descent is steeper than the ascent because the horizontal speed is smaller by then.
11. Gradient starts at about 9.8 m s^{-2} and decreases as drag grows, levelling at the first terminal speed (drag = weight). When the parachute opens, velocity drops steeply (drag > weight, so the resultant force is upward) and levels at a much lower terminal speed. Landing: velocity falls quickly to zero.

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Exam tips

- State your positive direction before you substitute; keep every sign consistent with it.
- "Gradient of a tangent" gives an instantaneous value; "gradient of a chord" gives an average value.
- List s, u, v, a, t, mark the three you know, and choose the equation that leaves out the one you do not need.
- Use the equations of motion only for constant acceleration; say so if a question asks you to justify them.
- In projectile questions, never put a horizontal velocity into a vertical equation. Time is the only quantity the two directions share.
- At the top of a vertical throw, $v = 0$ but $a = g$. At the top of a projectile path, only $v_y = 0$.
- Air-resistance answers are qualitative: compare with the no-drag path (range, height, symmetry, time) and explain each change with the direction of the drag force.

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Forces and momentum

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches A.2 Forces and momentum in full, for the course first assessed in 2025. There is no additional higher level content in A.2, but HL students must solve collisions and explosions in **two** dimensions; that section is marked **HL**.

Prerequisites: velocity, acceleration and the equations of motion from A.1; resolving a vector into components with sine and cosine. Use $g = 9.8\ \text{m s}^{-2}$.

Newton's laws and free-body diagrams

Why does a passenger lurch forward?

When a bus brakes hard, standing passengers fall forwards. Nothing pushes them forwards: their bodies simply keep moving while the bus slows down under them. A seat belt or a handrail supplies the force that changes their motion. Forces do not keep things moving; forces **change** how things move.

Forces are interactions

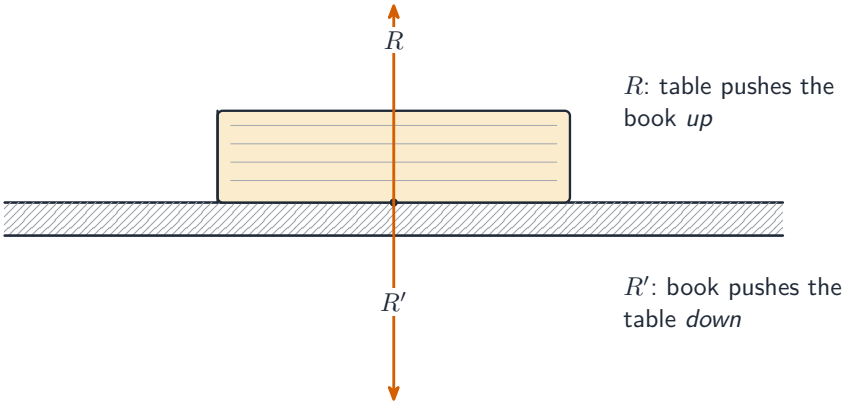
A **force** 力 is a push or pull that one body exerts on another. Every force is part of an **interaction** 相互作用 between two bodies, and is measured in newtons (N).

Newton's three laws

1. **First law:** a body stays at rest, or keeps moving at constant velocity, unless a resultant external force acts on it. Its tendency to keep its motion is called **inertia** 惯性.
2. **Second law:** for a fixed collection of particles, the resultant external force equals the instantaneous rate of change of its momentum. Over a finite interval, $\bar{F} = \dfrac{\Delta p}{\Delta t}$ gives the average resultant force. For constant mass, instantaneous force and acceleration obey $F = ma$.
3. **Third law:** if body A exerts a force on body B, then B exerts a force on A that is equal in size and opposite in direction.

A **third-law pair** acts on **two different** bodies and is always the **same type** of force. A book resting on a table has two forces on it: its weight (Earth pulls the book) and the normal force (table pushes the book). These are **not** a third-law pair: both act on the book. The partner of the book's weight is the book pulling the **Earth** upwards; the partner of the normal force is the book pushing **down** on the table.

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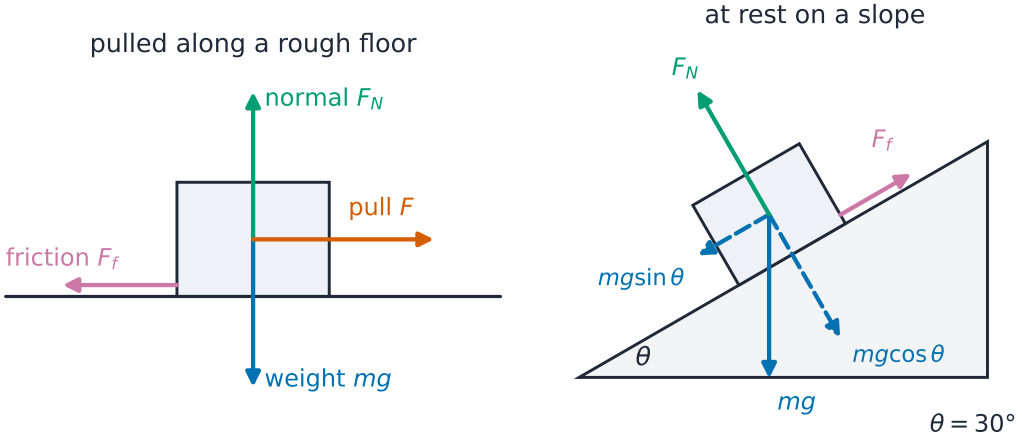
Newton's third-law pair: R on the book (up) and R' on the table (down)

Free-body diagrams

A **free-body diagram** 受力图 shows **one** body and **only** the forces acting on it, as labelled arrows starting from the body. Leave out forces the body exerts on other things.

The **resultant force** 合力 is the vector sum of all the forces. Resolve forces into two perpendicular directions and add each set of components.

- If the resultant force is zero, the body is in **translational equilibrium** 平动平衡: at rest, or at constant velocity (first law).
- If it is not zero, $a = F_{\text{net}}/m$ in the direction of the resultant (second law).



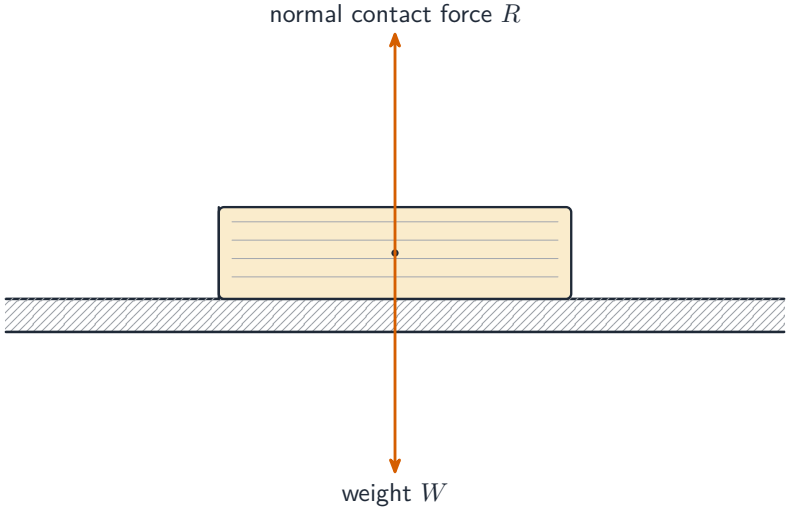
Draw only the forces on the chosen body; on a slope, resolve the weight along and perpendicular to the surface

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Contact and field forces

Contact forces

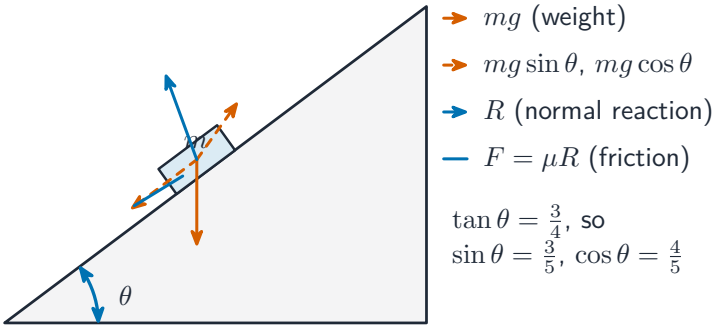
Force	Symbol	What it does
normal force	F_N	the component of the contact force **perpendicular** to the surface
surface friction	F_f	acts **parallel** to the surface, opposing sliding (or the tendency to slide)
tension	T	the pull of a string, rope or cable, along it
elastic restoring force	F_H	a spring's push or pull back towards its natural length
viscous drag	F_d	opposes a body's motion through a fluid
buoyancy	F_b	the upward push of a fluid on a body that displaces it



Weight and normal contact force on a book resting on a table

Friction

- On a body that is **not** sliding, **static friction** 静摩擦力 takes whatever value is needed to stop sliding, up to a maximum: $F_f \leq \mu_s F_N$.
- On a sliding body, **dynamic friction** 动摩擦力 is $F_f = \mu_d F_N$, where μ_d is usually a little smaller than μ_s.



resolve both ways: $R = \frac{4}{5}mg$ and $a = g(\sin\theta + \mu\cos\theta) = \frac{4}{5}g$ when $\mu = \frac{1}{4}$

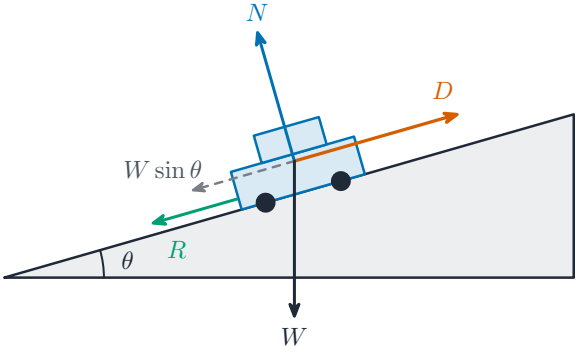
A block of mass m on a plane inclined at theta with tan theta = 3/4, showing the weight resolved into mg sin theta and mg cos theta, the normal reaction R, and friction F = mu R acting down the slope.

The coefficients μ_s and μ_d have no unit and depend on the two surfaces.

Worked example: a 5.0 kg box rests on a floor with $\mu_s = 0.50$ and $\mu_d = 0.40$. $F_N = mg = 49$ N.

- A horizontal 20 N push: the largest static friction is $0.50 \times 49 = 24.5$ N, so the box stays still and friction is exactly 20 N.
- A horizontal 30 N pull: more than 24.5 N, so the box slides. Friction $= 0.40 \times 49 = 19.6$ N; resultant $= 30 - 19.6 = 10.4$ N; $a = 10.4/5.0 = 2.1$ m s^{-2}.

For a body with no acceleration normal to a slope and no other force component in that direction, resolve the weight for slope angle θ: $mg\sin\theta$ down the slope and $mg\cos\theta$ into it, so $F_N = mg\cos\theta$. For the box at rest on a 30° slope, friction $= 49\sin 30° = 24.5$ N up the slope and $F_N = 49\cos 30° = 42$ N; it can stay still only if $\mu_s \geq \tan 30° = 0.58$.



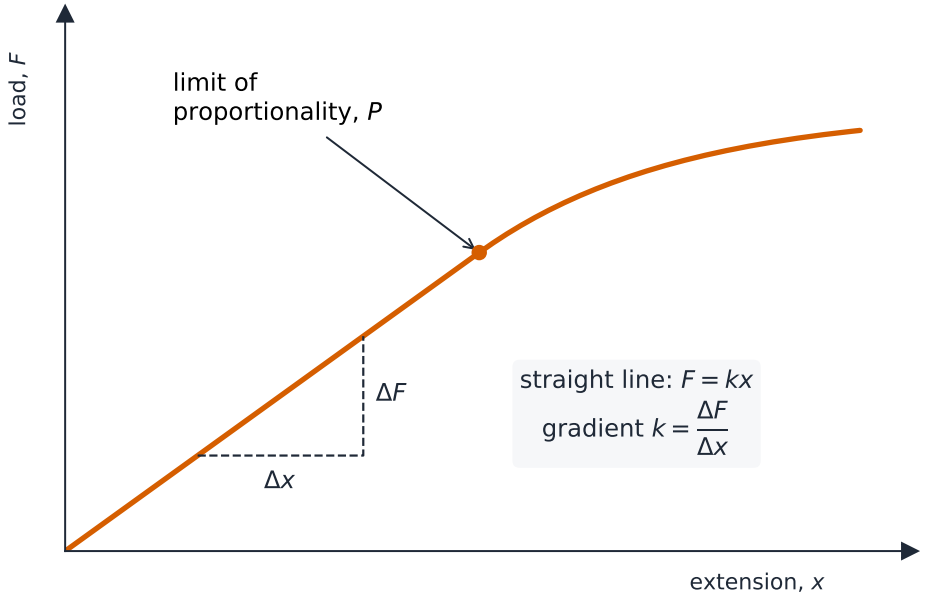
A car climbing a slope at constant speed: along the slope the driving force balances the resistive force plus the weight's component W sin theta

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Springs: Hooke's law

$$F_H = -kx$$

x is the extension from the natural length and k is the **spring constant** 劲度系数 (N m^{-1}). The minus sign shows that the force acts back towards the natural length. A 0.50 kg mass hung on a spring with $k = 200$ N m^{-1} stretches it by $x = mg/k = 4.9/200 = 0.025$ m.



A load–extension graph: straight up to the limit of proportionality P, then it curves

In the lab: add loads in equal steps, measure each extension from the **unloaded** position with a fixed pointer, and plot force against extension. The gradient of the straight part is k; stay within the linear, proportional range. A spring can return to its original length after a load is removed yet respond nonlinearly, so elastic behaviour alone does not establish constant k.

Viscous drag and buoyancy

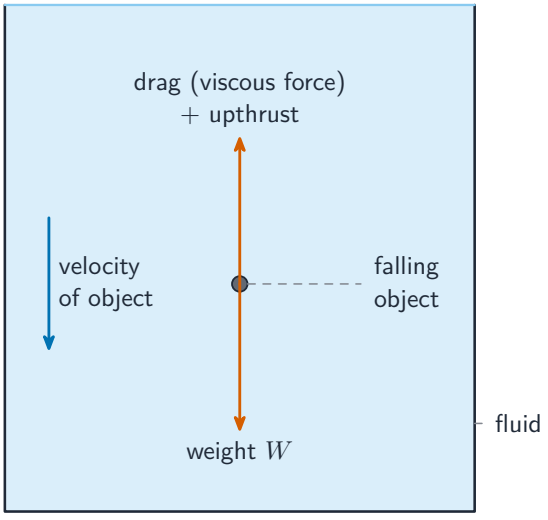
For a small sphere moving slowly through a fluid, **Stokes' law** 斯托克斯定律 gives the drag:

$$F_d = 6\pi\eta r v$$

η is the **viscosity** 黏度 of the fluid (Pa s), r the sphere's radius and v its speed.

Buoyancy 浮力 equals the weight of fluid displaced: $F_b = \rho V g$, where ρ is the density of the **fluid** and V the volume displaced.

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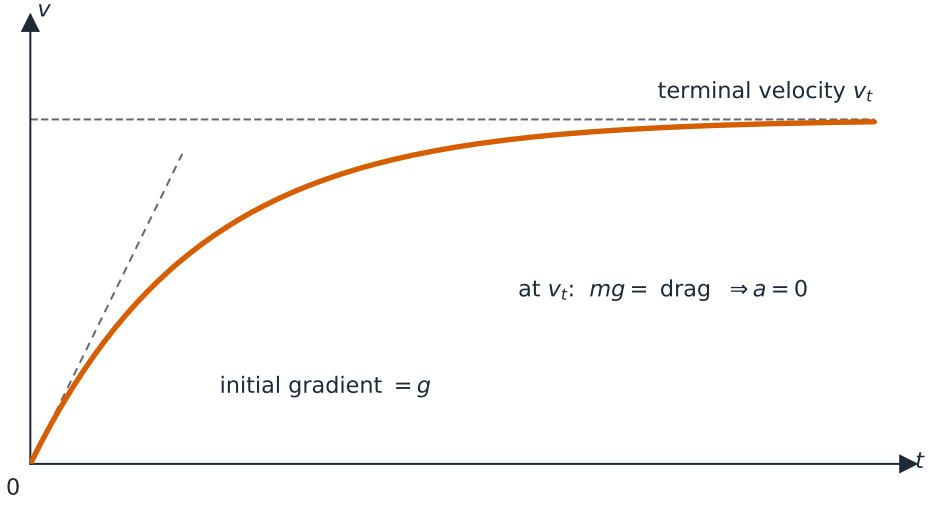


Forces on a falling object in a fluid

Worked example: a steel ball (radius 1.0 mm, density 7800 kg m^{-3}) falls through glycerol ($\rho = 1260$ kg m^{-3}, $\eta = 1.4$ Pa s). At terminal speed the forces balance: weight = buoyancy + drag.

- $V = \frac{4}{3}\pi r^3 = 4.19 \times 10^{-9}$ m^3.
- Weight $= 7800 \times 4.19 \times 10^{-9} \times 9.8 = 3.20 \times 10^{-4}$ N; buoyancy $= 1260 \times 4.19 \times 10^{-9} \times 9.8 = 0.52 \times 10^{-4}$ N.
- $v = \dfrac{(3.20 - 0.52) \times 10^{-4}}{6\pi(1.4)(1.0 \times 10^{-3})} = 0.010$ m s^{-1}.

This is how viscosity is measured: time the ball over a marked distance once it has reached terminal speed.



Velocity–time graph for an object falling through air

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Field forces

Field forces act without contact.

- **Gravitational force** (weight): $F_g = mg$, towards the centre of the Earth.
- **Electric force** F_e: between charges (D.2).
- **Magnetic force** F_m: on moving charges and currents (D.3).

Momentum and impulse

Linear momentum

Momentum 动量 is mass × velocity:

$$p = mv$$

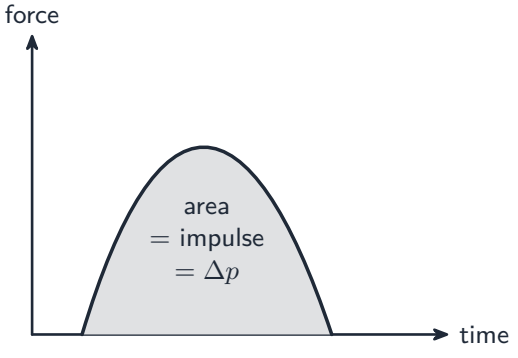
It is a vector, measured in kg m s^{-1}. A 1200 kg car at 15 m s^{-1} has $p = 18\,000$ kg m s^{-1}. The total momentum of a system stays constant unless a resultant **external** force acts on the system: the **conservation of momentum** 动量守恒.

Impulse

A resultant force F acting for a time Δt gives an **impulse** 冲量:

$$J = F\Delta t = \Delta p$$

F is the average resultant force. The impulse equals the change in momentum, and is measured in N s (the same as kg m s^{-1}). On a force–time graph, impulse is the **area under the graph**.



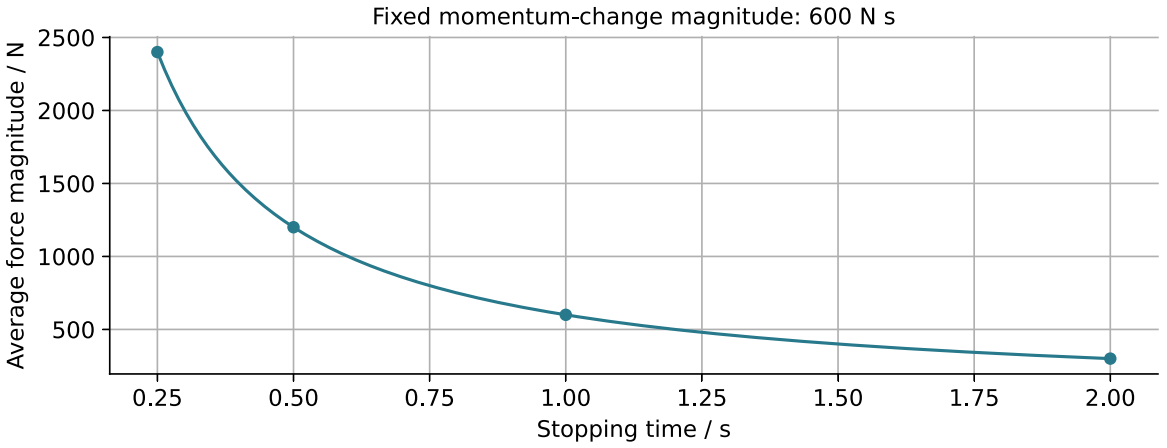
Impulse is the area under a force-time graph, equal to the change in momentum

Worked example: a 0.058 kg tennis ball hits a wall at 20 m s^{-1} and rebounds at 15 m s^{-1}. Take the rebound direction as positive.

- $\Delta p = 0.058 \times (15 - (-20)) = 2.0$ kg m s^{-1}. Forgetting the sign change would give only 0.29.

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- Contact lasts 0.010 s, so the average force = 2.0/0.010 = 200 N.



The same impulse, 600 N s, spread over a longer time needs a smaller force

Each point compares a different stopping event with the same momentum-change magnitude. The relationship is inverse: $|F_{\text{avg}}| = |\Delta p|/\Delta t$. At 0.75 s, the model gives $600/0.75 = 800$ N; a straight segment between the 0.50 s and 1.00 s points would give the wrong intermediate value. This curve is not the force history of one collision, so its area does not give that collision's impulse.

For the same change in momentum, a longer stopping time means a smaller force. Seat belts, air bags, crumple zones, bending your knees on landing and pulling your hands back to catch a ball all increase Δt.



In a crash, a large force acts over a very short time to change momentum.

When the mass changes

$F = ma$ assumes constant mass. For a steady mass flow, account for momentum transferred per unit time across the system boundary. If a hose sends 2.0 kg of water each second at 15 m s^{-1} against a wall, where it stops:

$$F = \frac{\Delta m}{\Delta t} v = 2.0 \times 15 = 30 \text{ N}$$

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A rocket uses the same momentum-flow idea: ejecting 50 kg of gas per second at 2000 m s^{-1} relative to the rocket gives a thrust of 1.0×10^5 N when the exhaust-pressure contribution is neglected. Ground-frame exhaust speed alone does not specify this thrust.



A rocket pushes gas down; by Newton's third law the gas pushes the rocket up.

Collisions and explosions

■ Conservation in a collision

In a collision the bodies push on each other with equal and opposite forces for the same time (third law), so their impulses are equal and opposite. Total momentum is conserved as long as external forces (like friction) are small during the collision.



A Newton's cradle shows momentum and kinetic energy passing through a line of balls in a near-elastic collision

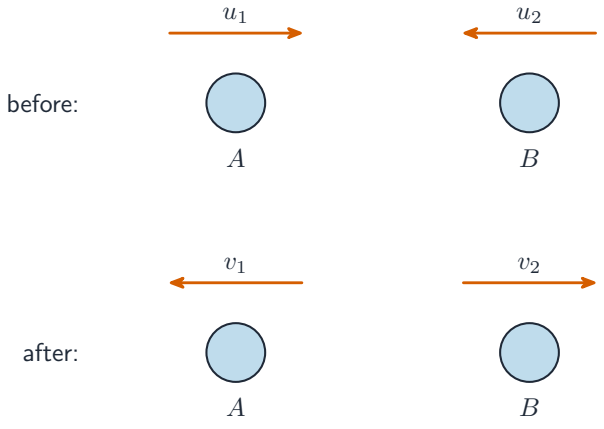
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$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Elastic and inelastic collisions

Type	Momentum	Kinetic energy
elastic collision 弹性碰撞	conserved	conserved
inelastic collision 非弹性碰撞	conserved	some transferred to thermal energy, sound and deformation
perfectly inelastic	conserved	the bodies stick together; the greatest possible KE loss

Total energy is always conserved; only the **kinetic** energy can decrease.



Head-on collision: velocities before and after

Worked example: a 2.0 kg trolley at 3.0 m s^{-1} hits a stationary 1.0 kg trolley and they stick together.

- Momentum: $2.0 \times 3.0 = 3.0v$, so $v = 2.0$ m s^{-1}.
- KE before $= \frac{1}{2}(2.0)(3.0^2) = 9.0$ J; KE after $= \frac{1}{2}(3.0)(2.0^2) = 6.0$ J. 3.0 J is transferred, so the collision is inelastic.

If a 1.0 kg trolley at 4.0 m s^{-1} hits an identical stationary trolley and stops, the second moves off at 4.0 m s^{-1}: KE is 8.0 J before and after, so that collision is elastic.

Explosions

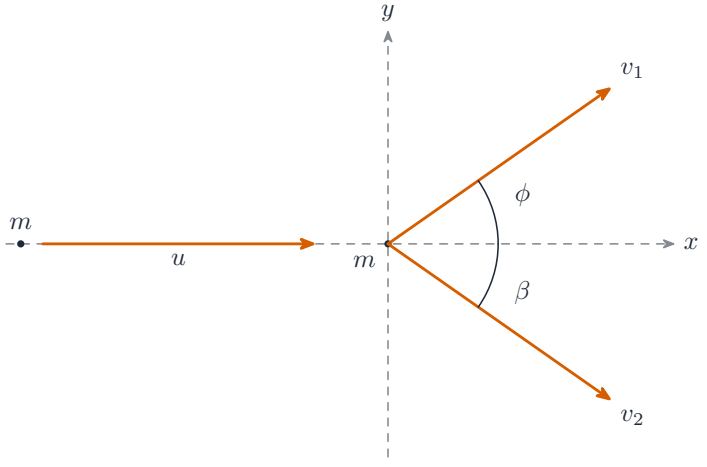
In a two-part **explosion** 爆炸 initially at rest, total momentum starts at zero. If external impulse is negligible, the parts acquire equal and opposite momenta, while stored chemical or elastic energy becomes kinetic energy, so the total KE **increases**.

A 4.0 kg rifle fires a 0.010 kg bullet at 400 m s^{-1}. $0 = 0.010(400) + 4.0v$, so the rifle recoils at $v = -1.0$ m s^{-1}. The bullet gets 800 J of kinetic energy and the rifle only 2.0 J: the lighter body takes almost all the energy.

HL: Collisions in two dimensions

Conserve each component

Momentum is a vector, so it is conserved separately in the x and y directions. Choose axes, resolve every momentum, and write two conservation equations.

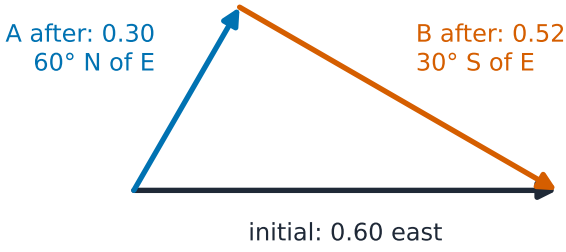


A glancing collision resolved along two perpendicular axes

Worked example: puck A (0.20 kg) moves east at 3.0 m s^{-1} and hits stationary puck B (0.30 kg). Afterwards A moves at 1.5 m s^{-1} at 60° north of east. Find B's velocity.

- x: $0.20\times3.0 = 0.20\times1.5\cos 60° + 0.30v_x$, so $0.60 = 0.15 + 0.30v_x$ and $v_x = 1.5$ m s^{-1}.
- y: $0 = 0.20 \times 1.5 \sin 60° + 0.30v_y$, so $v_y = -0.87$ m s^{-1} (south).
- $v = \sqrt{1.5^2 + 0.87^2} = 1.7$ m s^{-1} at $\tan^{-1}(0.87/1.5) = 30°$ south of east.
- KE: 0.90 J before, $0.225 + 0.45 = 0.68$ J after, so the collision is inelastic.

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momentum / kg m s^{-1}
x: 0.60 = 0.15 + 0.45
y: 0 = 0.26 − 0.26

The two final momenta add (tip to tail) to the initial momentum

A two-dimensional explosion works the same way: the vector sum of all the final momenta is zero, so the momentum vectors form a closed polygon.

Circular motion

■ Moving in a circle at constant speed

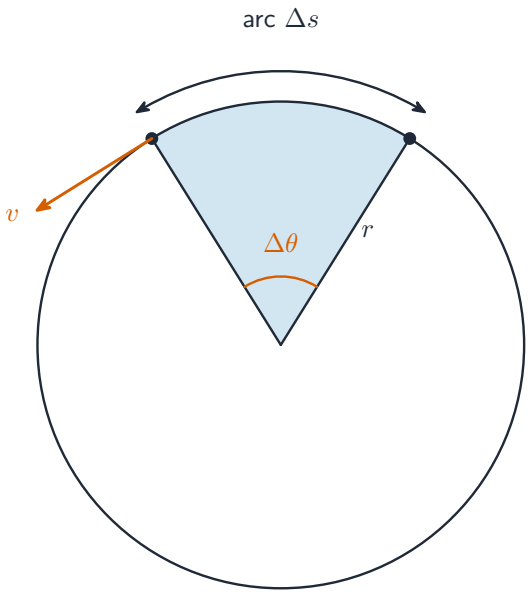
A body moving in a circle at constant speed is accelerating, because the **direction** of its velocity keeps changing. Describe the motion with:



A Ferris wheel: a centripetal force toward the centre keeps each car moving in a circle.

- the **period** 周期 T: the time for one revolution;
- the **angular velocity** 角速度 ω: the angle swept per second, in rad s^{-1}, $\omega = \dfrac{2\pi}{T}$;
- the linear speed $v = \dfrac{2\pi r}{T} = \omega r$.

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As the radius turns through $\Delta\theta$ the object moves an arc Δs at speed v

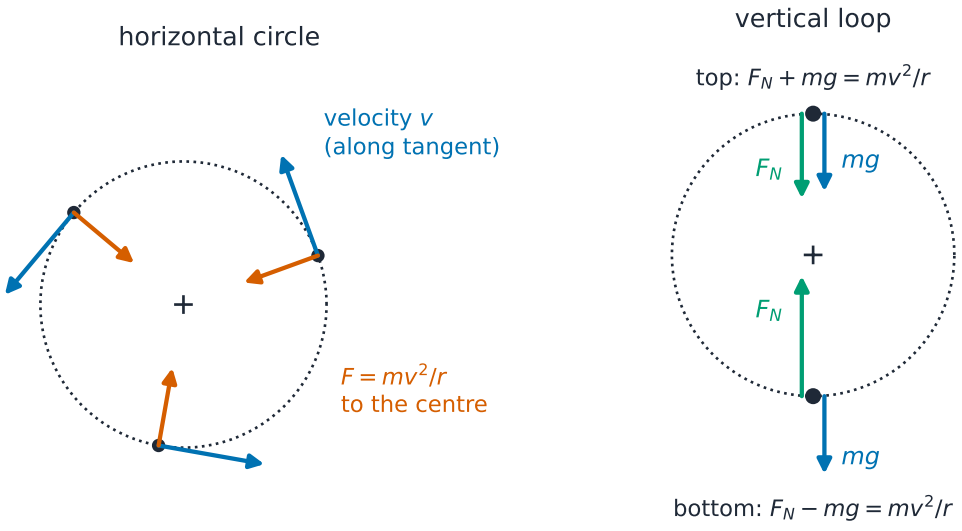
Centripetal acceleration and force

The acceleration points towards the centre of the circle: the **centripetal acceleration** 向心加速度.

$$a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$$

It needs a resultant force towards the centre, the **centripetal force** 向心力, $F = \frac{mv^2}{r}$. This is not a new kind of force: tension, friction, gravity, a normal force, or a combination of them supplies it. It acts **perpendicular** to the velocity, so it changes the direction but not the speed, and it does no work.

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Horizontal circle: force perpendicular to velocity. Vertical loop: the resultant towards the centre is mv^2/r at both top and bottom

Worked example: a stone on a string turns in a horizontal circle of radius 0.80 m, twice per second.

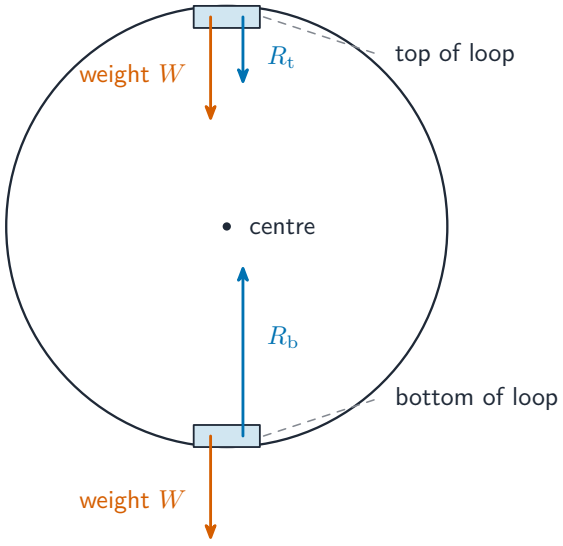
- $T = 0.50$ s; $\omega = 2\pi/0.50 = 12.6$ rad s^{-1}; $v = \omega r = 10$ m s^{-1}.
- $a = \omega^2 r = 12.6^2 \times 0.80 = 126$ m s^{-2}, about 13 times g.

A 1200 kg car rounds a level, unbanked bend of radius 20 m at 10 m s^{-1}: $a = 100/20 = 5.0$ m s^{-2}, so friction must supply $F = 6000$ N. The tyres must have $\mu_s \geq 5.0/9.8 = 0.51$; if horizontal friction were negligible, the car would continue along its instantaneous tangent instead of following the bend. Nonzero but insufficient friction can still curve its path without keeping it on the intended circle.

Vertical circles

In a vertical circle the speed changes (non-uniform circular motion), but at the top and the bottom the resultant force towards the centre is still mv^2/r.

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Forces on a person at the top and bottom of a vertical circle

- **Bottom** of a loop: $F_N - mg = \dfrac{mv^2}{r}$, so the seat pushes harder than your weight. A 500 kg roller-coaster car at 20 m s^{-1} at the bottom of a loop of radius 10 m: $F_N = 500(9.8 + 40) = 2.5 \times 10^4$ N.
- **Top** of a loop (inside): $F_N + mg = \dfrac{mv^2}{r}$. The smallest speed that keeps contact has $F_N = 0$: $v = \sqrt{gr}$. A bucket of water swung in a 1.0 m circle needs at least $\sqrt{9.8} = 3.1$ m s^{-1} at the top.
- **Over a hump** of radius r: $mg - F_N = \dfrac{mv^2}{r}$. Above $v = \sqrt{gr}$ (12 m s^{-1} for $r = 15$ m) a car leaves the road.

Practice

1. A book rests on a table. Name the third-law partner of (a) its weight, (b) the normal force on it.
2. A 3.0 kg block is pulled by a horizontal 25 N force across a floor with $\mu_d = 0.30$. Find its acceleration.
3. A spring stretches 6.0 cm under a 3.0 N load. Find k, and the extension under 5.0 N (assume the spring remains in its linear proportional range).
4. A 1.0 kg stone of volume 4.0×10^{-4} m^3 hangs from a string, fully under water. Find the buoyancy and the tension.
5. A 0.15 kg ball moving at 30 m s^{-1} is caught and stopped in 0.050 s. Find the average force, and explain why a catcher pulls the hands back.
6. A force–time graph for a kick is a triangle with a peak of 400 N lasting 0.020 s in total. Find the impulse, and the speed of a 0.080 kg ball kicked from rest.

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7. A 3000 kg railway truck at 2.0 m s^{-1} couples with a stationary 2000 kg truck. Find their common velocity and the kinetic energy transferred.
8. Two skaters (60 kg and 40 kg) stand together at rest and push apart. The 60 kg skater moves at 2.0 m s^{-1}. Find the other's velocity and the total kinetic energy gained.
9. A fire hose ejects 3.0 kg of water per second at 20 m s^{-1}. Find the force needed to hold the hose still.
10. A 1000 kg car takes a level, unbanked bend of radius 50 m at 15 m s^{-1}. Find the centripetal force and name what provides it.
11. A 0.20 kg ball on a 0.50 m string moves in a vertical circle at 4.0 m s^{-1} at the top. Find the tension there.
12. **HL.** A 2.0 kg ball moving east at 5.0 m s^{-1} hits a stationary 3.0 kg ball. Afterwards the 2.0 kg ball moves north at 2.0 m s^{-1}. Find the velocity of the 3.0 kg ball and decide whether the collision is elastic.

Answers.

1. (a) The book pulling the Earth upwards (gravitational). (b) The book pushing down on the table (contact).
2. $F_f = 0.30 \times 3.0 \times 9.8 = 8.8$ N; $a = (25 - 8.8)/3.0 = 5.4$ m s^{-2}.
3. $k = 3.0/0.060 = 50$ N m^{-1}; $x = 5.0/50 = 0.10$ m.
4. $F_b = 1000 \times 4.0 \times 10^{-4} \times 9.8 = 3.9$ N; $T = 9.8 - 3.9 = 5.9$ N.
5. $F = 0.15 \times 30/0.050 = 90$ N. Pulling back increases the stopping time, so the same momentum change needs a smaller force.
6. $J = \frac{1}{2}(400)(0.020) = 4.0$ N s; $v = 4.0/0.080 = 50$ m s^{-1}.
7. $v = 6000/5000 = 1.2$ m s^{-1}; KE $6000 - 3600 = 2400$ J transferred.
8. $0 = 60(2.0) + 40v$, $v = -3.0$ m s^{-1} (opposite direction); KE $= 120 + 180 = 300$ J, from the skaters' muscles.
9. $F = 3.0 \times 20 = 60$ N, forwards on the hose (the water pushes it back).
10. $F = 1000 \times 15^2/50 = 4500$ N, supplied by friction between the tyres and the road.
11. $T + mg = mv^2/r$: $T = 0.20 \times 16/0.50 - 0.20 \times 9.8 = 6.4 - 2.0 = 4.4$ N.
12. **HL.** x: $10 = 3.0v_x$, $v_x = 3.3$ m s^{-1}. y: $0 = 4.0 + 3.0v_y$, $v_y = -1.3$ m s^{-1}. $v = 3.6$ m s^{-1} at 22° south of east. KE: 25 J before, $4.0 + 19.3 = 23$ J after, so it is inelastic.

Exam tips

- On a free-body diagram, draw only forces **on** the chosen body; start each arrow on the body and label it with a standard name or symbol.
- A third-law pair acts on two different bodies and is the same type of force; weight and normal force on one body are never a pair.

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- Static friction is "up to" $\mu_s F_N$; use the equals sign only when sliding is about to start.
- Give momentum a sign. A rebound changes the sign of the velocity, so $\Delta p = m(v - u)$ is often larger than either momentum.
- After a collision question, check the kinetic energy: it can stay the same (elastic) or fall (inelastic), but an increase requires energy transferred from another store, as in an explosion.
- Centripetal force is the **resultant** towards the centre, never an extra force on the diagram.
- **HL.** In two dimensions, write one momentum equation for x and one for y; never add speeds in different directions.

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Work, energy and power

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches A.3 Work, energy and power in full, for the course first assessed in 2025. There is no additional higher level content in A.3, so SL and HL students study the same material.

Prerequisites: forces, momentum and free-body diagrams from A.2; the equations of motion from A.1; resolving a force into components. Use $g = 9.8\ \text{m s}^{-2}$.

Work and energy transfer

Where does the energy go?

A roller coaster is pulled slowly up a 20 m hill, then released. It races down, climbs the next hill and slows, and at the end brakes stop it completely. At every moment the energy is somewhere: in the motion, in the height, in the warm brakes and the noise. Energy is never made or destroyed, only moved between stores. This is the principle of **conservation of energy** 能量守恒: the total energy of an isolated system stays constant.



A coaster train hauled up the lift hill: the work done against gravity is stored as gravitational potential energy

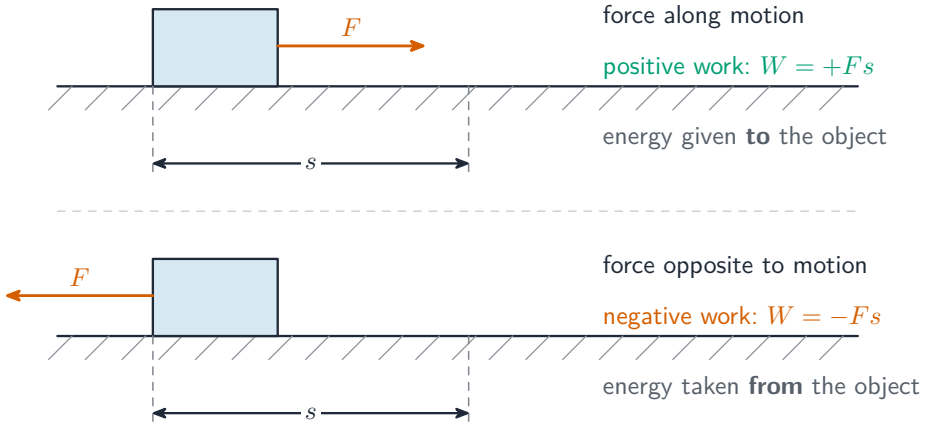
Work is energy transferred by a force

When a force moves its point of application, it does **work** 功 and transfers energy. For a constant force F that moves a body through a displacement s:

$$W = Fs\cos\theta$$

θ is the angle between the force and the displacement, so only the component of the force **along** the displacement does work. Work is a scalar measured in joules (1 J = 1 N m).

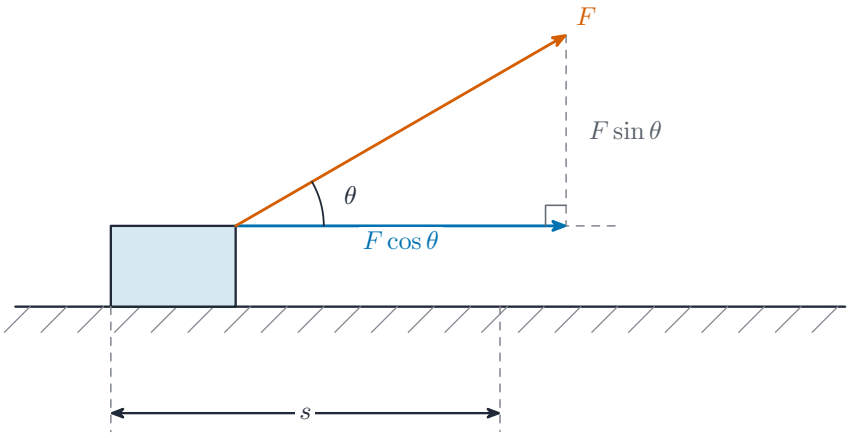
- $\theta = 0$: all of the force does work, $W = Fs$.
- $\theta = 90°$: no work. A centripetal force, or the normal force on a body sliding along a level floor, does no work.
- $\theta > 90°$: the work is negative; the force takes energy **away** from the body. Friction on a sliding body does negative work.



Positive work ($W = +Fs$): force along the motion (top). Negative work ($W = -Fs$): force opposite to the motion, e.g. friction (bottom)

Worked example: a suitcase is pulled 20 m along level ground by a 50 N force at 30° above the horizontal. $W = 50 \times 20 \times \cos 30° = 870$ J. The vertical component of the pull does no work, because the suitcase does not move vertically.

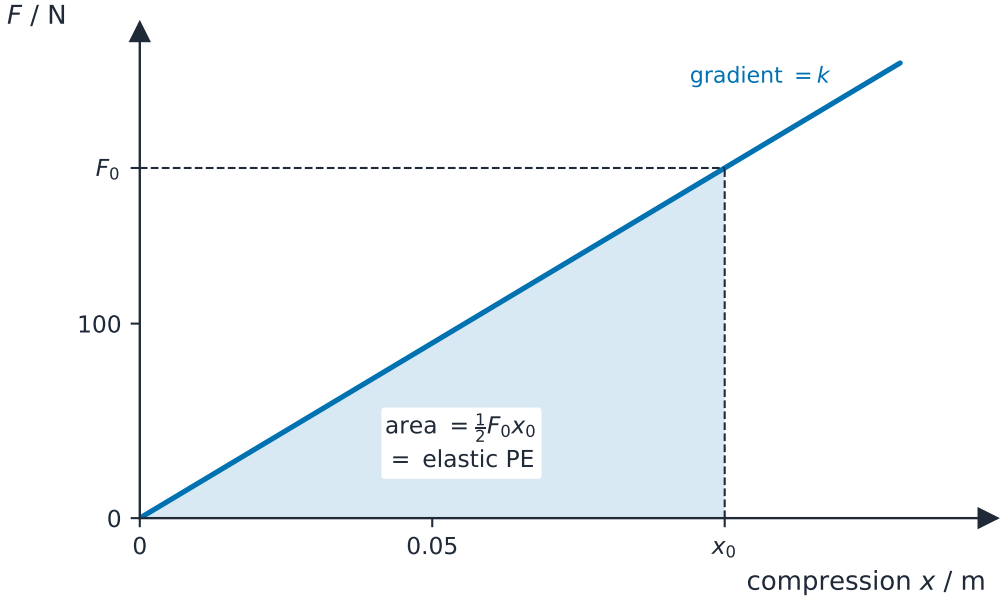
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Only the component of the force along the displacement ($F\cos\theta$) does work

■ Work from a force–displacement graph

For motion along a fixed axis, plot the force component parallel to that axis against displacement. Work is the **signed area under the force–displacement graph**: area below the axis contributes negative work. A graph of force magnitude alone can lose the direction information. For a Hookean spring, the magnitude of a slow stretching force rises linearly from zero, so the supplied work is a triangular area.



The energy stored in a spring is the area under its force–compression graph: $\frac{1}{2}F_0x_0$, which equals $\frac{1}{2}kx_0^2$

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The work–energy principle

For the constant-mass, non-relativistic translational model here, work done by the **resultant** force equals the change in kinetic energy. Include every force in this net work; the work of one applied force need not equal the kinetic-energy change:

$$W_{\text{net}} = \Delta E_k$$

A 1200 kg car speeds up from 10 m s^{-1} to 20 m s^{-1}. $\Delta E_k = \frac{1}{2}(1200)(20^2 - 10^2) = 1.8 \times 10^5$ J. If this happens over 100 m, the average resultant force is $1.8 \times 10^5 / 100 = 1800$ N, found without any time or acceleration.

Mechanical energy stores

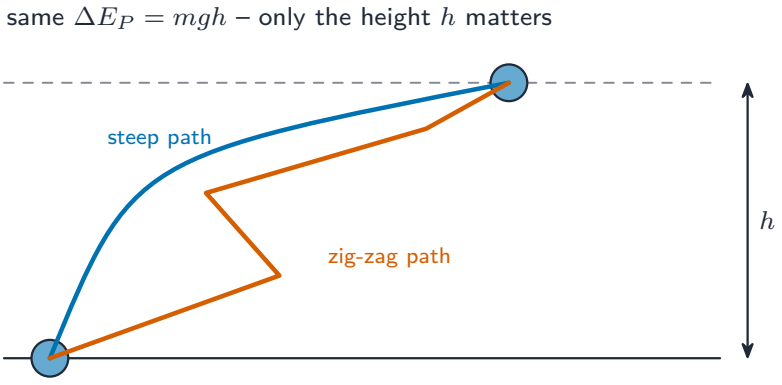
Three stores

Mechanical energy 机械能 is the sum of kinetic, gravitational potential and elastic potential energy.

Store	Equation	Notes
kinetic energy 动能	$E_k = \frac{1}{2}mv^2 = \dfrac{p^2}{2m}$	doubling speed quadruples E_k
gravitational potential energy 重力势能	$\Delta E_p = mg\Delta h$	only near the Earth's surface, where g is constant
elastic potential energy 弹性势能	$E_H = \frac{1}{2}k(\Delta x)^2$	linear elastic spring, measured from its unstretched length

The form $E_k = p^2/2m$ links energy to momentum: a 1200 kg car with $p = 18\,000$ kg m s^{-1} has $E_k = 18\,000^2/2400 = 1.35 \times 10^5$ J.

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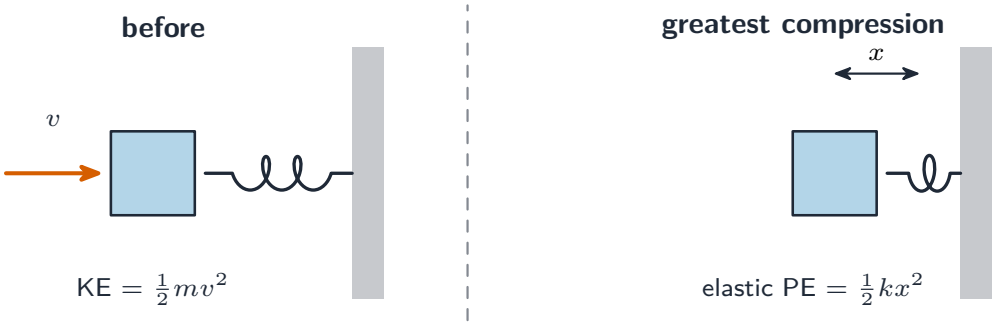


The change in gravitational potential energy depends only on the height change, not on the path

■ Elastic energy in a spring

The expression $E_H = \frac{1}{2}k(\Delta x)^2$ assumes a constant spring stiffness in the linear elastic range and zero stored energy at the unstretched length. During slow stretching from zero extension, the applied force does positive work; the spring restoring force does negative work. The magnitude of each is the stored-energy increase. Beyond the linear range, use the actual force–extension area rather than this formula.

A spring with $k = 400$ N m^{-1} is compressed by 0.10 m: $E_H = \frac{1}{2}(400)(0.10^2) = 2.0$ J. If it launches a 0.050 kg ball and all this energy becomes kinetic energy, $v = \sqrt{2(2.0)/0.050} = 8.9$ m s^{-1}.



Kinetic energy and elastic potential energy exchange through a spring

Conservation of mechanical energy

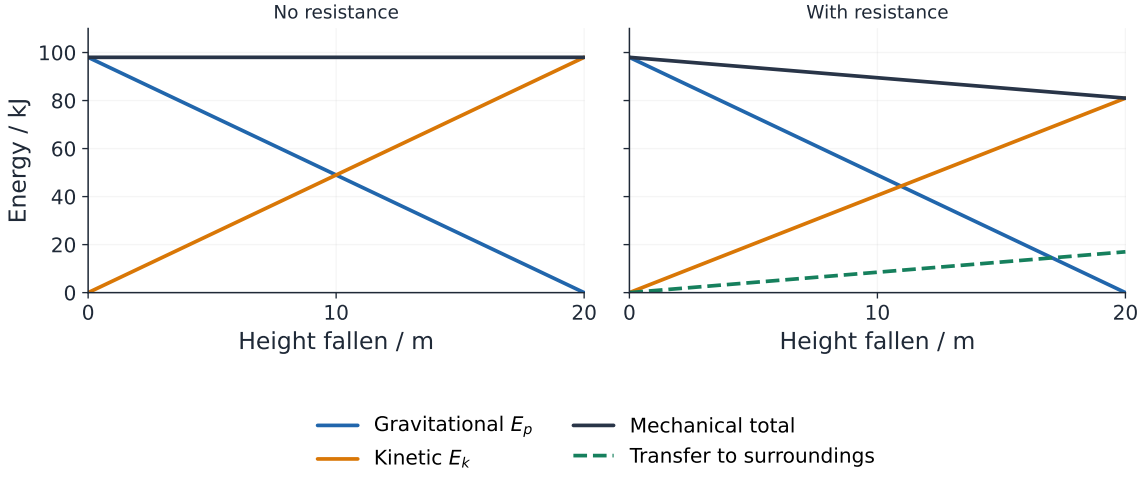
■ Without friction

Choose the mechanical system and include its relevant gravitational and spring potential energies. Mechanical energy stays constant when there is no net work from

forces outside those included conservative interactions. No friction is not enough if an engine, a person or another external agent supplies work. With no such transfer, gravity or a spring exchanges energy between the included stores.

$$E_k + E_p + E_H = \text{constant}$$

This method avoids needing a time or acceleration history. It can be especially useful on a curved path, but curvature alone does not invalidate kinematics: the constant-acceleration equations require constant acceleration in the chosen direction.

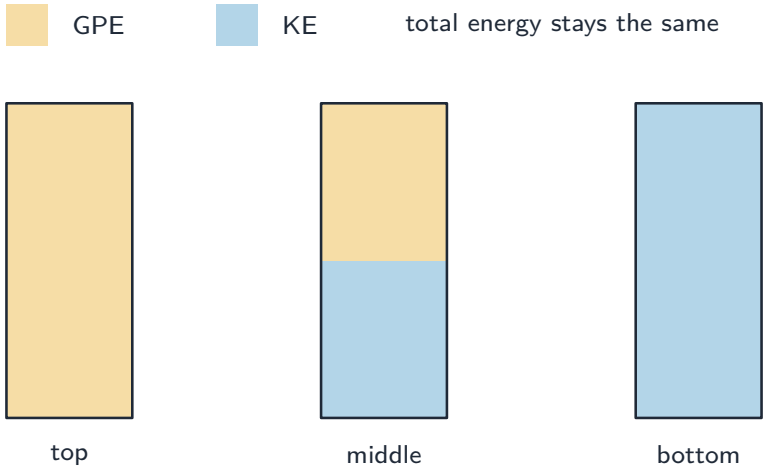


These illustrative models use the bottom as zero gravitational potential energy. The resistive case assumes dissipated energy proportional to height fallen; it is not a universal friction law.

Worked example: a roller-coaster car starts from rest at the top of a 20 m drop.

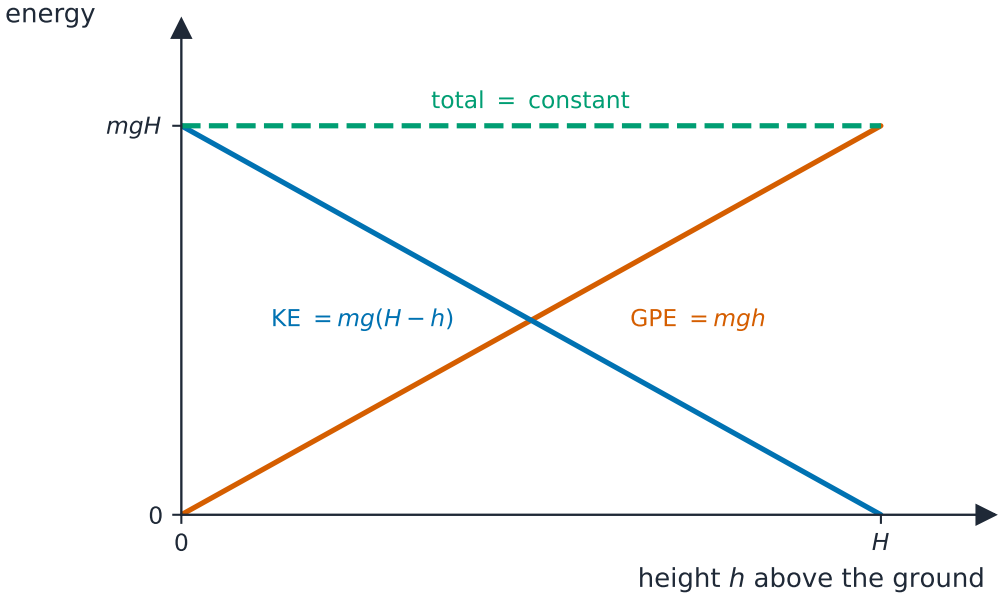
- Energy at the top $= mgh$; at the bottom $= \frac{1}{2}mv^2$. So $v = \sqrt{2gh} = \sqrt{2(9.8)(20)} = 20\ \text{m s}^{-1}$.
- The mass cancels: an empty car and a full car reach the same speed.

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Without resistance the total stays fixed: gravitational potential energy becomes kinetic energy

For a feasible release height of 0.20 m, a point-like pendulum bob with a taut string and negligible losses reaches $\sqrt{2(9.8)(0.20)} = 2.0$ m s^{-1} at the bottom. Length does not enter this energy calculation once the height change is specified.



A falling ball: GPE and KE both change in a straight line with height, and their sum is constant

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With friction and resistance

Friction and air resistance are **non-conservative** forces: they transfer mechanical energy to thermal energy in the body and its surroundings. For a system whose gravitational and elastic interactions are included, the mechanical-energy change equals work from the remaining interactions. Resistive work can reduce it; an external driving force can increase it. In the following descent, assume resistance is the only such transfer.

Worked example: the 500 kg car above actually reaches the bottom at 18 m s^{-1} after 60 m of track.

- Mechanical energy at the top: $mgh = 500 \times 9.8 \times 20 = 98\,000$ J.
- At the bottom: $\frac{1}{2}(500)(18^2) = 81\,000$ J.
- Work done against friction = 17 000 J; average resistive force = $17\,000/60 = 280$ N.

The 17 000 J is not lost: it is dissipated into thermal energy and sound. Total energy is still conserved.

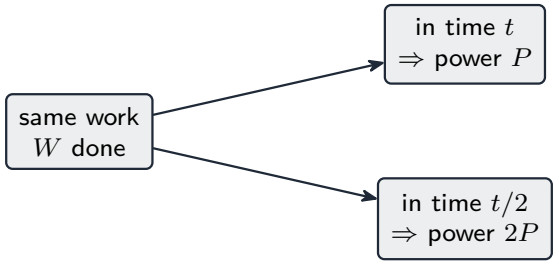
Power

The rate of energy transfer

Power 功率 is the rate at which work is done, or energy is transferred. Average power over a time interval is:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t}$$

It is measured in watts (1 W = 1 J s^{-1}). Power describes how **fast** energy moves, not how much: a 2 kW kettle and a 1 kW kettle can transfer the same energy, the second taking twice as long.



$P = \dfrac{W}{t}$ — less time, more power

The same work done in less time means more power

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Power and velocity

The instantaneous power of a force is its component along the point-of-application velocity multiplied by speed. If the angle is θ, $P = Fv\cos\theta$. For parallel force and velocity this becomes:

$$P = Fv$$

Constant velocity is not required for this instantaneous relation. Average power over an interval is total work divided by elapsed time; do not generally substitute average force and average speed into a product and assume the same result.

A car moving at a steady 25 m s^{-1} against 600 N of resistive forces needs a driving force of 600 N (zero resultant force), so its engine delivers $P = 600 \times 25 = 15$ kW to the wheels. Since air resistance rises steeply with speed, the power needed rises even faster.



A hydroelectric station is rated in megawatts: the energy of the stored water, divided by the time it takes to release it

In the lab: to measure the output power of a small electric motor, let it lift a known load through a measured height, time the lift with a stopwatch or light gates, and record the current and potential difference supplied to it (B.5). Repeat the trials. The energy that does not lift the load is transferred as heating and friction; account for it, do not call it "missing".

Efficiency and Sankey diagrams

Efficiency

Efficiency 效率 compares the useful output with the total input:

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$$\eta = \frac{E_{\text{output}}}{E_{\text{input}}} = \frac{P_{\text{output}}}{P_{\text{input}}}$$

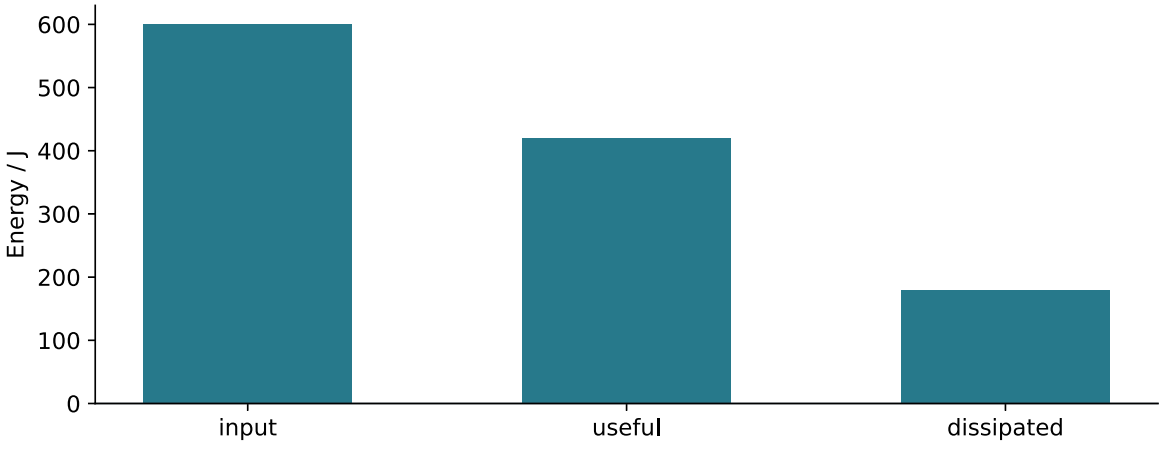
It has no unit. For useful output divided by all relevant energy input over the same interval, $0 \le \eta \le 1$. A chosen ideal model, or a boundary counting all input as useful output, can give 100%; real machines often give less for the specified useful transfer. Energy transferred elsewhere is not destroyed. Decide what counts as **useful** before calculating: the useful output of a lamp is light, of a heater is thermal energy.

Worked example: a crane motor lifts a 500 kg load 12 m in 20 s while taking 4.2 kW of electrical power.

- Useful power $= \dfrac{mgh}{t} = \dfrac{500 \times 9.8 \times 12}{20} = 2940$ W.
- $\eta = 2940/4200 = 0.70$, or 70%.

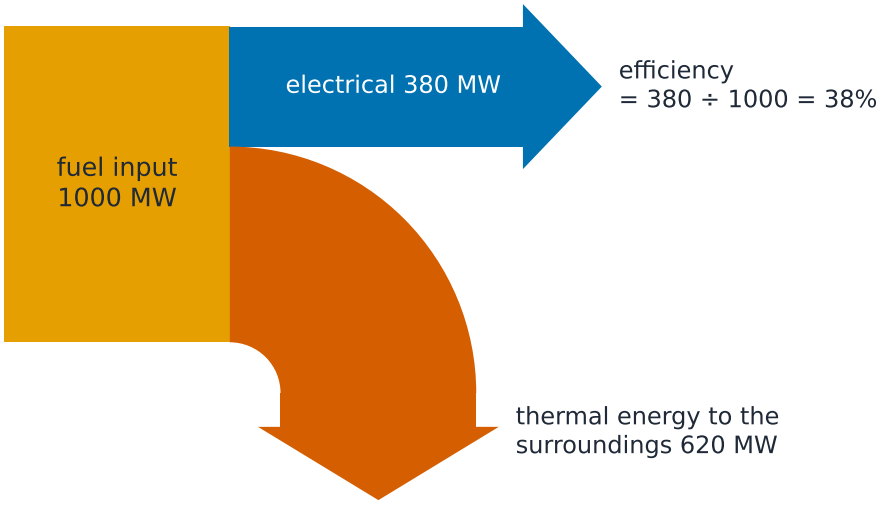
Sankey diagrams

A **Sankey diagram** 桑基图 shows energy transfers as arrows whose widths are proportional to the energy (or power). The input arrow splits into a useful output arrow and arrows for the wasted transfers, which usually bend away. The widths must add up: the diagram shows conservation of energy at a glance.



Efficiency = useful ÷ input = 420 ÷ 600 = 70%; the other 180 J is dissipated

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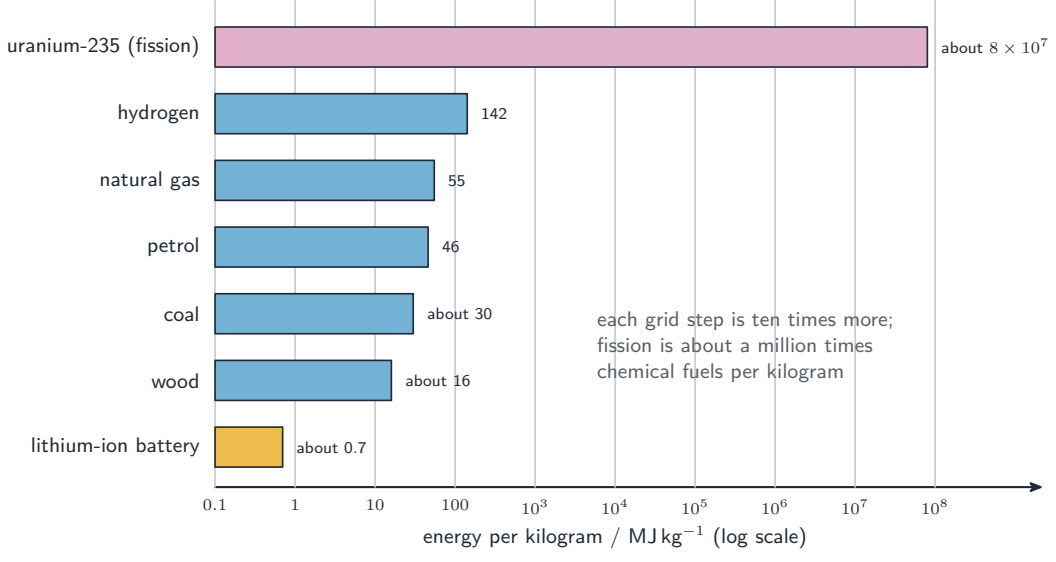


Arrow widths are to scale: 1000 MW in = 380 MW useful + 620 MW to the surroundings

Energy density of fuels

Energy stored per kilogram

The **energy density** 能量密度 of a fuel is the energy released per kilogram (J kg^{-1}). Some books call energy per kilogram *specific energy* and use energy density for energy per cubic metre (J m^{-3}); check the unit in the question.



On a log scale each grid step is ten times more: fission releases about a million times more energy per kilogram than chemical fuels

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Fuel or store	Energy per kg / MJ kg^{-1}
wood	about 16
coal	about 30
petrol	46
natural gas (methane)	55
hydrogen	142
lithium-ion battery	about 0.7
uranium-235 (fission)	about 8×10^7

These rounded values illustrate magnitudes, not universal sample specifications. Fuel composition and the water-product phase affect combustion energy; battery charge state, chemistry and the mass boundary affect stored energy per kilogram. Use the supplied value and definition in a calculation.

Fission energy per kilogram is roughly a million times chemical-fuel values because nuclear binding-energy changes are much larger per particle. A battery stores less per kilogram than petrol in these examples, but whole-vehicle mass also depends on drivetrain efficiency, design and required range; fuel energy alone does not determine it.

Worked example: a coal power station delivers 500 MW of electrical power at 38% efficiency, burning coal with an energy density of 30 MJ kg^{-1}.

- Input power $= 500/0.38 = 1316$ MW.
- Mass of coal per second $= 1316 \times 10^6/(30 \times 10^6) = 44$ kg s^{-1}, about 3800 tonnes per day.

Practice

1. A box is pushed 5.0 m across a floor by a 40 N force. Find the work done when the force is (a) horizontal, (b) at 60° to the horizontal.
2. In an ideal model a bag moves horizontally at fixed height and constant speed. Its supporting force from the person is vertical. Find the work done by that supporting force. Does this imply the person expends no metabolic energy?
3. A 0.50 kg ball is thrown vertically up at 12 m s^{-1}. Use energy to find its maximum height.
4. A 70 kg skier starts from rest and descends a slope with a vertical drop of 40 m. Friction and air resistance do 6000 J of work against the skier. Find the speed at the bottom.
5. A spring ($k = 250$ N m^{-1}) is compressed by 8.0 cm and launches a 20 g dart horizontally. Find the dart's speed.
6. A 2.0 kg trolley has momentum 6.0 kg m s^{-1}. Find its kinetic energy without finding its speed.

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7. The magnitude of a slow stretching force rises linearly from zero to 12 N at 6.0 cm extension, within the elastic range. Find the energy stored and the work done by the spring during that extension.
8. A motor lifts an 80 kg mass through 6.0 m in 12 s. The electrical input power is 600 W. Find the useful power and the efficiency.
9. A cyclist developing 300 W rides at a steady 10 m s^{-1}. Find the total resistive force.
10. A power station takes in 1000 MW from fuel and delivers 380 MW of electrical power. Find the efficiency and sketch a Sankey diagram.
11. A 25%-efficient car engine delivers 15 kW for one hour. Petrol releases 46 MJ kg^{-1}. Find the mass of petrol burned.
12. A 3.0 kg trolley starts from rest on a horizontal frictionless track. A constant 10 N horizontal push acts through 4.0 m. Find the mechanical-energy change and final speed. Explain why "no friction means constant mechanical energy" fails for the trolley system.
13. At an instant an accelerating body moves horizontally at 5.0 m/s. A 120 N applied force acts 30° above the horizontal. Find the instantaneous power of this force. Is it necessarily the net power changing kinetic energy? Explain.
14. For a 2.0 kg trolley moving along +x, the parallel force component is +12 N from x = 0 to 4.0 m and −4.0 N from x = 4.0 to 6.0 m. Initial speed is 4.0 m/s. Find the signed net work and the final speed, neglecting other work.

Answers.

1. (a) $40 \times 5.0 = 200$ J. (b) $40 \times 5.0 \times \cos 60° = 100$ J.
2. The specified supporting force is perpendicular to the horizontal displacement: $W = Fs\cos 90° = 0$. This is mechanical work on the bag, not the person's metabolic energy expenditure. Real walking can include vertical motion, which this ideal model excludes.
3. $\frac{1}{2}mv^2 = mgh$, so $h = 12^2/(2 \times 9.8) = 7.3$ m; the mass cancels.
4. $mgh = 70 \times 9.8 \times 40 = 27\,440$ J; $E_k = 27\,440 - 6000 = 21\,440$ J; $v = \sqrt{2(21\,440)/70} = 25$ m s^{-1}.
5. $E_H = \frac{1}{2}(250)(0.080^2) = 0.80$ J; $v = \sqrt{2(0.80)/0.020} = 8.9$ m s^{-1}.
6. $E_k = p^2/2m = 36/4.0 = 9.0$ J.
7. Stored energy equals positive stretching work: $E_H = \frac{1}{2}F_{\max}x = \frac{1}{2}(12)(0.060) = 0.36$ J. The spring force opposes extension, so its work is −0.36 J.
8. Useful power $= 80 \times 9.8 \times 6.0/12 = 390$ W; $\eta = 392/600 = 0.65$ (65%).
9. At constant speed the driving force equals the resistance: $F = P/v = 300/10 = 30$ N.
10. $\eta = 380/1000 = 38\%$. Sankey: a 1000 MW input arrow splitting into a 380 MW electrical arrow and a 620 MW arrow to the surroundings, widths in the ratio 38 : 62.

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11. Useful energy $= 15\,000 \times 3600 = 54$ MJ; input $= 54/0.25 = 216$ MJ; mass $= 216/46 = 4.7$ kg.
12. Work supplied is $W = Fs = 10(4.0) = 40$ J. With no height or spring change, mechanical energy rises by 40 J. From $\frac{1}{2}mv^2 = 40$, $v = \sqrt{80/3.0} = 5.2$ m/s. The person or device pushing transfers energy into the chosen trolley system despite the absence of friction.
13. $P = Fv\cos\theta = 120(5.0)\cos 30° = 5.2 \times 10^2$ W. This is an instantaneous relation, valid during acceleration. It is the power of the stated force; other forces may also do work, so net kinetic-energy power requires the resultant or the sum of individual powers.
14. Signed work is $W = 12(4.0) + (-4.0)(2.0) = 40$ J. Initial $E_k = \frac{1}{2}(2.0)(4.0)^2 = 16$ J; final $E_k = 16 + 40 = 56$ J. Thus $v = \sqrt{2(56)/2.0} = 7.5$ m/s. Adding the two unsigned areas would incorrectly give 56 J of work.

Exam tips

- In $W = Fs\cos\theta$, θ is the angle between the force and the **displacement**, not the angle of a slope.
- Net work changes kinetic energy. To conserve mechanical energy, also rule out external driving work and include the relevant potential-energy interactions.
- If resistance is the only transfer out of the chosen mechanical stores, write: initial mechanical energy = final mechanical energy + energy dissipated. Include any driving work separately.
- Energy is never "lost". Say it is **dissipated** or transferred to thermal energy of the surroundings.
- Define useful output, all relevant inputs and the system boundary before finding efficiency; the stated conversion-efficiency ratio cannot exceed 100%.
- Instantaneous $P = Fv\cos\theta$ does not require constant speed. A driving force equals the opposing resistance only when the relevant resultant force is zero.
- Check units: kJ, MJ and kW h appear in questions, and 1 kW h = 3.6 MJ.

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Rigid body mechanics

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches A.4 Rigid body mechanics, for the course first assessed in 2025. The whole unit is additional higher level (AHL); there is no SL counterpart.

Prerequisites: Newton's laws, momentum and circular motion ($v = \omega r$) from A.2; kinetic and potential energy from A.3; the equations of motion from A.1. Use $g = 9.8\ \text{m s}^{-2}$. Formulas for the moment of inertia of particular shapes are given in questions when needed.

AHL: Torque and rotational equilibrium

Why is a door handle far from the hinge?

Push a door near its hinge and it barely moves; push the same force at the handle and it swings open easily. What matters for turning is not only the force but **where** and **in which direction** it acts. A **rigid body** 刚体 is an extended object whose shape does not change, so it can rotate as well as move along.



A tower crane is a moments problem: the counterweight's moment about the mast balances the moment of the load

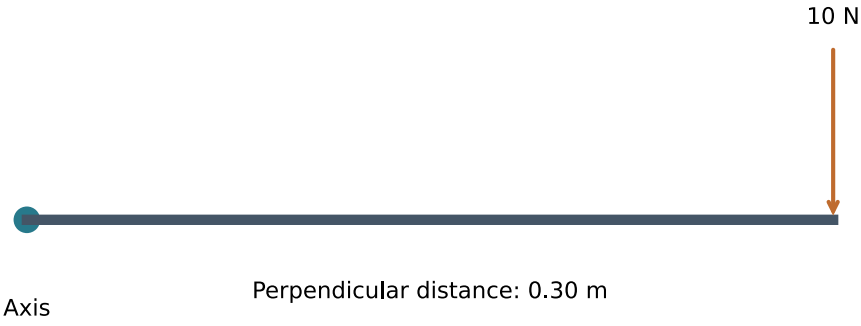
Torque

The **torque** 力矩 τ of a force about an axis is its turning effect:

$$\tau = Fr\sin\theta$$

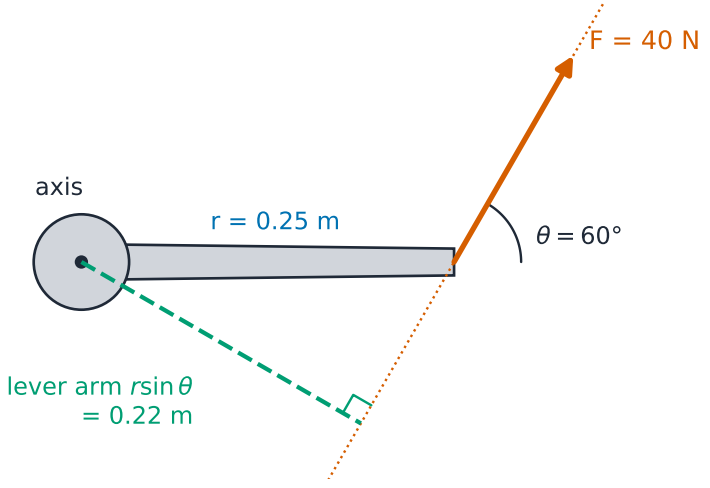
r is the distance from the axis to the point where the force acts, and θ is the angle between r and the force. The product $r\sin\theta$ is the **perpendicular distance** from the axis to the line of action of the force, the **lever arm** 力臂. Torque is measured in N m. Always state its **sense**: clockwise or anticlockwise.

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Torque = 3.0 N m

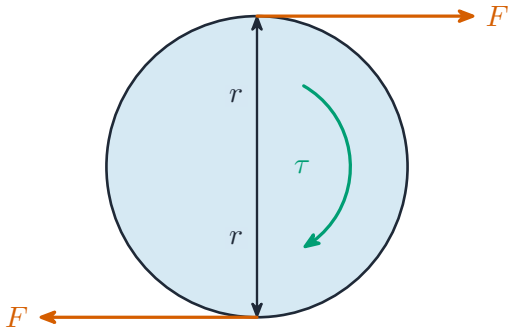
$$Torque = force \times perpendicular\ distance = 10\ N \times 0.30\ m = 3.0\ N\ m$$



Only the perpendicular distance to the line of action counts: $\tau = 40 \times 0.25 \times \sin 60° = 8.7\ N\ m$

A force whose line of action passes through the axis ($\theta = 0$) has no turning effect, however large it is. Two equal and opposite forces with different lines of action form a **couple** 力偶: they give a torque but no resultant force.

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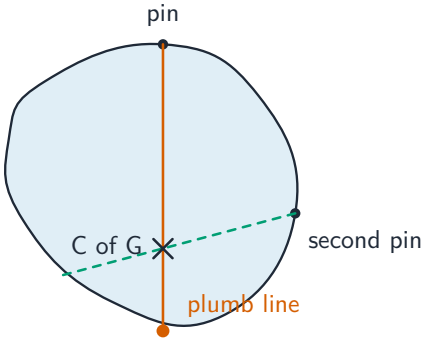
A couple: two equal and opposite forces, a distance apart, producing a torque

Rotational equilibrium

A body in **rotational equilibrium** 转动平衡 about a chosen fixed axis has zero resultant torque about that axis: clockwise and anticlockwise torques balance. Complete equilibrium requires both conditions below; with zero resultant force, torque balance then holds about **any** axis:

- zero resultant force (translational equilibrium, A.2), and
- zero resultant torque.

For forces such as weight, the whole mass of a body can be treated as acting at its **centre of mass** 质心. For a uniform beam that is the middle.



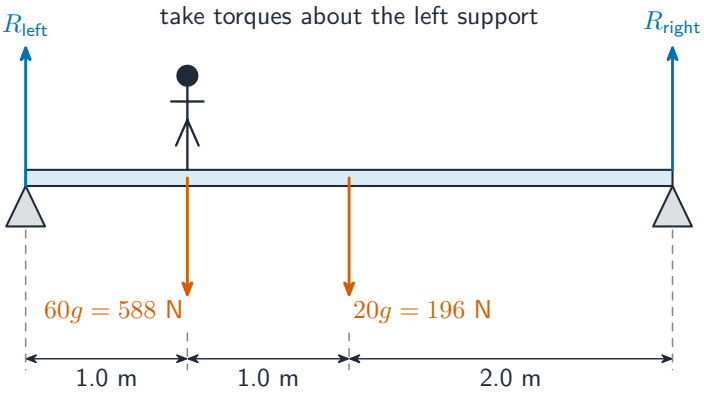
hanging freely, the sheet settles with its centre of gravity vertically below the pin

hang it from a second point: the two plumb lines cross at the centre of gravity

Finding a centre of gravity: hung freely, an object settles with its centre of gravity vertically below the pin, so two plumb lines from two pins cross at it

Worked example: a uniform 4.0 m plank of mass 20 kg rests on supports at each end. A 60 kg person stands 1.0 m from the left end. Find the support forces.

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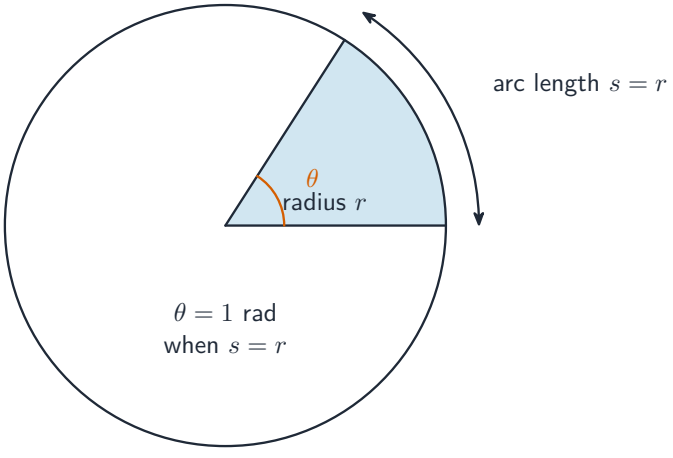
Taking torques about the left support removes R_{left} from the torque equation

- Take torques about the **left** support, so its unknown force has no torque.
- Clockwise: $(20 \times 9.8)(2.0) + (60 \times 9.8)(1.0) = 392 + 588 = 980$ N m.
- Anticlockwise: $R_{\text{right}} \times 4.0$. Equilibrium gives $R_{\text{right}} = 245$ N.
- Forces: $R_{\text{left}} = 80 \times 9.8 - 245 = 539$ N. The support nearer the person carries more.

AHL: Describing rotation

Angular quantities

Every point of a rotating rigid body turns through the same angle in the same time, so rotation is described with angular quantities, measured in **radians** (one revolution $= 2\pi$ rad).



One radian is the angle whose arc length equals the radius

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Linear	Angular	Link for a point at radius r
displacement s	**angular displacement** 角位移 $\Delta\theta$ (rad)	$s = r\Delta\theta$
velocity v	angular velocity ω (rad s^{-1})	$v = \omega r$
acceleration a	**angular acceleration** 角加速度 α (rad s^{-2})	$a = \alpha r$ (along the path)

For rotation about a fixed axis, choose one sense as positive. Angular velocity has a sign in that convention; angular speed is its magnitude. The link $v = \omega r$ gives tangential speed when the magnitudes are used.

Equations of motion for uniform angular acceleration

Each linear equation has an angular twin. They hold only when α is constant.

$$\omega_f = \omega_i + \alpha t \qquad \Delta\theta = \frac{\omega_f + \omega_i}{2}\, t$$

$$\Delta\theta = \omega_i t + \tfrac{1}{2}\alpha t^2 \qquad \omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Worked example: a ceiling fan turning at 30 rad s^{-1} is switched off and slows uniformly to rest over 40 revolutions.

- $\Delta\theta = 40 \times 2\pi = 251$ rad; $\omega_f = 0$.
- $0 = 30^2 + 2\alpha(251)$, so $\alpha = -1.8$ rad s^{-2} (decelerating).

AHL: Moment of inertia and Newton's second law for rotation

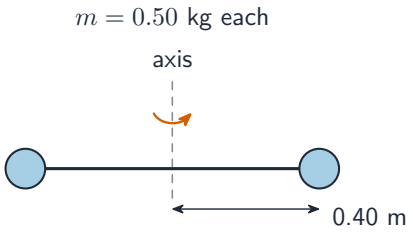
Moment of inertia

Mass resists changes in linear motion. For rotation, what matters is how the mass is **distributed** about the axis. The **moment of inertia** 转动惯量 I of a set of point masses is

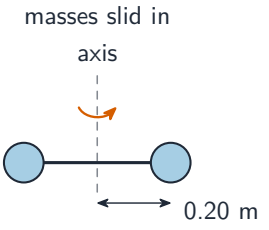
$$I = \sum mr^2$$

measured in kg m^2. Mass far from the axis counts much more than mass near it, because r is squared. Two 0.50 kg masses fixed 0.40 m either side of an axis on a light rod give $I = 2(0.50)(0.40^2) = 0.16$ kg m^2. Slide them in to 0.20 m and I falls to a quarter, 0.040 kg m^2.

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$I = 2mr^2 = 0.16$ kg m^2



$I = 2mr^2 = 0.040$ kg m^2

Halving r quarters I, because $I = \sum mr^2$

For a body of mass M and radius (or length) R, the formula depends on the shape and the axis, for example: thin ring about its centre MR^2; solid disc or cylinder $\frac{1}{2}MR^2$; solid sphere $\frac{2}{5}MR^2$; thin rod about its centre $\frac{1}{12}ML^2$. Questions give the formula you need.

■ Newton's second law for rotation

An unbalanced torque gives a rigid body an angular acceleration:

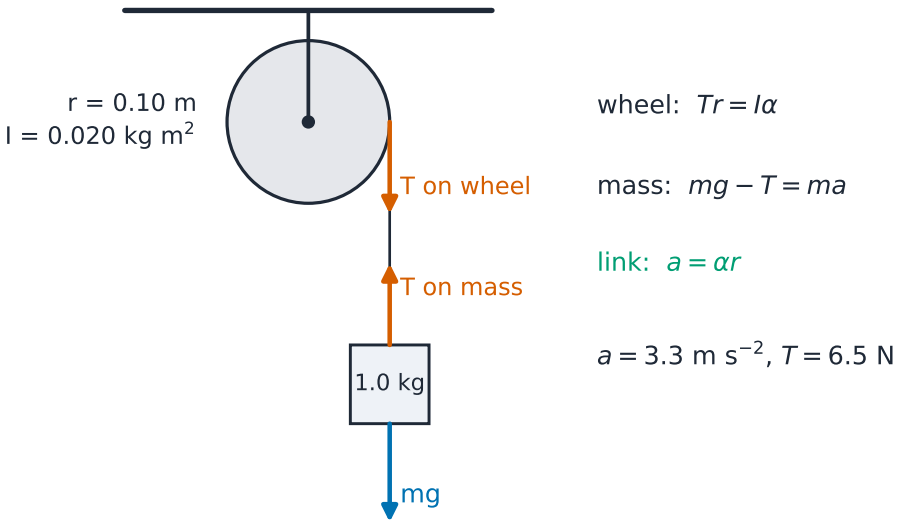
$$\tau = I\alpha$$

For a rigid body with constant I about a fixed axis, τ is the instantaneous resultant torque; average torque gives average angular acceleration over an interval. A 3.0 N m torque on a flywheel with $I = 0.60$ kg m^2 gives $\alpha = 3.0/0.60 = 5.0$ rad s^{-2}. The same torque on a body with a larger moment of inertia gives a smaller angular acceleration.

■ Coupled bodies: a mass on a pulley

A 1.0 kg mass hangs from a light string wrapped round a pulley wheel of radius 0.10 m and moment of inertia 0.020 kg m^2. Treat the two bodies separately and link them: the string does not slip, so $a = \alpha r$.

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The tension gives the wheel its torque and slows the mass: solve the two bodies together

- Mass (down positive): $mg - T = ma$, so $9.8 - T = 1.0a$.
- Wheel: $Tr = I\alpha = I\dfrac{a}{r}$, so $T = \dfrac{Ia}{r^2} = \dfrac{0.020a}{0.010} = 2.0a$.
- Add: $9.8 = 3.0a$, so $a = 3.3$ m s^{-2} and $T = 6.5$ N.

The mass falls with less than g because part of the energy goes into spinning the wheel.

In the lab: use a guarded pulley with its axle clamped securely. Fasten hanging loads and place a tray below them to catch a fall; keep hands and loose clothing away from rotating parts.

AHL: Angular momentum and angular impulse

Angular momentum

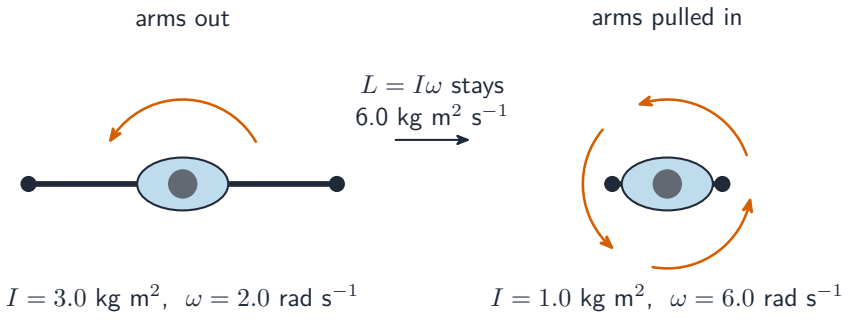
A rotating body has **angular momentum** 角动量:

$$L = I\omega$$

measured in kg m^2 s^{-1}. Angular momentum stays constant unless a resultant **external torque** acts: the **conservation of angular momentum** 角动量守恒.

A spinning ice skater pulls in their arms. The skater's moment of inertia falls from 3.0 to 1.0 kg m^2, so the angular velocity rises from 2.0 to $\dfrac{3.0 \times 2.0}{1.0} = 6.0$ rad s^{-1}. The rotational kinetic energy rises from 6.0 J to 18 J: the extra comes from the work the skater's muscles do pulling the arms inwards. Divers and gymnasts tuck to spin faster for the same reason.

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With no external torque, $L = I\omega$ is conserved: a smaller I means a larger ω

Coupled discs

A disc ($I = 0.20$ kg m^2) spinning at 30 rad s^{-1} is dropped onto a stationary disc ($I = 0.10$ kg m^2) on the same axle, and they turn together. The torques between them are internal, so L is conserved:

- $0.20 \times 30 = (0.20 + 0.10)\,\omega$, so $\omega = 20$ rad s^{-1}.
- Kinetic energy falls from 90 J to 60 J: like a perfectly inelastic collision, 30 J is transferred to thermal energy by friction between the discs.

Angular impulse

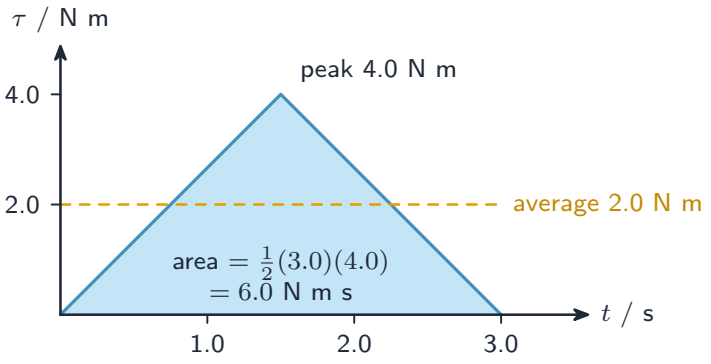
A resultant torque acting for a time Δt gives an **angular impulse** 角冲量 equal to the change in angular momentum:

$$\Delta L = \tau \Delta t = \Delta(I\omega)$$

Here τ is constant or is the average resultant external torque over the interval. For a varying torque, angular impulse is the signed area under its torque–time graph.

Take the initial rotation as positive. A brake applies a torque of -15 N m to a flywheel ($I = 2.0$ kg m^2) for 4.0 s. Its angular impulse is $\Delta L = (-15)(4.0) = -60$ kg m^2 s^{-1}. If its initial angular velocity was 50 rad s^{-1}, it slows to $50 + (-60)/2.0 = 20$ rad s^{-1}. The impulse magnitude is 60; the directed change is negative.

Worked example with invented data: a net external torque about a fixed axis rises linearly from zero to +4.0 N m, then falls linearly to zero, over a total of 3.0 s. The torque–time graph is a triangle. Its signed area is $\frac{1}{2}(3.0)(4.0) = +6.0$ N m s, so the average torque is +2.0 N m, not the peak +4.0 N m. For a constant inertia of 0.50 kg m^2, $\Delta\omega = \Delta L/I = +12$ rad s^{-1}. If the initial angular velocity is +8.0 rad s^{-1}, the final value is +20 rad s^{-1}. Using peak torque for the whole interval would double the impulse incorrectly.



Angular impulse is the area under the torque–time graph; use the average torque, not the peak.

AHL: Rotational kinetic energy and rolling

Kinetic energy of rotation

A rotating body stores **rotational kinetic energy** 转动动能:

$$E_k = \tfrac{1}{2} I\omega^2 = \frac{L^2}{2I}$$

The second form is useful when L is conserved: if I falls, E_k rises.

Rolling without slipping

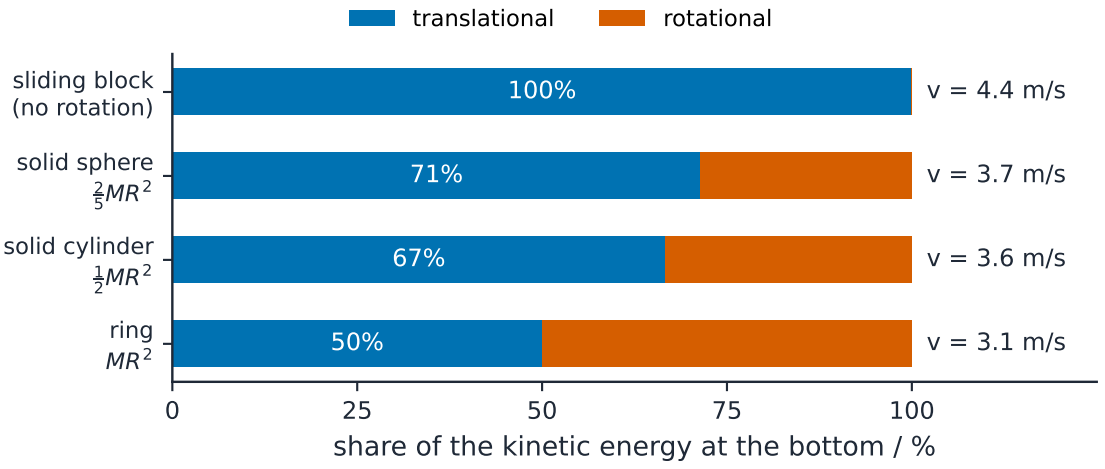
A wheel that rolls without slipping moves forward one circumference per turn, so its centre moves at $v = \omega r$. Its kinetic energy has two parts:

$$E_k = \tfrac{1}{2} M v^2 + \tfrac{1}{2} I\omega^2$$

Here v is the speed of the centre of mass and I is the moment of inertia about the centre-of-mass axis. Do not combine translational energy with a moment of inertia about the contact point: that would count the translation twice.

Worked example: a solid cylinder ($I = \tfrac{1}{2}MR^2$) rolls without slipping from rest down a slope with a vertical drop of 1.0 m. Neglect rolling resistance and other dissipative losses.

- $Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\tfrac{v}{R}\right)^2 = \tfrac{3}{4}Mv^2$.
- $v = \sqrt{\tfrac{4}{3}gh} = \sqrt{\tfrac{4}{3}(9.8)(1.0)} = 3.6$ m s^{-1}, whatever its mass and radius.



The more of the energy that goes into spinning, the slower the body arrives

A frictionless sliding block would reach $\sqrt{2gh} = 4.4$ m s^{-1}; a ring ($I = MR^2$) only $\sqrt{gh} = 3.1$ m s^{-1}. For these shapes rolling without slip from rest down the same straight slope, the smaller ratio $I/(MR^2)$ gives the greater acceleration and final speed. Mass or radius alone does not decide the result.

Practice

1. A 40 N force is applied to the end of a 0.25 m spanner. Find the torque when the force is (a) perpendicular to the spanner, (b) at 30° to it.
2. A metre rule is balanced at its 50 cm mark. A 2.0 N weight hangs at the 20 cm mark. Where must a 3.0 N weight hang to balance it?
3. A wheel speeds up uniformly from 5.0 to 25 rad s^{-1} in 4.0 s. Find its angular acceleration and the number of revolutions it makes.
4. Point masses of 0.20 kg, 0.30 kg and 0.50 kg sit at 0.10 m, 0.20 m and 0.30 m from an axis. Find the moment of inertia.
5. A constant 2.0 N m torque acts for 6.0 s on a disc ($I = 0.40$ kg m^2) starting from rest. Find its final angular velocity, angular momentum and kinetic energy.
6. A diver leaves the board turning at 2.0 rad s^{-1} with $I = 12$ kg m^2, then tucks to $I = 4.0$ kg m^2. Find the new angular velocity and explain where the extra kinetic energy comes from.
7. A turntable ($I = 0.050$ kg m^2) turns freely at 3.0 rad s^{-1}. A 0.10 kg lump of clay is dropped onto it 0.15 m from the axis and sticks. Find the new angular velocity.
8. A motor applies 0.80 N m for 5.0 s to a wheel ($I = 0.20$ kg m^2) at rest. Find the angular impulse and the final angular velocity.
9. A flywheel has $L = 6.0$ kg m^2 s^{-1} and $I = 1.5$ kg m^2. Find its kinetic energy without finding ω.

10. A solid disc and a ring roll from rest down the same slope with a 0.50 m drop. Find each speed at the bottom and explain which arrives first.
11. In an invented model, a 1.0 kg mass falls from rest through 0.60 m while unwinding a light string from a fixed-axis pulley with radius 0.10 m and moment of inertia 0.020 kg m^2. Neglect axle friction and string slip. Use energy to find the final speed and the two kinetic-energy contributions; check against the acceleration method.

Answers.

1. (a) $40 \times 0.25 = 10$ N m. (b) $40 \times 0.25 \times \sin 30° = 5.0$ N m.
2. Anticlockwise $2.0 \times 0.30 = 0.60$ N m; $3.0d = 0.60$, $d = 0.20$ m: at the 70 cm mark.
3. $\alpha = (25 - 5.0)/4.0 = 5.0$ rad s^{-2}; $\Delta\theta = \frac{(5.0+25)}{2} \times 4.0 = 60$ rad $= 9.5$ revolutions.
4. $I = 0.20(0.10^2) + 0.30(0.20^2) + 0.50(0.30^2) = 0.002 + 0.012 + 0.045 = 0.059$ kg m^2.
5. $\alpha = 2.0/0.40 = 5.0$ rad s^{-2}; $\omega = 30$ rad s^{-1}; $L = 0.40 \times 30 = 12$ kg m^2 s^{-1} (also $\tau\Delta t = 12$); $E_k = \frac{1}{2}(0.40)(30^2) = 180$ J.
6. $\omega = 12 \times 2.0/4.0 = 6.0$ rad s^{-1}. E_k rises from 24 J to 72 J; the diver's muscles do work pulling the body inwards.
7. New $I = 0.050 + 0.10(0.15^2) = 0.0523$ kg m^2; $\omega = 0.050 \times 3.0/0.0523 = 2.9$ rad s^{-1}.
8. $\Delta L = 0.80 \times 5.0 = 4.0$ kg m^2 s^{-1}; $\omega = 4.0/0.20 = 20$ rad s^{-1}.
9. $E_k = L^2/2I = 36/3.0 = 12$ J.
10. Disc: $v = \sqrt{\frac{4}{3}gh} = 2.6$ m s^{-1}. Ring: $v = \sqrt{gh} = 2.2$ m s^{-1}. The disc arrives first: its mass is nearer the axis, so a smaller share of the energy goes into rotation and more into forward motion.
11. With $\omega = v/r$, $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I(v/r)^2$. Thus $5.88 = \frac{1}{2}(1.0 + 0.020/0.10^2)v^2 = 1.5v^2$, giving $v^2 = 3.92$ and $v = 2.0$ m s^{-1}. Translational energy is 1.96 J; rotational energy is 3.92 J, summing to 5.88 J. Independently, $a = mg/(m + I/r^2) = 9.8/3.0 = 3.27$ m s^{-2} and $v^2 = 2as = 3.92$, agreeing before rounding.

Exam tips

- Torque uses the **perpendicular** distance to the line of action; with $Fr\sin\theta$, θ is the angle between r and F.
- State the sense of each torque (clockwise or anticlockwise) and take torques about the point with the most unknown forces.
- Work in radians: convert revolutions with 2π and rpm with $2\pi/60$.
- $\tau = I\alpha$ uses the **resultant** torque; subtract a frictional torque first.
- For coupled bodies, write one equation for each body and link them with $a = \alpha r$.
- When I changes with no external torque, conserve $L = I\omega$, not kinetic energy.
- A rolling body has translational **and** rotational kinetic energy; use $v = \omega r$ to combine them.

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Galilean and special relativity

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches A.5 Galilean and special relativity, for the course first assessed in 2025. The whole unit is additional higher level (AHL); there is no SL counterpart.

Prerequisites: velocity and displacement from A.1, Newton's laws from A.2, squares and square roots. Use $c = 3.00\times10^{8}$ m s^{-1}. Derivations of the Lorentz transformations, velocity addition, time dilation and length contraction are not required.

AHL: Reference frames and Galilean relativity

Who is moving?

Sit in a train waiting at a station while the train on the next track pulls away: for a moment you are sure that **you** are moving. Without looking out of a window, no experiment inside a smoothly moving train can tell you whether it is moving or at rest. Physics describes motion only **relative to a reference frame**.

Reference frames and events

- A **reference frame** 参考系 is a set of rulers and synchronized clocks used by an observer to give every event a position and a time.
- An **event** 事件 is something that happens at one place at one instant, with coordinates (x, t) in a frame.
- An **inertial reference frame** 惯性参考系 is a frame that is not accelerating. In it, a body with no resultant force moves at constant velocity.

Galilean relativity 伽利略相对性: Newton's laws of motion are the same in every inertial frame.

The Galilean transformation

Frame S′ moves at speed v along the x-axis of frame S, and the origins coincide at $t = 0$. An event at (x, t) in S has coordinates in S′:

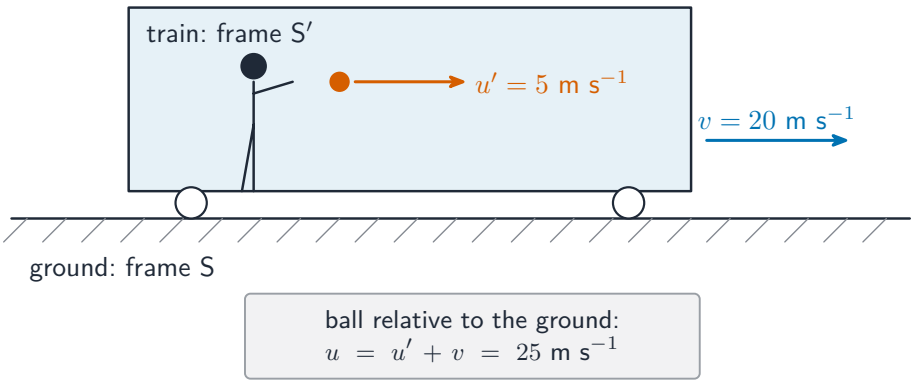
$$x' = x - vt \qquad t' = t$$

Time is the same for everyone. Differentiating gives the **velocity addition** rule: an object moving at u in S moves at

$$u' = u - v$$

in S′. A ball thrown forwards at 5 m s^{-1} inside a train moving at 20 m s^{-1} moves at 25 m s^{-1} relative to the ground.

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Galilean addition: velocities in the two frames differ by the frame speed v

Where Galilean relativity fails

Maxwell's equations of electromagnetism predict that light travels at c, with no mention of a frame. Galilean addition says that light from a moving source should move faster or slower than c. Experiments (such as Michelson and Morley's, and measurements of light from fast-moving particles) always find c. Galilean relativity is only an approximation for speeds much smaller than c.

AHL: The postulates of special relativity

Einstein built **special relativity** 狭义相对论 on two postulates:

1. The laws of physics are the same in all inertial reference frames.
2. The speed of light in a vacuum is the same, c, for all inertial observers, whatever the motion of the source or the observer.

Together these force space and time to depend on the frame. Two observers in relative motion can disagree about time intervals, lengths and whether two events are simultaneous, yet both are right in their own frame.

AHL: The Lorentz transformations

Transforming the coordinates of an event

The **Lorentz transformations** 洛伦兹变换 replace the Galilean equations:

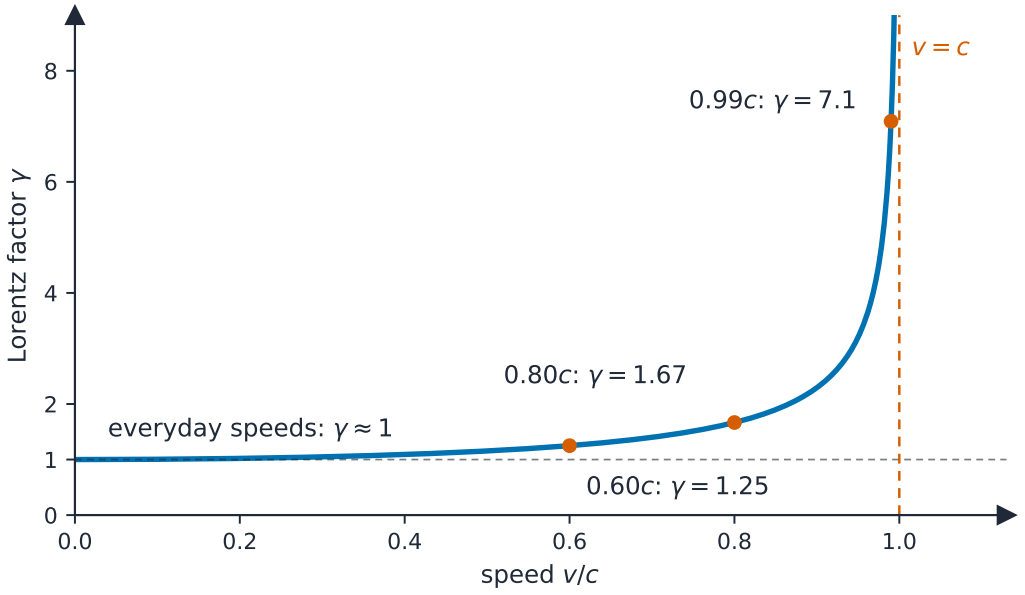
$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

with the **Lorentz factor** 洛伦兹因子

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

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γ is 1 at rest and grows without limit as v approaches c: $\gamma = 1.25$ at $0.60c$, 1.67 at $0.80c$, 7.1 at $0.99c$. At everyday speeds $\gamma \approx 1$ and the Lorentz equations reduce to the Galilean ones. They also apply to intervals: $\Delta x' = \gamma(\Delta x - v\Delta t)$ and $\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$.



Relativistic effects are tiny at everyday speeds and grow without limit near c

Worked example: S′ moves at $0.60c$ relative to S ($\gamma = 1.25$). An event in S happens at $x = 3.0 \times 10^8$ m, $t = 2.0$ s.

- $x' = 1.25\,(3.0 \times 10^8 - 0.60 \times 3.0 \times 10^8 \times 2.0) = -7.5 \times 10^7$ m.
- $t' = 1.25\left(2.0 - \dfrac{0.60c \times 3.0 \times 10^8}{c^2}\right) = 1.25\,(2.0 - 0.60) = 1.75$ s.

Relativistic velocity addition

An object moving at u in S moves at

$$u' = \frac{u - v}{1 - \dfrac{uv}{c^2}}$$

in S′. Two spacecraft approach each other, each at $0.80c$ relative to the Earth. In A's frame ($v = +0.80c$), B ($u = -0.80c$) moves at

$$u' = \frac{-0.80c - 0.80c}{1 - \dfrac{(-0.80c)(0.80c)}{c^2}} = \frac{-1.60c}{1.64} = -0.98c$$

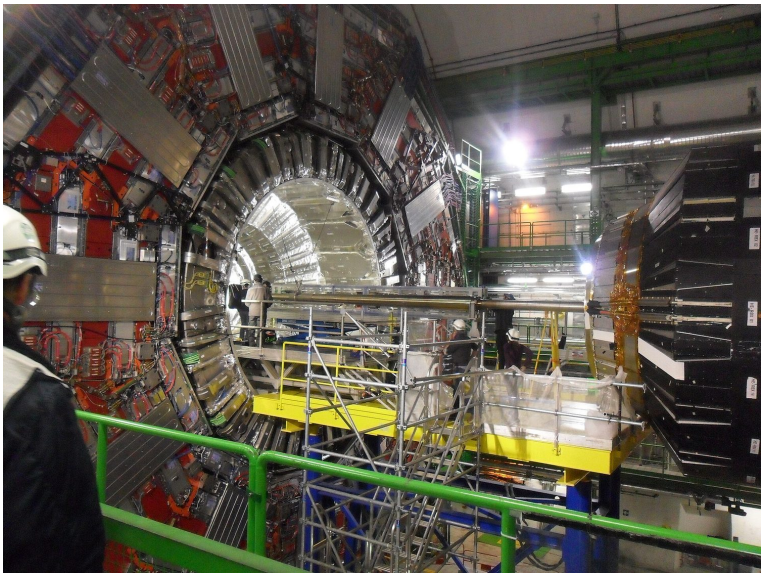
not $1.6c$. No combination of speeds below c gives a speed above c; if $u = c$, then $u' = c$ in every frame, as postulate 2 requires.

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AHL: Time dilation and length contraction

Proper time and time dilation

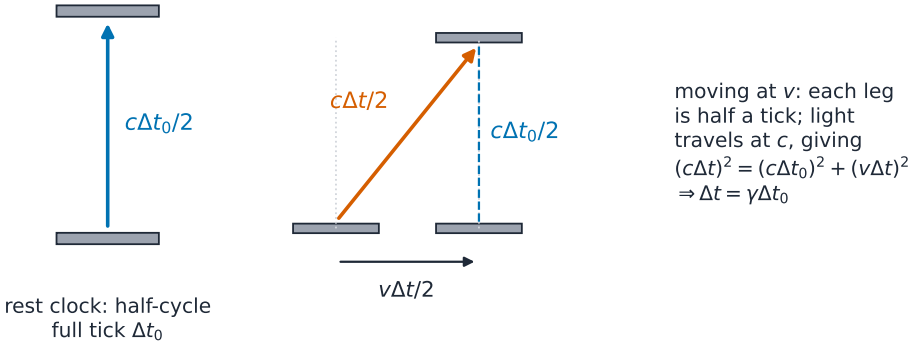
The **proper time interval** 固有时间 Δt_0 between two events is the time measured in the frame where both events happen at the **same place**, for example by a single clock that travels with them. Every other inertial observer measures a longer interval:



The CMS particle detector at CERN: particle lifetimes measured in such experiments match the time-dilation formula

$$\Delta t = \gamma \Delta t_0$$

This is **time dilation** 时间膨胀: a moving clock runs slow, as seen by the observer it moves past.



The shown path is half a tick; a full tick returns light to the same lower-mirror clock, defining its proper interval Δt_0. Both half-cycles give $\Delta t = \gamma \Delta t_0$

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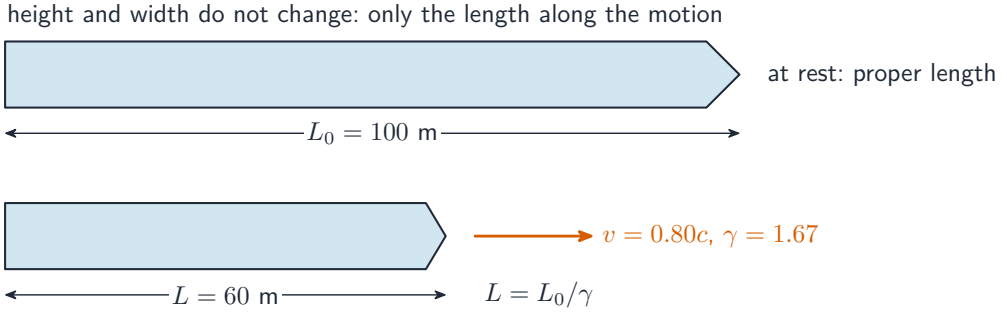
A spacecraft passes the Earth at $0.60c$. Its clock measures 4.0 μs between two flashes made on board (proper time, same place in the craft). Earth observers measure $1.25 \times 4.0 = 5.0$ μs.

■ Proper length and length contraction

The **proper length** 固有长度 L_0 of an object is its length measured in the frame where it is at rest. An observer who sees it moving along its length measures it shorter:

$$L = \frac{L_0}{\gamma}$$

This is **length contraction** 长度收缩; it happens only along the direction of motion. A 100 m spacecraft moving at $0.80c$ is 60 m long in the Earth's frame.



$L = L_0/\gamma$: the proper length is measured in the object's rest frame

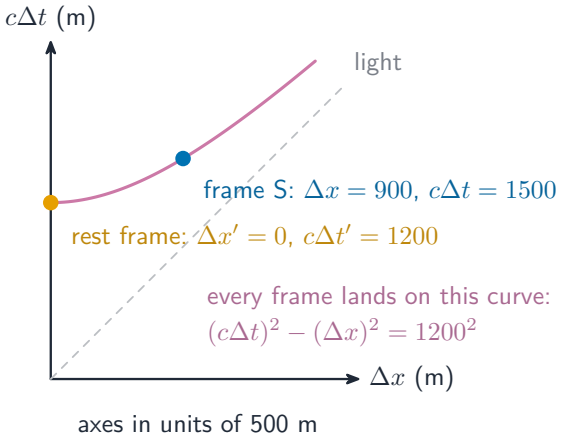
Before substituting, label which frame measures the proper quantity. The proper time is measured by the observer who sees both events in one place; the proper length by the observer who sees the object at rest. Do not mix a length from one frame with a time from another.

AHL: Space–time interval and simultaneity

■ The invariant interval

Observers disagree about Δt and Δx, but they all agree on the **space–time interval** 时空间隔 between two events:

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Every frame lands on the same curve

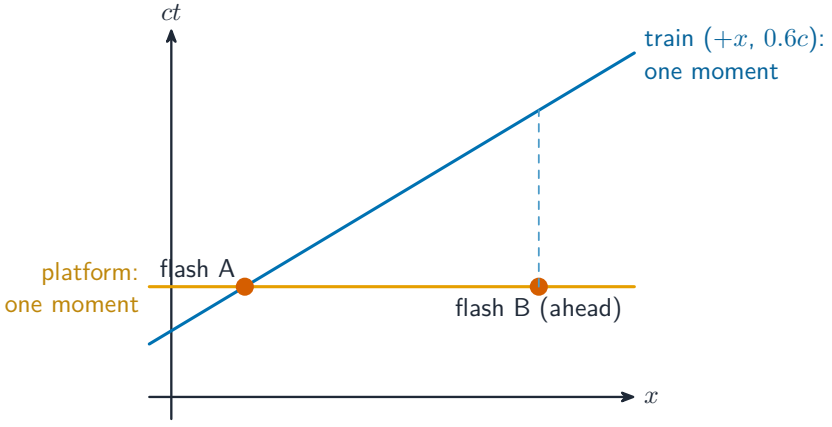
$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

Two events are 900 m apart and 5.0 μs apart in S ($c\Delta t = 1500$ m). $(\Delta s)^2 = 1500^2 - 900^2$, so $\Delta s = 1200$ m, the same in every frame. In the frame where both events happen at one place, $\Delta x' = 0$, so $c\Delta t' = 1200$ m: the proper time is 4.0 μs, matching $\Delta t = \gamma \Delta t_0$ with $v = 900/(5.0 \times 10^{-6}) = 0.60c$.

The interval also checks Lorentz calculations: in the worked example above, $(c \times 2.0)^2 - (3.0 \times 10^8)^2 = (c \times 1.75)^2 - (7.5 \times 10^7)^2 = 2.7 \times 10^{17}$ m^2.

The relativity of simultaneity

Events that are simultaneous in one frame are generally **not** simultaneous in another frame moving along the line joining them. This is the **relativity of simultaneity** 同时的相对性. From the Lorentz transformation with $\Delta t = 0$:



Equal-time lines tilt in a moving frame

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$$\Delta t' = -\gamma \frac{v \Delta x}{c^2}$$

Lamps at the two ends of a 3.0 km platform flash at the same time in the platform frame. For a train passing at $0.60c$, $\Delta t' = -1.25 \times 0.60 \times 3000/(3.0 \times 10^8) = -7.5\ \mu$s: the train frame assigns the lamp ahead (further along $+x$) an emission time 7.5 μs earlier. These are event times assigned using synchronized clocks and light-travel corrections, not the times at which the two light signals reach one passenger. Events at the **same place** that are simultaneous in one frame are simultaneous in all frames.

AHL: Space–time diagrams

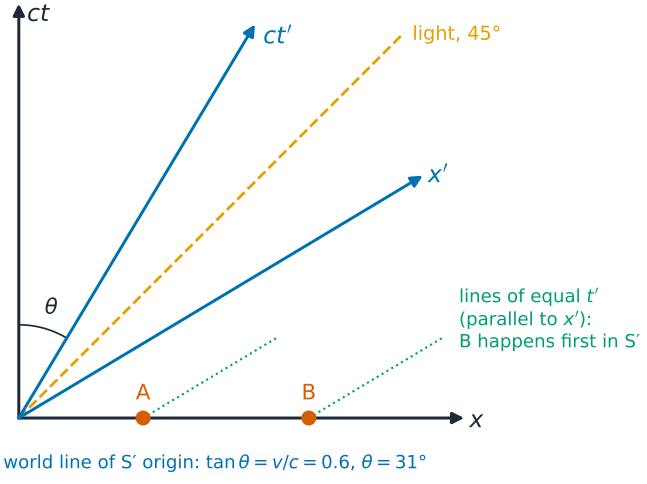
Drawing world lines

A **space–time diagram** 时空图 plots events with x horizontal and ct vertical, so both axes are in metres. The path of an object is its **world line** 世界线.

- An object at rest is a vertical line.
- Light is a line at 45°, since $x = ct$.
- An object at constant speed v is a straight line at angle θ to the ct axis, with

$$\tan\theta = \frac{v}{c}$$

At $0.60c$, $\theta = \tan^{-1}(0.60) = 31°$. A future-directed world line of a massive particle stays within the light cone; light lies on its boundary. With equal axis scales, this means at most 45° from the time axis. A line joining spacelike-separated events can be steeper away from that axis, but is not a possible material-particle trajectory.

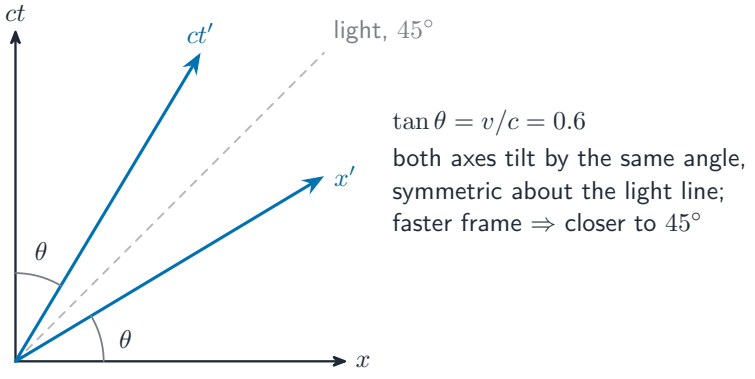


The moving frame's axes close in towards the light line; lines of simultaneity in S′ are parallel to the x′ axis

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The axes of a moving frame

On the same diagram, the axes of S′ (moving at v) are both tilted by θ towards the light line:



Moving axes close in on the light line

- the ct' axis is the world line of the origin of S′, at θ from ct;
- the x' axis (all events with $t' = 0$) is at θ above the x axis.

To read an event's coordinates in S′, draw lines **parallel** to the S′ axes. Events on a line parallel to x' are simultaneous in S′; events on a horizontal line are simultaneous in S. The scales on the S′ axes differ from those on the S axes; they are fixed by the invariant interval.

The diagram shows each effect without equations: two events on the x axis (simultaneous in S) lie on different x'-parallel lines, so they are not simultaneous in S′.

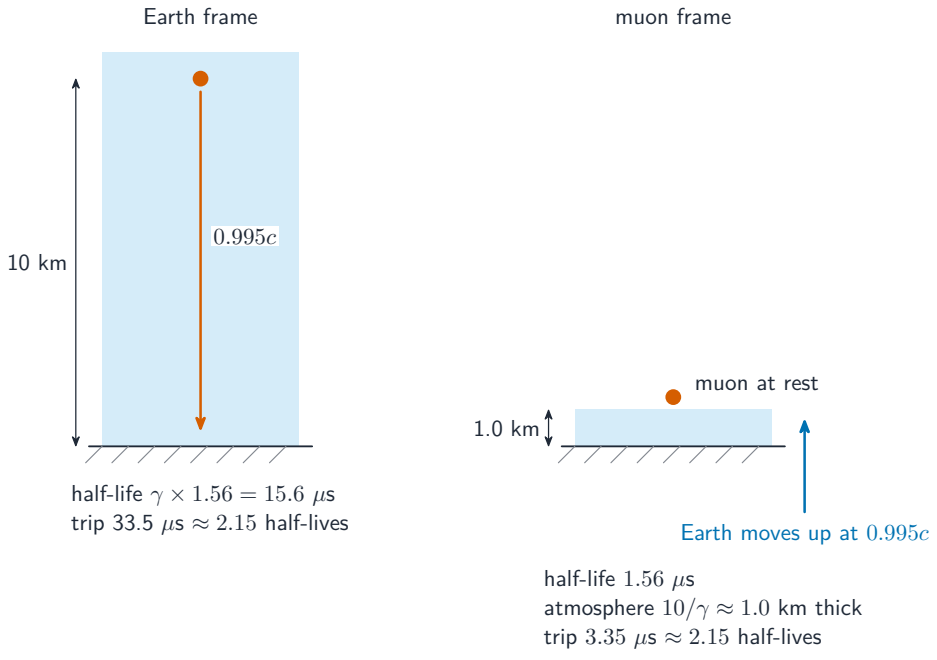
AHL: Muon decay evidence

Muons that should not reach the ground

Consider an illustrative **muon** 缪子 model: particles start 10 km above the ground and travel at constant $0.995c$ ($\gamma \approx 10$), neglecting energy loss and a spread of production heights or speeds. Use an at-rest population half-life of 1.56 μs. The following numbers are model predictions, not measurements from a particular experiment.

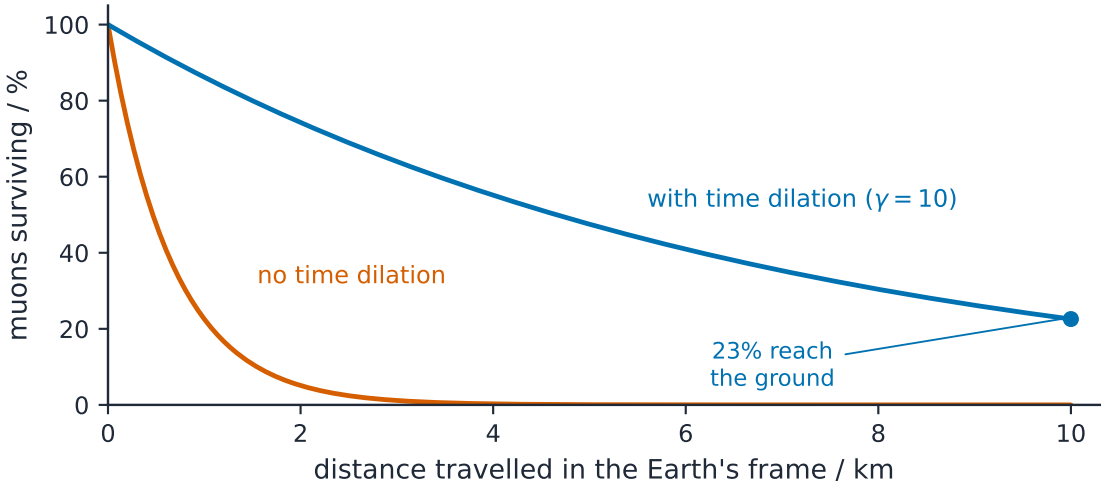
- Without relativity: the trip takes $10\,000/(0.995 \times 3.0 \times 10^8) = 33.5$ μs, about 21.5 half-lives, so only $2^{-21.5} \approx 3 \times 10^{-7}$ of the muons should survive.
- **Earth frame (time dilation):** the muons' half-life is $\gamma \times 1.56 = 15.6$ μs. The trip is only 2.15 half-lives, so about 23% survive.
- **Muon frame (length contraction):** the muons are at rest and their half-life is 1.56 μs, but the atmosphere rushes past contracted to $10\,000/10 = 1.0$ km. That takes 3.35 μs = 2.15 half-lives: the same 23%.

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Time dilation in one frame, length contraction in the other: the same number of half-lives

Both frames predict the same surviving fraction: time dilation and length contraction are descriptions of one model. For actual evidence, the Rossi–Hall paper abstract reports a faster decay rate in a less penetrating particle group, consistent with the relativistic clock-rate prediction. That comparison does not make this illustrative 23% curve measured data.



Illustrative survival predictions for the stated model; neither curve represents measured counts

Practice

1. Two cars travel towards each other at 30 m s^{-1} and 25 m s^{-1} relative to the road. Use Galilean relativity to find the speed of one relative to the other.
2. State the two postulates of special relativity.
3. Calculate γ for $0.50c$, $0.90c$ and $0.99c$.
4. S′ moves at $0.80c$ relative to S. An event in S has $x = 600$ m and $ct = 900$ m. Find x' and t', and check your answer with the space–time interval.
5. Two protons move in opposite directions in a laboratory, each at $0.90c$. Find the speed of one relative to the other.
6. A spacecraft travels at $0.80c$. Its clock records 1.0 hour for a journey. How long does the journey take in the Earth's frame?
7. A rod has a proper length of 2.0 m. Find its length when it moves along its length at $0.60c$.
8. Pions have a proper mean lifetime of 26 ns. A beam moves at $0.95c$. Find the mean lifetime in the laboratory and the average distance travelled.
9. Muons ($t_{1/2} = 1.56$ μs at rest) move at $0.98c$ through 2.0 km of atmosphere in the Earth's frame. Find the fraction that survive, using (a) the Earth's frame, (b) the muons' frame.
10. On a space–time diagram, a world line makes 25° with the ct axis. Find the object's speed.
11. Explain, using a space–time diagram, why two events simultaneous in S are not simultaneous in S′.

Answers.

1. $30 + 25 = 55$ m s^{-1}.
2. The laws of physics are the same in all inertial frames; the speed of light in a vacuum is the same for all inertial observers.
3. 1.15; 2.29; 7.09.
4. $\gamma = 5/3$. $x' = \frac{5}{3}(600 - 0.80 \times 900) = -200$ m; $ct' = \frac{5}{3}(900 - 0.80 \times 600) = 700$ m, so $t' = 2.3$ μs. Check: $900^2 - 600^2 = 700^2 - 200^2 = 4.5 \times 10^5$ m^2.
5. $u' = \dfrac{-0.90c - 0.90c}{1 + 0.81} = -0.99c$; speed $0.99c$.
6. $\gamma = 1.67$: $\Delta t = 1.67 \times 1.0 = 1.7$ hours.
7. $L = 2.0/1.25 = 1.6$ m.
8. $\gamma = 3.20$: lifetime $= 3.20 \times 26 = 83$ ns; distance $= 0.95 \times 3.0 \times 10^8 \times 83 \times 10^{-9} = 24$ m.
9. $\gamma = 5.03$. (a) Time $= 2000/(0.98c) = 6.80$ μs; dilated half-life $= 7.84$ μs; 0.868 half-lives, fraction $= 2^{-0.868} = 0.55$. (b) Contracted distance $= 2000/5.03 = 398$ m; time $= 1.35$ μs $= 0.868$ half-lives; fraction 0.55.
10. $v = c \tan 25° = 0.47c$.

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11. In S, the events lie on a horizontal line (same ct). Lines of simultaneity in S′ are parallel to the tilted x' axis, so the two events lie on different x'-parallel lines and have different t'; the event further along $+x$ happens first in S′.

Exam tips

- Decide which frame measures the **proper** time (both events at one place) or the **proper** length (object at rest) before you substitute.
- For the same timelike-separated events and inertial-frame comparison, proper time is the shortest coordinate-time interval. Proper length is measured at rest; a moving length requires simultaneous endpoint readings in the measuring frame.
- Use the Lorentz transformations when events are at different places **and** different times; time dilation alone fails there.
- Check any pair of transformed coordinates with the invariant interval $(c\Delta t)^2 - (\Delta x)^2$.
- In velocity addition, keep the signs: objects moving in opposite directions have opposite signs of u.
- On space–time diagrams, label the time axis ct; light is at 45°, and $\tan\theta = v/c$ is measured from the ct axis.
- For muon questions, say which frame you use: time dilation in the Earth's frame, length contraction in the muon's frame.

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Thermal energy transfers

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches B.1 Thermal energy transfers in full, for the course first assessed in 2025. There is no additional higher level content in B.1, so SL and HL students study the same material.

Prerequisites: energy stores, work and power from A.3; area and volume; powers of ten. Use $k_B = 1.38 \times 10^{-23}$ J K^{-1}, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} and $c_{\text{water}} = 4200$ J kg^{-1} K^{-1}.

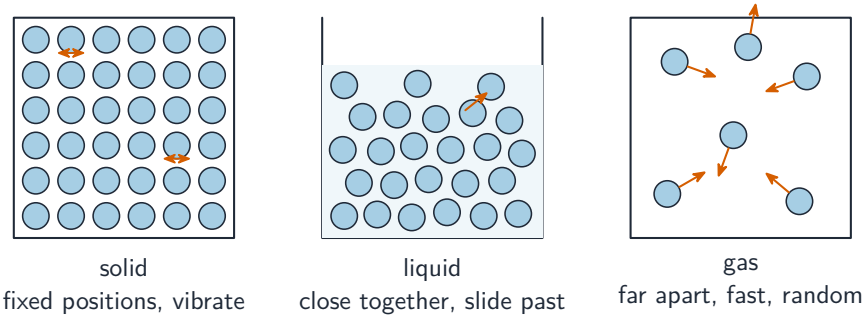
Particles in solids, liquids and gases

Why does a puddle disappear?

On a warm afternoon a puddle dries up long before it could ever boil. A metal bench in the sun feels much hotter than a wooden one at the same temperature, and a cup of tea cools faster with the lid off. All of these follow from one idea: matter is made of particles that are always moving, and energy moves between them. This unit builds that model and then uses it to measure energy transfers.

The molecular model

The **molecular model** 分子模型 explains the three states of matter by the arrangement, spacing and motion of the particles.



The three states differ in particle spacing, arrangement and motion

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State	Arrangement and spacing	Motion	Forces between particles
solid	closely packed; crystalline solids are regular, amorphous solids lack long-range order	vibrate about fixed positions	strong; fixed shape and volume
liquid	irregular, still close together	vibrate and slide past each other	weaker; fixed volume, takes the container's shape
gas	random, widely spaced in a dilute gas; spacing depends on conditions	move fast in straight lines between collisions	almost none; fills any container

A dilute gas is readily compressed because of its large particle spacing. The gas-to-liquid volume ratio for the same mass depends on temperature, pressure and substance; a factor of order 1000 is an illustration at familiar pressures, not a fixed law.

■ Density

Density 密度 is mass per unit volume:

$$\rho = \frac{m}{V}$$

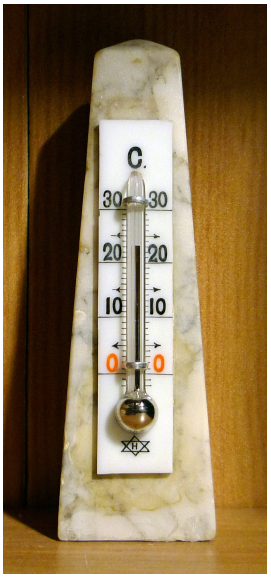
in kg m^{-3}. A block of aluminium measuring 10 cm × 10 cm × 2.0 cm has mass 0.54 kg, so $\rho = 0.54/(0.10 \times 0.10 \times 0.020) = 2700$ kg m^{-3}. Water has density 1000 kg m^{-3} and air about 1.2 kg m^{-3}: the large gap between them is the wide particle spacing in a gas. Convert to metres before calculating: 1 cm^3 = 10^{-6} m^3.

Temperature and internal energy

■ Kelvin and Celsius

At standard atmospheric pressure, pure water melts near 0 °C and boils near 100 °C on the **Celsius scale** 摄氏温标. The **Kelvin scale** 开尔文温标 starts at **absolute zero** 绝对零度, the lowest possible temperature, where the particles have their minimum energy:

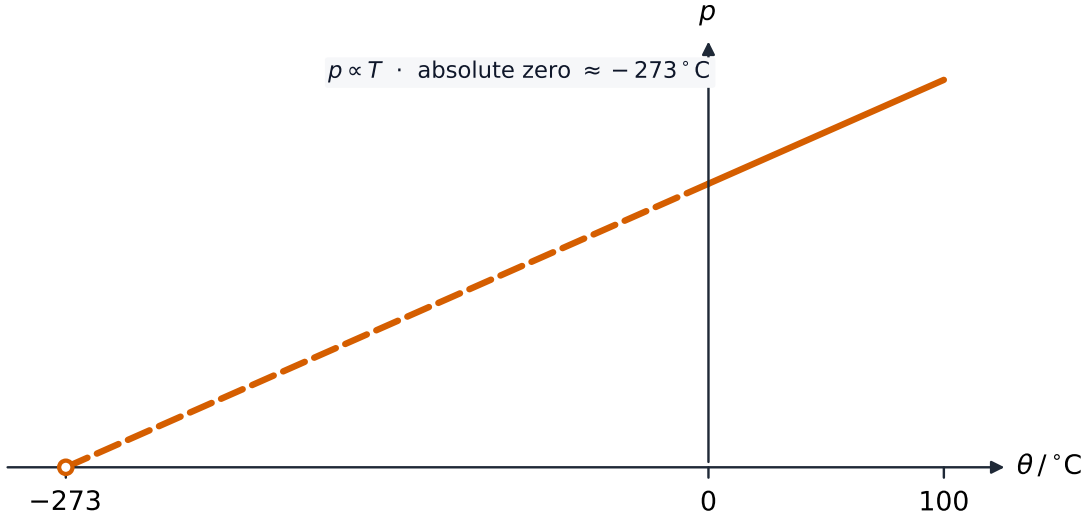
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A thermometer measures temperature on a defined scale.

$$T/\mathrm{K} = \theta/^\circ\mathrm{C} + 273.15$$

Use the rounded offset 273 in the numerical examples here: 27 °C is about 300 K and liquid nitrogen at −196 °C about 77 K. One kelvin is the same size as one degree Celsius, so a **change** in temperature is the same number on both scales: warming from 20 °C to 50 °C is a rise of 30 °C and of 30 K. Always use kelvin in an equation that contains T itself rather than ΔT.



Extrapolating the pressure–temperature line back to zero pressure gives absolute zero, about −273 °C

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Temperature measures particle kinetic energy

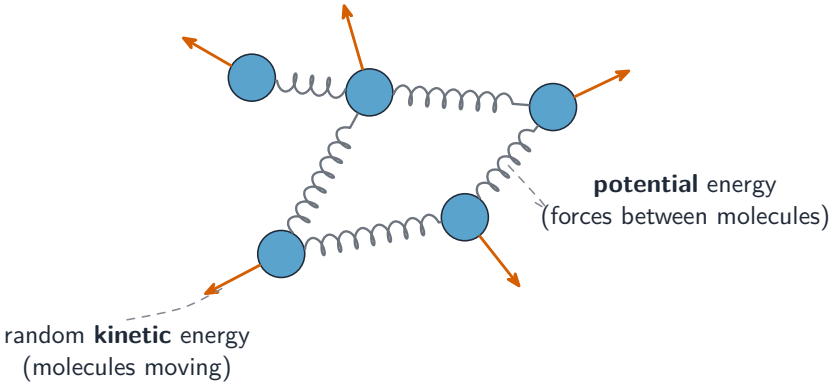
In the classical ideal-gas model, kelvin temperature sets the **average kinetic energy** 平均动能 of translational motion per particle:

$$\overline{E}_k = \frac{3}{2}k_B T$$

where k_B is the **Boltzmann constant** 玻尔兹曼常数. At 300 K, $\overline{E}_k = 1.5 \times 1.38 \times 10^{-23} \times 300 = 6.2 \times 10^{-21}$ J. Doubling the kelvin temperature to 600 K doubles this mean translational kinetic energy; doubling a Celsius temperature does not. This relation does not include every possible rotational, vibrational or interaction-energy contribution in all materials.

Internal energy

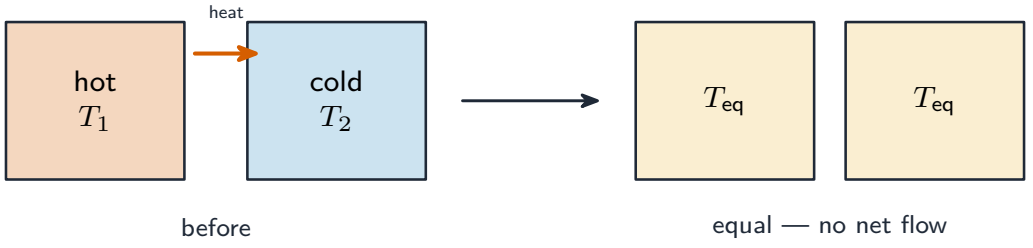
The **internal energy** 内能 of a system is the total intermolecular potential energy, from the forces between the particles, plus the total random kinetic energy of the particles. In the classical ideal-gas model, temperature determines the average translational kinetic energy per particle; this is not a general equation for every contribution in every material. A bath of warm water has much more internal energy than a cup of boiling water, because it has so many more particles, even though its temperature is lower.



Internal energy is the molecules' random kinetic energy (the arrows) plus the potential energy of the forces between them (the springs)

The direction of thermal energy transfer

For spontaneous thermal exchange between two bodies in contact, the net transfer is from the body at the higher temperature to the body at the lower temperature, whatever their sizes or internal energies. At equal temperatures, opposing microscopic exchanges balance; the net thermal transfer is zero: the bodies are then in **thermal equilibrium** 热平衡. A small hot coin dropped into a large cold bath gives energy to the bath, even though the bath has more internal energy.



Heat flows from hot to cold until both reach the same temperature – thermal equilibrium

Specific heat capacity

Energy and temperature change

The **specific heat capacity** 比热容 c of a substance is the energy needed to raise the temperature of 1 kg of it by 1 K:

$$Q = mc\Delta T$$

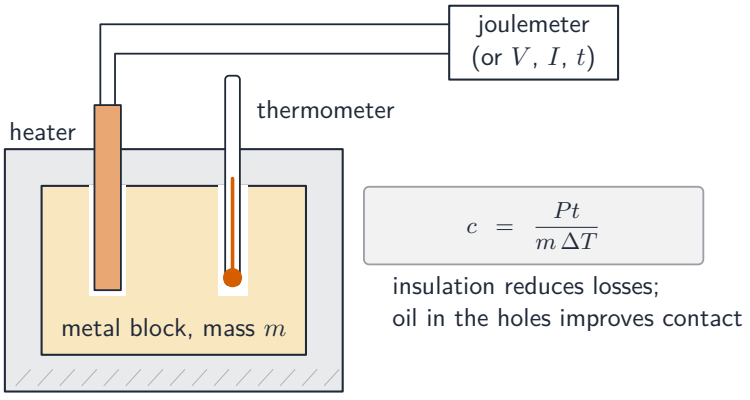
in J kg^{-1} K^{-1}. Water has a very large value, 4200 J kg^{-1} K^{-1}; copper only 390 J kg^{-1} K^{-1}. The same energy warms 1 kg of copper about 11 times more than 1 kg of water, which is why a metal pan heats up quickly while the water in it heats slowly.

Worked example: a kettle heats 1.5 kg of water from 20 °C to 100 °C.

- $Q = 1.5 \times 4200 \times 80 = 5.0 \times 10^5$ J.
- With a 2.2 kW element and no losses, $t = Q/P = 504\,000/2200 = 230$ s, nearly 4 minutes.

Measuring a specific heat capacity

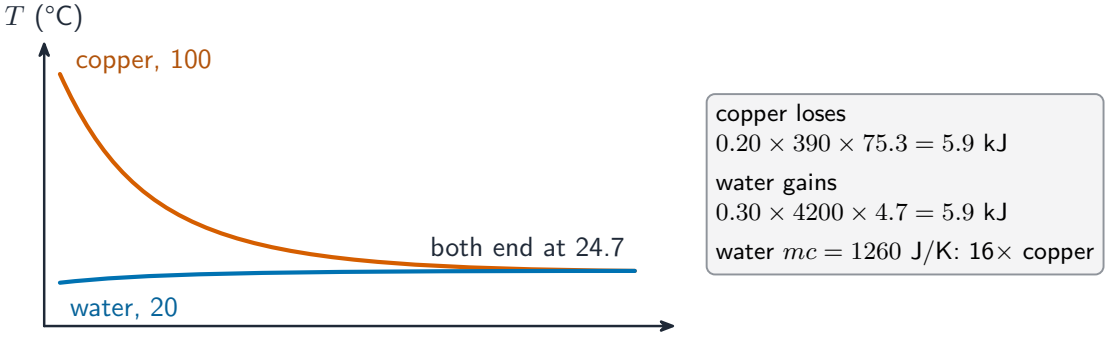
In the lab: an electric heater of known power P sits in an insulated metal block of mass m. Record the temperature every minute while the heater is on for a time t. Then $c = Pt/(m\Delta T)$, or $c = P/(m \times \text{gradient})$ from a temperature–time graph. Energy lost to the surroundings makes the measured ΔT too small, so the value of c is **too large**. Reduce the error with insulation, good thermal contact (a drop of oil around the thermometer) and by starting a few degrees below room temperature.



Energy in from the joulemeter, temperature rise from the thermometer: $c = Pt/(m\,\Delta T)$

■ Mixing and the energy account

When a hot body and a cold body reach thermal equilibrium with no losses:



Energy lost equals energy gained

energy lost by the hot body = energy gained by the cold body

Worked example: 0.20 kg of copper ($c = 390$) at 100 °C is dropped into 0.30 kg of water at 20 °C. Let the final temperature be T.

- Copper loses $0.20 \times 390 \times (100 - T)$; water gains $0.30 \times 4200 \times (T - 20)$.
- $7800 - 78T = 1260T - 25\,200$, so $T = 33\,000/1338 = 24.7$ °C.

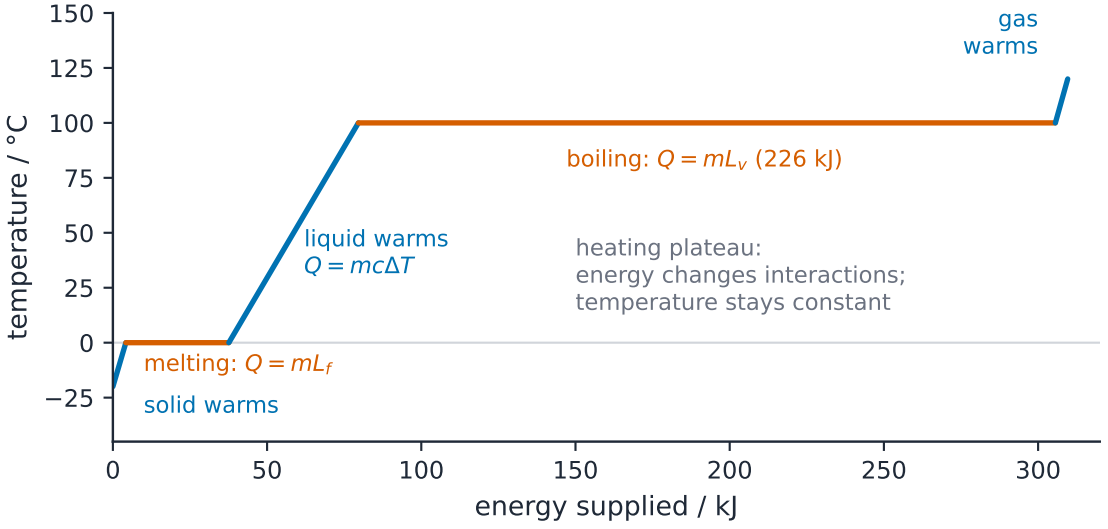
The water barely warms because its heat capacity, $mc = 1260$ J K^{-1}, is 16 times that of the copper.

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Phase changes and latent heat

Energy without a temperature change

A **phase change** 相变 is a change of state: **melting** 熔化, **freezing** 凝固, **boiling** 沸腾, **condensing** 凝结 or evaporation. For a pure substance changing phase at fixed pressure, energy can be transferred while temperature stays constant. In the particle model, melting or vaporization raises interaction potential energy; freezing or condensation releases energy and lowers it. The modeled mean kinetic energy stays unchanged during the ideal plateau. Melting ice and boiling water change intermolecular interactions, not the covalent bonds inside each water molecule.



Each flat step is a phase change; boiling 0.10 kg of water takes more than five times the energy of heating it from 0 °C to 100 °C

Specific latent heat

The **specific latent heat** 比潜热 L is the energy transferred per kilogram for a stated phase change at constant temperature. With L written as a positive magnitude:



Boiling water: at 100 °C the energy supplied changes liquid into steam instead of raising the temperature, and the energy per kilogram is the specific latent heat

$$Q = mL$$

in J kg^{-1}. This is energy absorbed for melting or vaporization and released for the reverse changes under the same conditions. Use the **specific latent heat of fusion** 熔化比潜热 L_f for melting or freezing and the **specific latent heat of vaporization** 汽化比潜热 L_v for boiling or condensing. For water, $L_f = 3.34 \times 10^5$ J kg^{-1} and $L_v = 2.26 \times 10^6$ J kg^{-1}. L_v is much larger because the particles must be pulled completely apart into a gas, not just loosened.

The heating curve above is an energy account for 0.10 kg of ice:

Stage	Equation	Energy / kJ
ice −20 °C → 0 °C	$0.10 \times 2100 \times 20$	4.2
melt at 0 °C	$0.10 \times 3.34 \times 10^5$	33.4
water 0 °C → 100 °C	$0.10 \times 4200 \times 100$	42
boil at 100 °C	$0.10 \times 2.26 \times 10^6$	226

At constant power the length of each plateau is proportional to mL, and the slope of each rising part is proportional to $1/(mc)$: the ice line is steeper than the water line because ice has the smaller c.

■ Ice in a drink

Worked example: 0.050 kg of ice at 0 °C is added to 0.25 kg of water at 30 °C. Find the final temperature T.

- Energy to melt the ice: $0.050 \times 3.34 \times 10^5 = 16\,700$ J.

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- The warm water loses $0.25 \times 4200 \times (30 - T)$; the melted ice then gains $0.050 \times 4200 \times T$.
- $31\,500 - 1050T = 16\,700 + 210T$, so $T = 14\,800/1260 = 11.7$ °C.

Forgetting the melting step would give 25 °C, far too warm.

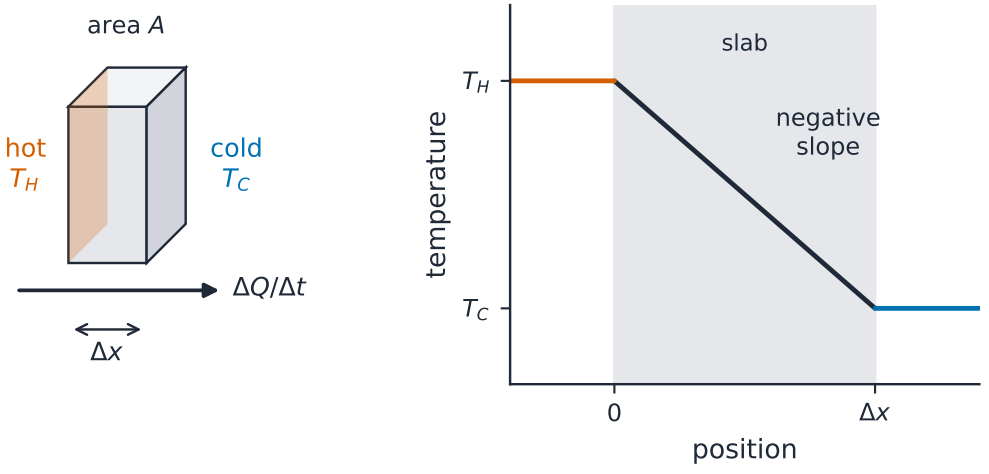
Evaporation and boiling

Evaporation 蒸发 happens at any temperature, only at the surface: particles with sufficient energy escape from the surface, tending to reduce the mean kinetic energy of those left. The liquid cools unless another transfer supplies the energy. This is why sweating cools you and why the puddle dries. **Boiling** occurs throughout the liquid, with vapour bubbles forming inside it. Its temperature depends on pressure and composition; for a pure substance at fixed pressure, the ideal boiling stage has constant temperature.

Conduction

Particles passing on energy

The three mechanisms of thermal energy transfer are conduction, convection and thermal radiation. In **conduction** 热传导, particles at the hot end have more kinetic energy; they collide with their slower neighbours and pass energy along. Many metals conduct well because their free electrons move quickly through the whole lattice and carry energy with them. Dilute gases generally conduct less effectively than good metal conductors; microscopic collisions still transfer energy.



The rate of conduction depends on the material, the area and the temperature gradient across the slab

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The rate of conduction

For a uniform slab at steady state, with constant conductivity, use the magnitude of the face-to-face temperature difference, $\Delta T = T_H - T_C > 0$. The temperature gradient along the hot-to-cold direction is negative; this positive formula gives the power magnitude:

$$\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$$

- $\Delta Q/\Delta t$ is the rate of energy transfer, in W.
- k is the **thermal conductivity** 热导率 of the material, in W m^{-1} K^{-1}: about 390 for copper, 0.80 for glass, 0.025 for still air.
- A is the cross-sectional area and $\Delta T/\Delta x$ is the **temperature gradient** 温度梯度.

Worked example: a window of glass 4.0 mm thick and area 1.5 m^2 has a difference of 20 K between its two surfaces.

- $\Delta Q/\Delta t = 0.80 \times 1.5 \times 20/0.0040 = 6000$ W.
- A 12 mm layer of still air with 15 K across it conducts only $0.025 \times 1.5 \times 15/0.012 = 47$ W. Double glazing traps a layer of air for this reason.

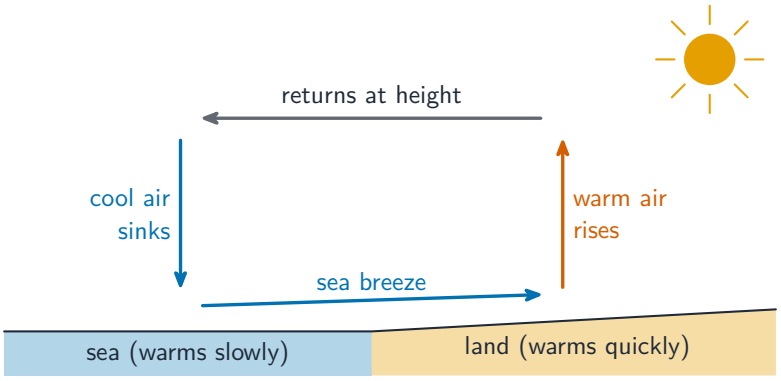
Real windows lose much less than 6000 W because a layer of still air clings to each surface and takes most of the temperature difference.

Convection

Convection 对流 transfers energy by the bulk movement of a fluid, a liquid or a gas. In ordinary free convection under gravity, fluid near a heater warms, expands and becomes less dense, so buoyancy makes it rise; cooler fluid replaces it, forming a **convection current** 对流环流. A fan or pump can drive forced convection without relying on buoyant rise. A material remaining solid cannot form a bulk fluid circulation internally.

- A low heater can establish room-air circulation. Some refrigerators use buoyant circulation; others use fans and may place the freezer below.
- Sea breezes: by day the land warms faster than the sea, warm air rises over the land and cooler air flows in from the sea.
- A lid reduces evaporation and some convective transfer from a drink; conduction and radiation can still remove energy.

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A convection current: the sea breeze is its lower branch

Thermal radiation

Emission by every surface

Every body above absolute zero emits **thermal radiation** 热辐射, electromagnetic waves (mostly infrared at everyday temperatures) from its surface. Radiation needs no material medium and carries the main solar energy transfer across space to Earth. A **black body** 黑体 is an ideal surface that absorbs all radiation falling on it and is also the best possible emitter. Stars are close to black bodies.

The Stefan–Boltzmann law

The total power radiated by a black body is its **luminosity** 光度 L:

$$L = \sigma A T^4$$

where σ is the **Stefan–Boltzmann constant** 斯特藩玻尔兹曼常量, A is the surface area and T is the absolute temperature. Because of the fourth power, doubling the kelvin temperature multiplies the power by 16.

Worked example: the Sun has radius 6.96×10^8 m and surface temperature 5800 K.

- $A = 4\pi r^2 = 6.09 \times 10^{18}$ m^2.
- $L = 5.67 \times 10^{-8} \times 6.09 \times 10^{18} \times 5800^4 = 3.9 \times 10^{26}$ W.

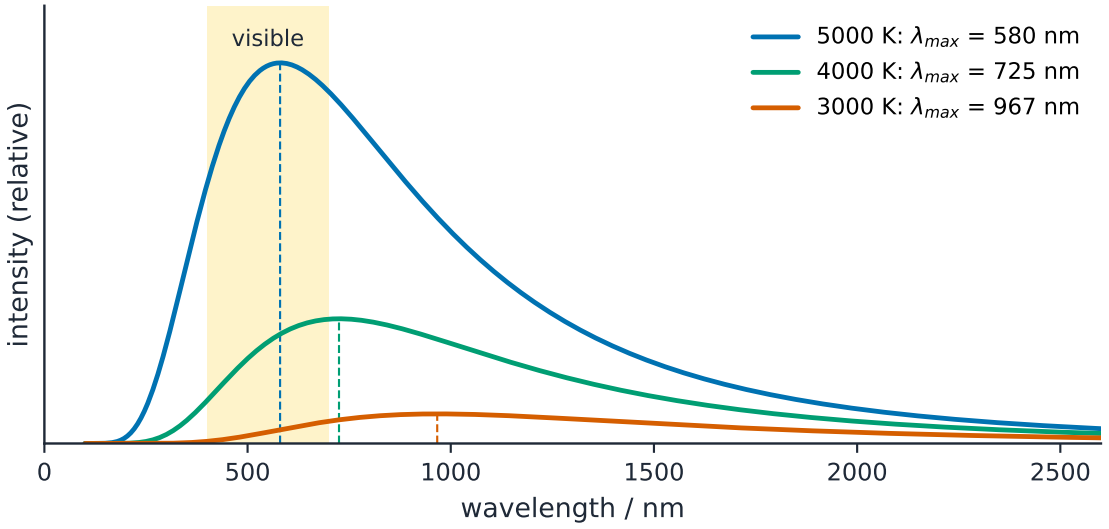
A body also absorbs radiation from its surroundings. In an illustrative black-surface enclosure model, a 1.8 m^2 surface at 306 K emits about 890 W and absorbs about 750 W from 293 K surroundings, giving about 140 W net radiative loss. Emission does not stop at thermal equilibrium: absorption then balances it. These are approximate radiative terms, not a person's total heat loss; real surfaces, clothing and geometry affect them.

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The black-body spectrum and Wien's law

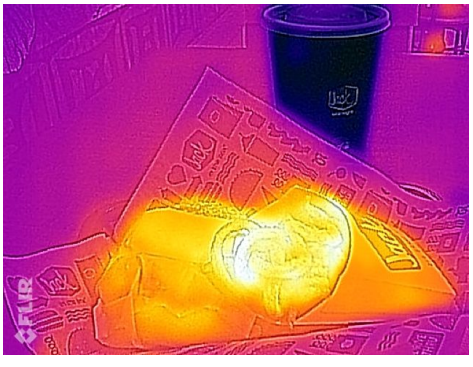
A black body emits a continuous spectrum with a peak. As the temperature rises the whole curve grows, and the peak moves to shorter wavelength. **Wien's displacement law** 维恩位移定律 links the peak wavelength to the temperature:

$$\lambda_{\max} T = 2.9 \times 10^{-3} \text{ m K}$$



Hotter bodies emit more at every wavelength, and their peak moves to shorter wavelength

- The Sun at 5800 K peaks at $2.9 \times 10^{-3}/5800 = 5.0 \times 10^{-7}$ m = 500 nm, in the visible.
- A filament lamp at 2900 K peaks at 1.0 μm, in the infrared, so most of its energy is not light.
- The human body at 310 K peaks at 9.4 μm; thermal cameras detect this.



A thermal (infrared) camera maps temperature to colour: the hot fries glow bright orange, the cold drink stays dark

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The colour of a star tells its temperature: red stars are cooler, blue-white stars are hotter.

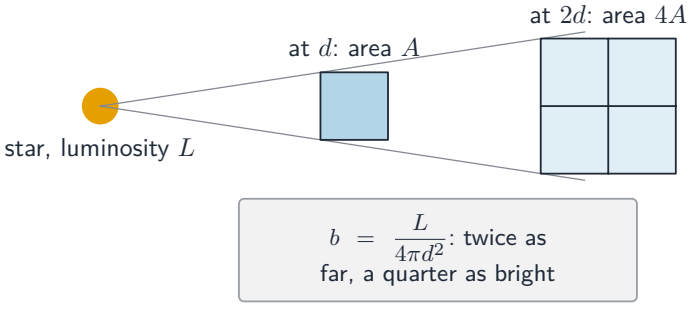
Luminosity and apparent brightness

■ Brightness falls with distance

The **apparent brightness** 视亮度 b of a star is the power received per unit area at the observer, in W m^{-2}. For isotropic emission, negligible intervening absorption and distance much larger than the source, luminosity spreads over a sphere of radius d:

$$b = \frac{L}{4\pi d^2}$$

This is an inverse-square law: twice as far away, a quarter as bright. At the Earth, $d = 1.5 \times 10^{11}$ m, so the Sun's apparent brightness is $3.9 \times 10^{26} / (4\pi \times (1.5 \times 10^{11})^2) = 1.4 \times 10^3$ W m^{-2}.



The same power spreads over a sphere of area $4\pi d^2$

■ Using brightness, colour and luminosity together

A star's luminosity is often given as a multiple of the Sun's, $L_\odot = 3.8 \times 10^{26}$ W. Astronomers measure b and $\lambda_{\max}$, then combine the laws:

- Wien's law gives the surface temperature T.
- If the distance is known, $L = 4\pi d^2 b$.
- Then $L = \sigma 4\pi R^2 T^4$ gives the radius.

Worked example: a red star has $T = 3500$ K and $L = 1.0 \times 10^5 L_\odot$. Compare its radius with the Sun's ($T_\odot = 5800$ K).

- $\dfrac{L}{L_\odot} = \left(\dfrac{R}{R_\odot}\right)^2 \left(\dfrac{T}{T_\odot}\right)^4$.
- $\dfrac{R}{R_\odot} = \sqrt{1.0 \times 10^5} \times \left(\dfrac{5800}{3500}\right)^2 = 870$.

A cool star can only be so luminous if it is enormous: this is a red supergiant.

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Practice

1. Convert (a) 25 °C to kelvin, (b) 4.0 K to degrees Celsius. A liquid warms from 15 °C to 60 °C: state the temperature rise in kelvin.
2. Explain, using the molecular model, why a gas is easy to compress but a liquid is not.
3. A gold ring has mass 7.7 g and volume 0.40 cm^3. Find its density in kg m^{-3}.
4. Find the average kinetic energy of the particles of a gas at 127 °C.
5. A 2.0 kW heater warms 0.80 kg of oil from 20 °C to 70 °C in 40 s. Find the specific heat capacity of the oil, and state whether heat loss makes your value too high or too low.
6. 0.30 kg of water at 80 °C is mixed with 0.20 kg of water at 20 °C. Find the final temperature.
7. Find the energy needed to turn 0.25 kg of water at 100 °C into steam at 100 °C.
8. Explain why the temperature stays constant while a solid melts, even though energy is still being supplied.
9. A copper rod of length 0.50 m and cross-sectional area 1.0×10^{-4} m^2 has its ends at 100 °C and 20 °C. Find the rate of conduction ($k = 390$ W m^{-1} K^{-1}).
10. Explain how a convection current forms above a radiator.
11. Model a hotplate as an ideal black body of radiating area 0.040 m^2 is at 500 K. Find the power it radiates. Find the factor by which this changes if it is heated to 1000 K.
12. A star has $\lambda_{\max} = 290$ nm. Find its surface temperature and state its colour.
13. A star of luminosity 3.8×10^{28} W has apparent brightness 2.0×10^{-9} W m^{-2} at the Earth. Find its distance in metres.
14. Star X has twice the surface temperature of star Y and the same luminosity. Find the ratio of their radii R_X/R_Y.

Answers.

1. (a) 298 K. (b) −269 °C. A rise of 45 K.
2. Gas particles are far apart with mostly empty space between them, so pressing moves them closer. Liquid particles are already touching, so the volume barely changes.
3. $\rho = \frac{7.7\times10^{-3}}{0.40\times10^{-6}} = 1.9 \times 10^4$ kg m^{-3}.
4. $T = 400$ K; $\overline{E}_k = 1.5 \times 1.38 \times 10^{-23} \times 400 = 8.3 \times 10^{-21}$ J.
5. $c = Pt/(m\Delta T) = 2000 \times 40/(0.80 \times 50) = 2000$ J kg^{-1} K^{-1}. Too high: some of the 80 kJ went to the surroundings, so the oil's real energy gain was smaller.
6. $0.30(80 - T) = 0.20(T - 20)$, so $T = 28/0.50 = 56$ °C.
7. $Q = 0.25 \times 2.26 \times 10^6 = 5.7 \times 10^5$ J.

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8. During an ideal pure-substance melting stage at fixed pressure, energy changes interparticle arrangements and raises interaction potential energy while temperature and modeled mean kinetic energy remain unchanged. For ice, this does not break the covalent bonds inside water molecules.
9. $\Delta Q/\Delta t = 390 \times 1.0 \times 10^{-4} \times 80/0.50 = 6.2$ W.
10. Air next to the radiator warms, expands and becomes less dense, so it rises. Cooler, denser air moves in to replace it, is warmed in turn, and a circulating current carries energy around the room.
11. $L = 5.67 \times 10^{-8} \times 0.040 \times 500^4 = 140$ W. Doubling T multiplies the power by $2^4 = 16$, to about 2300 W.
12. $T = \frac{2.9\times10^{-3}}{290\times10^{-9}} = 1.0 \times 10^4$ K; blue-white.
13. $d = \sqrt{L/(4\pi b)} = \sqrt{3.8 \times 10^{28}/(4\pi \times 2.0 \times 10^{-9})} = 1.2 \times 10^{18}$ m (about 130 light-years).
14. $L \propto R^2T^4$, so $R^2 \propto 1/T^4$ at equal L: $R_X/R_Y = 1/2^2 = 1/4$.

Exam tips

- Use kelvin for absolute temperature in gas-energy, radiation and Wien equations. Temperature changes have the same numerical value in K and °C.
- In the classical ideal-gas model, temperature tracks mean translational kinetic energy per particle; internal energy includes total microscopic kinetic and interaction potential energies.
- In mixing, balance all gains and losses, including each phase change and any significant container heat capacity.
- A fixed-pressure pure-substance phase-change plateau has continuing input; a measured plateau can also reflect balanced input and losses.
- Unaccounted heat losses make an experimentally inferred specific heat capacity too large.

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Greenhouse effect

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches B.2 Greenhouse effect in full, for the course first assessed in 2025. There is no additional higher level content in B.2, so SL and HL students study the same material.

Prerequisites: the Stefan–Boltzmann law, Wien's law and luminosity from B.1; the electromagnetic spectrum; resonance as a driven oscillation at the natural frequency. Use $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} and $S = 1360$ W m^{-2} for the Earth.

Energy balance and the solar constant

Why does an atmosphere change the energy balance?

A planet's orbital distance alone does not determine its surface temperature: reflection, heat redistribution and atmospheric radiation also matter. This unit first builds an ideal uniform-temperature radiative model, then adds a layer that absorbs and emits infrared. The numerical layer diagram is an illustrative calculation, not a complete measured Earth energy budget.

Conservation of energy for a planet

A planet gains energy as radiation from its star and loses energy as radiation to space. When the two powers are equal the planet is in **radiative equilibrium** 辐射平衡 and its temperature stays constant:

$$\text{power absorbed} = \text{power radiated}$$

If the planet absorbs more than it radiates, its temperature rises. A warmer planet radiates more ($\propto T^4$), so it settles at a new, higher equilibrium temperature.

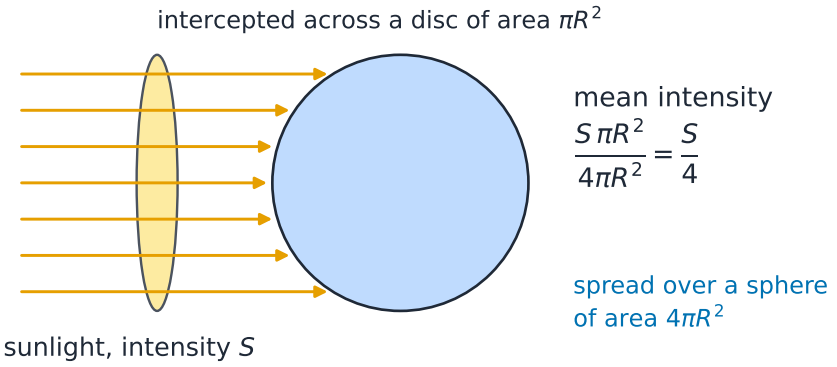
The solar constant

The **solar constant** 太阳常数 S is the intensity of the Sun's radiation at the Earth's distance, measured above the atmosphere on a surface perpendicular to the rays: $S \approx 1360$ W m^{-2}. It is the apparent brightness of the Sun from B.1, $S = L_\odot / 4\pi d^2$. At Mars, 1.52 times further from the Sun, it is $1360/1.52^2 = 590$ W m^{-2}.

Why the mean intensity is S/4

The Earth intercepts sunlight across a disc of area πR^2 but has full surface area $4\pi R^2$. The whole-sphere mean incoming intensity is $S\pi R^2/(4\pi R^2) = S/4$, about 340 W m^{-2} here, irrespective of rotation. This geometric mean does not imply uniform local temperatures.

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The Earth intercepts sunlight across a disc but radiates from its whole surface

Albedo and emissivity

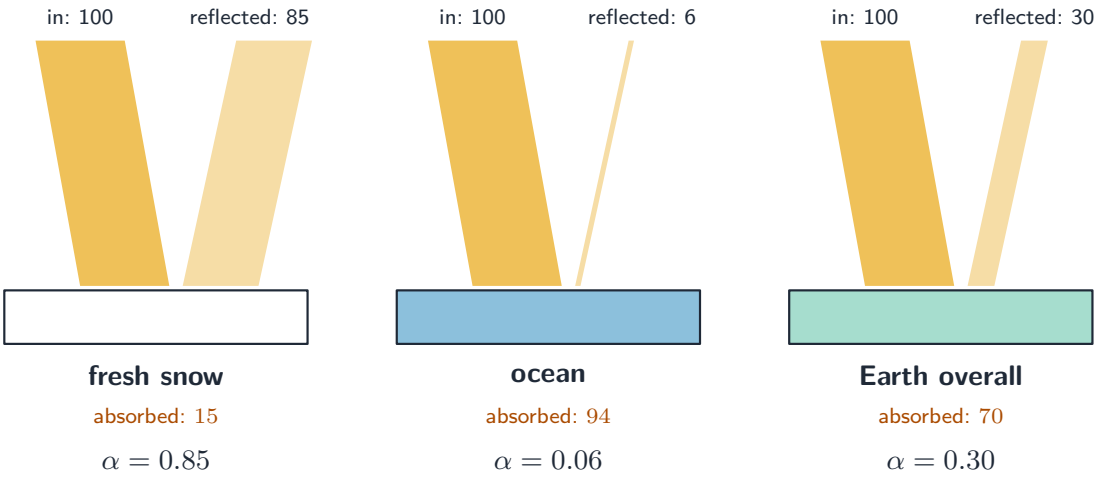
Albedo

The **albedo** 反照率 α of a body is the fraction of the incident power it scatters or reflects:



Melt ponds on Arctic sea ice: dark water absorbs far more sunlight than bright ice, so a lower albedo speeds further melting

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Albedo is the fraction reflected; the fraction absorbed is $1 - \alpha$

$$\alpha = \frac{\text{total scattered power}}{\text{total incident power}}$$

Fresh snow has an albedo of about 0.85, ocean about 0.06 and the Earth as a whole about 0.30. The fraction absorbed is $1 - \alpha$. The Earth's albedo varies from day to day and with latitude: it is higher where there is cloud cover, ice or snow, and where sunlight strikes at a low angle near the poles.

■ Emissivity

Real surfaces radiate less than a black body at the same temperature. The **emissivity** 发射率 e is

$$e = \frac{\text{power radiated per unit area}}{\sigma T^4}$$

so a body of emissivity e radiates $P = e\sigma AT^4$. A black body has $e = 1$; every real surface has $e < 1$. A surface at 300 K radiating 410 W m^{-2} has $e = 410/(5.67 \times 10^{-8} \times 300^4) = 0.89$.

Do not confuse the two: albedo is about **reflecting incoming** radiation (mostly visible), emissivity about **emitting** radiation (mostly infrared).

Equilibrium temperature of a planet

■ A planet with no atmosphere

Per square metre of surface, the absorbed intensity is $(1 - \alpha)S/4$ and the emitted intensity is $e\sigma T^4$. In equilibrium:

$$(1 - \alpha)\frac{S}{4} = e\sigma T^4$$

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Worked example: the Earth with no greenhouse effect, $\alpha = 0.30$ and $e = 1$.

- Absorbed: $0.70 \times 340 = 238$ W m^{-2}.
- $T = \left(\dfrac{238}{5.67 \times 10^{-8}}\right)^{1/4} = 255$ K, about -18 °C.

A conventional representative surface value of 288 K is about 33 K above this 255 K effective-temperature comparison, illustrating the **greenhouse effect** 温室效应. This keeps albedo fixed and is not a claim that removing the atmosphere would leave every other property unchanged.

Body	S / **W m^{-2}**	**Albedo**	**Predicted T / K**	**Measured mean surface T / K**
Earth	1360	0.30	255	288
Mars	590	0.25	210	about 210
Venus	2600	0.75	231	737

Mars, with a very thin atmosphere, matches the prediction. Venus, with a thick carbon dioxide atmosphere, is 500 K hotter than predicted, even though it reflects most sunlight.

Using an effective emissivity

One way to include the atmosphere is to treat the Earth as a body with an **effective emissivity** below 1: the atmosphere stops some of the surface's radiation from reaching space. For a 288 K surface to radiate only 238 W m^{-2} to space:

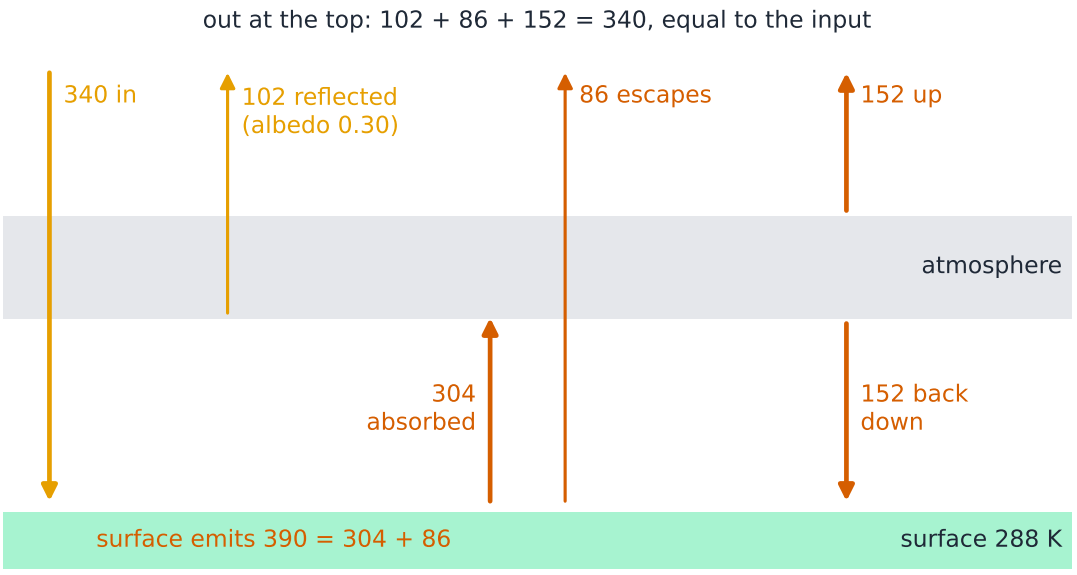
$$e = \frac{238}{5.67 \times 10^{-8} \times 288^4} = 0.61$$

Surface and atmosphere energy balance

A one-layer model

A better model treats the surface and the atmosphere as two bodies that exchange energy. All values are mean intensities in W m^{-2}.

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Every layer is in balance: each arrow into a layer is matched by arrows out of it

- Sunlight: 340 arrives, 102 is reflected ($\alpha = 0.30$), 238 is absorbed by the surface.
- The surface at 288 K emits $\sigma T^4 = 390$. The atmosphere absorbs 78% of it, 304; 86 escapes through to space.
- The atmosphere radiates the 304 it absorbs equally in all directions: 152 up to space and 152 back down to the surface.

Check each part:

Part	In	Out
surface	$238 + 152 = 390$	390
atmosphere	304	$152 + 152 = 304$
top of atmosphere	340	$102 + 86 + 152 = 340$

The surface receives energy from the Sun **and** from the atmosphere, so it must be warmer than 255 K to radiate it all away. That back-radiation is the greenhouse effect.

■ What changes when the atmosphere absorbs more

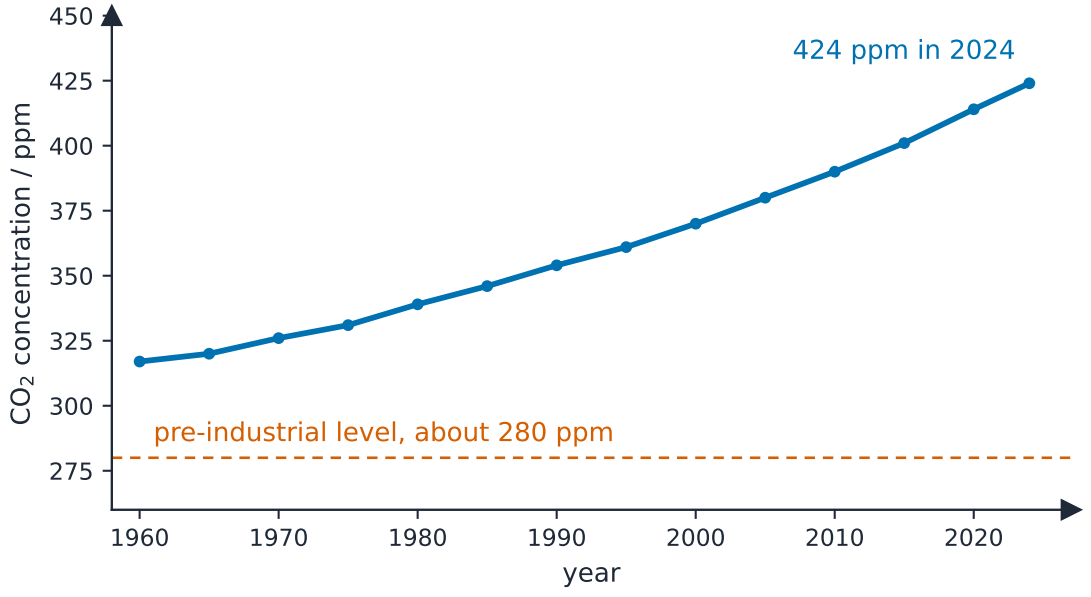
If more greenhouse gas makes the atmosphere absorb 80% of the surface's radiation, it absorbs 312 and sends 156 back down. The surface then receives $238+156 = 394 > 390$: it gains energy and warms until it radiates enough to restore the balance, at about 289 K in this model.

Greenhouse gases

A **greenhouse gas** 温室气体 absorbs infrared radiation. The main ones are water vapour, carbon dioxide, methane and nitrous oxide. Each has both natural and human sources.

Gas	Natural sources	Human sources
water vapour H_2O	evaporation from oceans, lakes and plants	irrigation; more evaporation in a warmer climate
carbon dioxide CO_2	respiration, volcanoes, forest fires	burning fossil fuels, deforestation, cement
methane CH_4	wetlands, termites	rice paddies, cattle, landfill, gas leaks
nitrous oxide N_2O	bacteria in soil and oceans	nitrogen fertilizers, industry

Nitrogen and oxygen, 99% of dry air, are not greenhouse gases: they barely absorb infrared radiation.



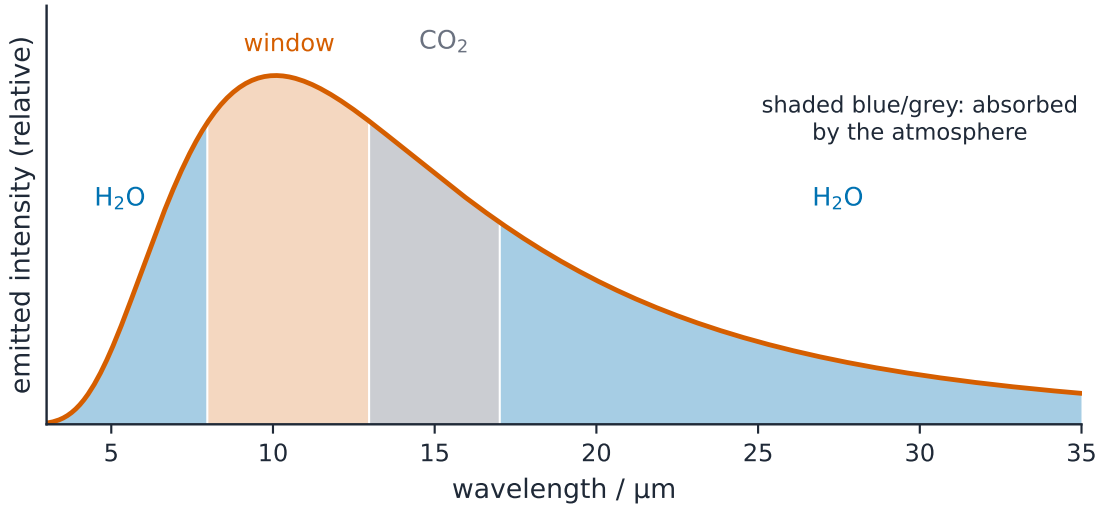
Carbon dioxide measured at Mauna Loa, Hawaii (rounded annual means)

How greenhouse gases absorb infrared

■ Sunlight in, infrared out

The Sun (5800 K) peaks in the visible at 500 nm, and the atmosphere is largely transparent to this radiation. The Earth's surface (288 K) peaks in the infrared at about 10 μm, and greenhouse gases absorb strongly in parts of this range. Energy gets in easily but gets out with difficulty.

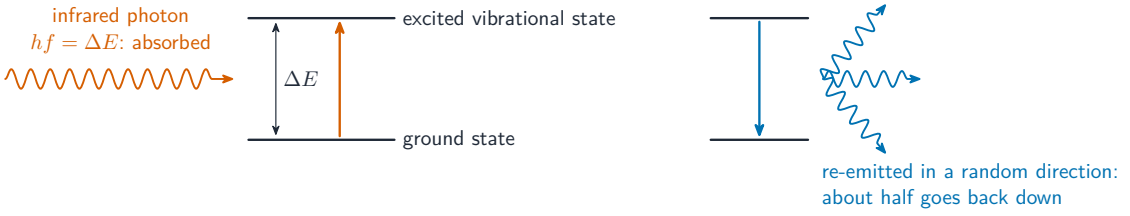
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Schematic: most of the Earth's infrared falls in bands absorbed by water vapour and carbon dioxide

The molecular energy level model

Molecules have discrete vibrational energy levels. A molecule absorbs an infrared **photon** 光子 only if the photon's energy hf equals the gap between two levels. The absorbed energy can be redistributed through collisions as well as exchanged radiatively. A warm atmosphere emits infrared both upward and downward; the one-layer model assigns equal powers to those directions. This does not require every absorbed surface photon to be immediately re-emitted at the same energy by the same molecule.



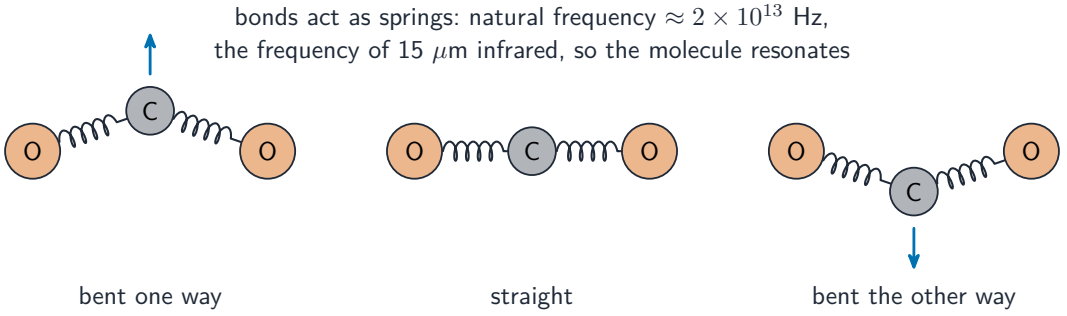
Absorption needs $hf = \Delta E$; re-emission in a random direction sends about half back down

Worked example: carbon dioxide absorbs strongly at 15 μm. The energy gap is $E = hc/\lambda = 6.63 \times 10^{-34} \times 3.00 \times 10^{8}/(15 \times 10^{-6}) = 1.3 \times 10^{-20}$ J.

The resonance model

A molecule acts like masses (atoms) on springs (bonds). The bending of a CO_2 molecule has a **natural frequency** 固有频率 of about 2.0×10^{13} Hz. Infrared radiation of the same frequency, $c/\lambda = 3.00 \times 10^{8}/(15 \times 10^{-6}) = 2.0 \times 10^{13}$ Hz, drives the vibration at **resonance** 共振, so the molecule absorbs energy strongly at that

frequency and hardly at all at others. Both models predict absorption at the same frequencies: the resonant frequency of the classical model corresponds to the energy gap $E = hf$ of the quantum model.



The bending mode of CO_2 resonates with 15 μm infrared

The enhanced greenhouse effect

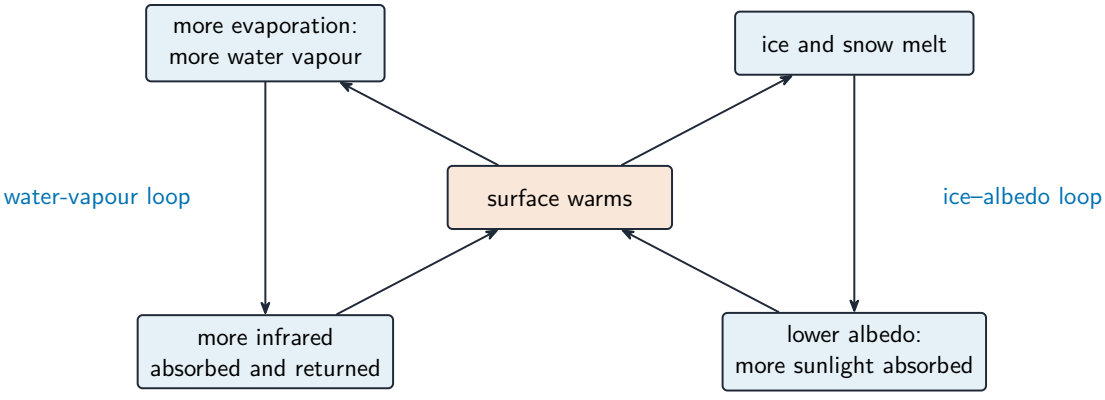
The conventional 255 K versus 288 K comparison illustrates natural greenhouse warming within the stated fixed-albedo model; it does not describe a fully modeled removal of every atmospheric influence. The **enhanced greenhouse effect** 增强温室效应 is the extra warming caused by human activities that increase the concentration of greenhouse gases. The primary cause is the burning of fossil fuels, which has raised the CO_2 concentration from roughly 280 ppm in the preindustrial comparison to much higher modern levels. The NOAA Annual Greenhouse Gas Index reports a 2025 global remote-marine-boundary-layer annual mean of 425.62 ± 0.05 ppm; this is a dated network average, not every local concentration.

- More greenhouse gas absorbs more of the surface's infrared and sends more back down.
- Less energy leaves at the top of the atmosphere, so the Earth absorbs more than it radiates.
- The surface warms until it radiates enough to restore the balance.

Feedback loops amplify the change. A warmer ocean evaporates more water vapour, itself a greenhouse gas. Melting ice exposes darker land and sea, lowering the albedo so that more sunlight is absorbed.



Melting land and sea ice lowers the albedo of the polar regions

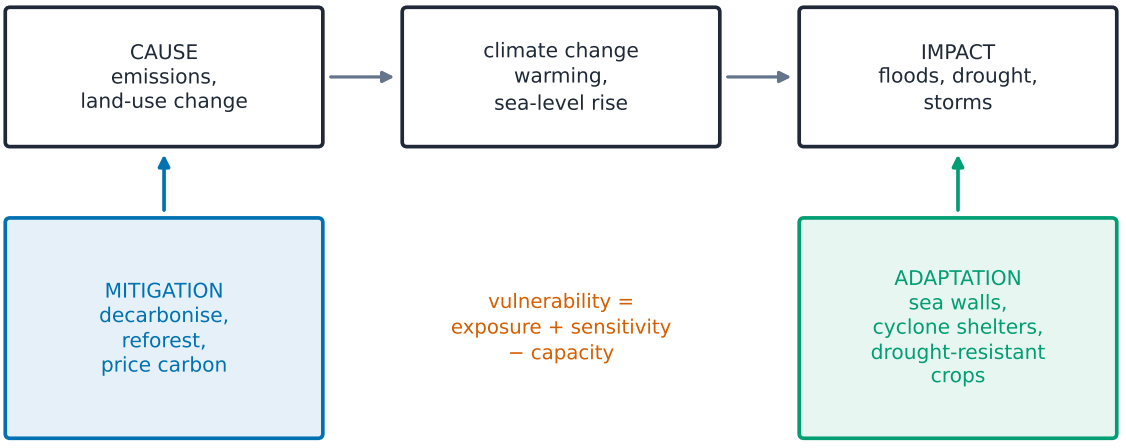


Positive feedback: each loop returns to warm the surface further

Responding to climate change and judging evidence

Mitigation addresses the causes of climate change, for example replacing fossil-fuel electricity with a lower-emission supply. **Adaptation** reduces harm from its impacts, for example protecting buildings against flooding. A flood barrier can reduce damage without reducing greenhouse gas emissions.

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Adaptation raises capacity; mitigation shrinks the hazard itself.

In an invented town decision, compare replacing diesel buses with electric buses against building a flood barrier. Estimate the buses' lifetime emissions using the electricity supply as well as vehicle use; estimate the barrier's protection using local flood evidence. Compare effectiveness, construction and running costs, and who pays and benefits. For an HL assessment, connect the choice to law (required permits and protection standards), economics (costs and avoided losses) and ethics (whether poorer households receive protection). A cheap proposal alone is not necessarily the best decision.

For a climate-data comparison, state the region, the years and the uncertainty. Compare multi-year averages using the same reference period for temperature changes; do not compare anomalies based on different baselines without converting them. A local hot year alone does not establish a global long-term trend.

Practice

1. Explain why the mean intensity of sunlight over the Earth's surface is $S/4$.
2. A snowfield receives 400 W m^{-2} and reflects 340 W m^{-2}. Find its albedo and the power per square metre it absorbs.
3. A surface at 300 K radiates 410 W m^{-2}. Find its emissivity.
4. Find the equilibrium temperature of Mars ($S = 590$ W m^{-2}, albedo 0.25, emissivity 1).
5. Melting ice lowers the Earth's albedo from 0.30 to 0.28. Find the new equilibrium temperature with no atmosphere, and explain the direction of the change.
6. A planet receives $S = 1000$ W m^{-2}, has albedo 0.20 and emissivity 0.90. Find its equilibrium temperature.
7. In the one-layer model, show that the surface energy balance holds for a surface at 288 K.

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8. Name the four main greenhouse gases, and give one natural and one human source of methane.
9. Explain, using molecular energy levels, how a greenhouse gas warms the surface below it.
10. Find the energy of a 15 μm infrared photon, and the frequency of the radiation.
11. Venus has an albedo of 0.75 and a predicted equilibrium temperature of 231 K, yet its surface is at 737 K. Explain.
12. Outline how burning fossil fuels leads to a rise in the Earth's mean surface temperature.

Answers.

1. The Earth intercepts sunlight over a disc of area πR^2, and dividing by the whole surface area $4\pi R^2$ gives the global geometric mean, irrespective of rotation: $S\pi R^2/4\pi R^2 = S/4$.
2. $\alpha = 340/400 = 0.85$; absorbed $= 60$ W m^{-2}.
3. $e = 410/(5.67 \times 10^{-8} \times 300^4) = 410/459 = 0.89$.
4. $T = (0.75 \times 590/4 \,/\, 5.67 \times 10^{-8})^{1/4} = 210$ K.
5. Absorbed $= 0.72 \times 340 = 245$ W m^{-2}; $T = \left(\frac{244.8}{5.67 \times 10^{-8}}\right)^{1/4} = 256$ K. A lower albedo means more sunlight is absorbed, so the planet must be warmer to radiate it all.
6. $(1 - 0.20) \times 1000/4 = 200$ W m^{-2}; $T = (200/(0.90 \times 5.67 \times 10^{-8}))^{1/4} = 250$ K.
7. In: 238 from the Sun + 152 from the atmosphere $= 390$ W m^{-2}. Out: $\sigma(288)^4 = 390$ W m^{-2}. They are equal.
8. Water vapour, carbon dioxide, methane, nitrous oxide. Methane: wetlands (natural); cattle, rice paddies or landfill (human).
9. The surface emits infrared. A photon whose energy matches the gap between vibrational levels of the molecule is absorbed. Absorbed energy can be redistributed by collisions, and the warm atmosphere emits infrared upward and downward. Downward radiation contributes to the surface balance; it is not necessary for each absorbed photon to be promptly re-emitted unchanged by the same molecule.
10. $E = hc/\lambda = 1.3 \times 10^{-20}$ J; $f = c/\lambda = 2.0 \times 10^{13}$ Hz.
11. The prediction ignores the atmosphere. Venus's thick CO_2 atmosphere absorbs almost all the infrared from the surface and radiates much of it back down, a very strong greenhouse effect.
12. Burning fossil fuels releases CO_2. A higher concentration absorbs more of the surface's infrared and re-emits more back to the surface, so less energy leaves the Earth than it absorbs. The surface warms until it radiates enough to restore the balance.

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Exam tips

- Use $S/4$, not S, for the mean intensity over a planet's surface, and multiply by $1 - \alpha$ for the absorbed part.
- Albedo is about reflected incoming radiation; emissivity is about emitted radiation. Do not swap them.
- In an equilibrium question write power in = power out for each layer, then solve for T^4 before taking the fourth root.
- In energy-flow diagrams check each layer separately: surface, atmosphere and top of the atmosphere.
- When explaining the greenhouse effect, say the surface emits **infrared**, greenhouse gases **absorb** in infrared bands and the warm atmosphere **emits** upward and downward. Collisions can redistribute absorbed energy; check each layer's balance.
- Distinguish the natural greenhouse effect (keeps the Earth habitable) from the enhanced greenhouse effect (human-caused extra warming).

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Gas laws

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches B.3 Gas laws in full, for the course first assessed in 2025. There is no additional higher level content in B.3, so SL and HL students study the same material.

Prerequisites: momentum and Newton's second law from A.2; kelvin temperature, $\overline{E}_k = \frac{3}{2}k_BT$ and internal energy from B.1. Use $R = 8.31$ J K^{-1} mol^{-1}, $k_B = 1.38 \times 10^{-23}$ J K^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1}.

Pressure and amount of substance

Why does a bicycle tyre feel harder on a hot day?

Pump a tyre up in the cool morning and it feels firmer by the afternoon, though no air has been added. A sealed bag of crisps carried up a mountain swells; a plastic bottle closed at the top of the mountain is crushed by the time it reaches the bottom. The pressure, volume and temperature of a gas are linked, and the link comes from billions of molecules hitting the walls.

Pressure

Pressure 压强 is the force per unit area acting perpendicular to a surface:

$$P = \frac{F}{A}$$

in pascals, 1 Pa = 1 N m^{-2}. A 600 N person standing on shoes of total area 0.030 m^2 exerts $600/0.030 = 2.0 \times 10^4$ Pa on the floor. Atmospheric pressure at sea level is about 1.0×10^5 Pa (100 kPa). Gas laws use absolute pressure; a gauge reads the difference from its reference pressure, often the atmosphere. Add that atmospheric pressure before forming a gas-law ratio.

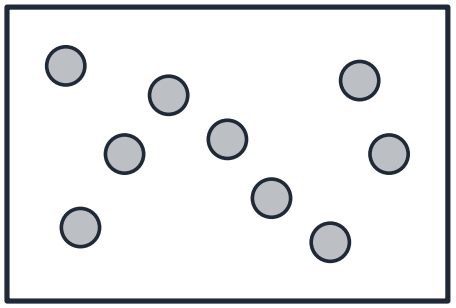
Amount of substance

The **amount of substance** 物质的量 n is measured in moles:

$$n = \frac{N}{N_A}$$

where N is the number of molecules and N_A is the **Avogadro constant** 阿伏伽德罗常数. One mole of any substance contains 6.02×10^{23} particles. The **molar mass** 摩尔质量 is the mass of one mole: 32 g of oxygen gas (O_2, molar mass 32 g mol^{-1}) is 1.0 mol, or 6.02×10^{23} molecules.

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$= 6.02 \times 10^{23}$ particles (N_A)

1 mole

One mole contains the Avogadro number of particles

The empirical gas laws

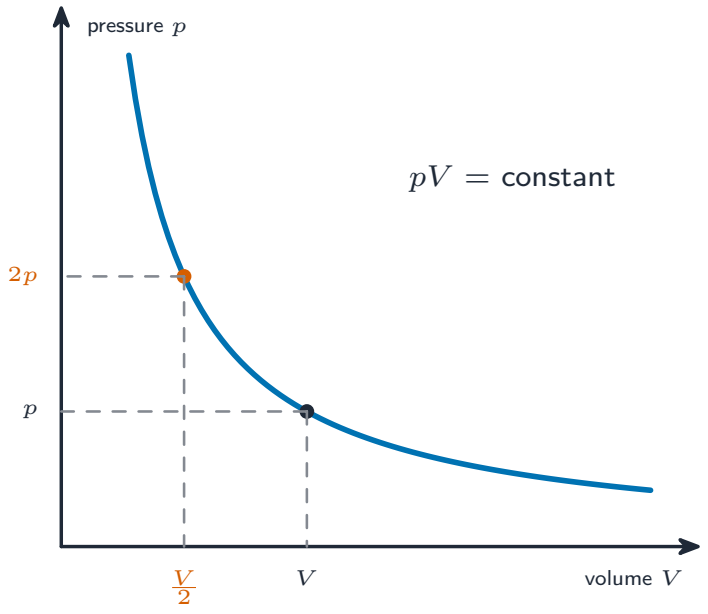
Three laws from experiment

For a fixed amount of gas in a regime well approximated by the ideal model, experiments hold one of P, V and T constant and vary the other two. Temperatures must be in kelvin.



Heating the air in a balloon at constant pressure makes it expand (Charles's law), so its density falls and it rises

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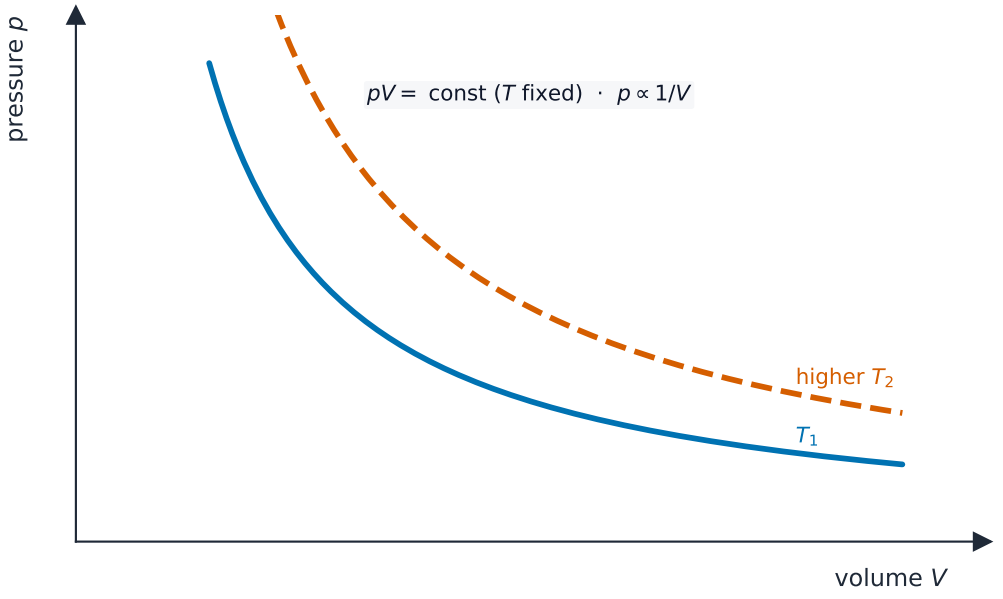
Boyle's law: at constant temperature, pressure times volume is constant

Law	**Held constant**	**Relationship**	**Graph**
Boyle's law	temperature	$PV = \text{constant}$	P against $1/V$ is a straight line through the origin
Charles's law	pressure	$V/T = \text{constant}$	V against T is a straight line through 0 K
pressure law	volume	$P/T = \text{constant}$	P against T is a straight line through 0 K

Worked examples:

- Boyle: a syringe holds 60 cm^3 of air at 100 kPa. Pushed slowly to 20 cm^3, $P = 100 \times 60/20 = 300$ kPa.
- Charles: 2.0 L of gas is heated from 27 °C (300 K) to 127 °C (400 K) at constant pressure: $V = 2.0 \times 400/300 = 2.7$ L.
- Pressure law: a constant-volume tyre model at an absolute pressure of 200 kPa and 7 °C (280 K) warms to 35 °C (308 K): $P = 200 \times 308/280 = 220$ kPa.

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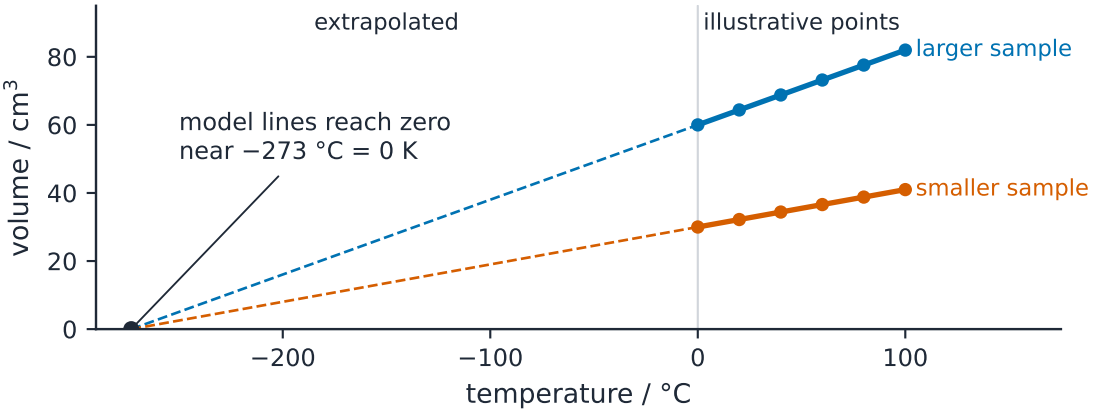


Boyle's law: at constant temperature pV is constant, so a p–V graph is a hyperbola

In the lab: investigate the pressure law using teacher-approved constant-volume apparatus with a pressure sensor in a water bath. Keep the amount of gas fixed; the teacher sets the temperature range and pressure limit. At each bath temperature, wait for the gas to reach thermal equilibrium before recording its absolute pressure and temperature in kelvin. Plot pressure against kelvin temperature and repeat readings to check consistency. Never heat an improvised sealed vessel.

■ Absolute zero from the gas laws

For a fixed amount of gas at constant pressure, volume is approximately linear with Celsius temperature over a range where ideal-gas behaviour is a good approximation. Extrapolating that model gives a zero-volume intercept near −273 °C: **absolute zero** 绝对零度, 0 K. Real gases depart from the ideal model and can condense before that extrapolated state; the graph is not a claim that a real gas reaches zero volume.



Invented ideal-model points illustrate a common extrapolated intercept; these are not measured data

Combining the laws

The three laws combine into one:

$$\frac{PV}{T} = \text{constant} \qquad \text{so} \qquad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

for a fixed amount in the ideal-gas approximation. Worked example: 50 cm^3 of gas at 100 kPa and 300 K is compressed to 20 cm^3 and warms to 360 K. $P_2 = 100 \times \frac{50}{20} \times \frac{360}{300} =$ 300 kPa.

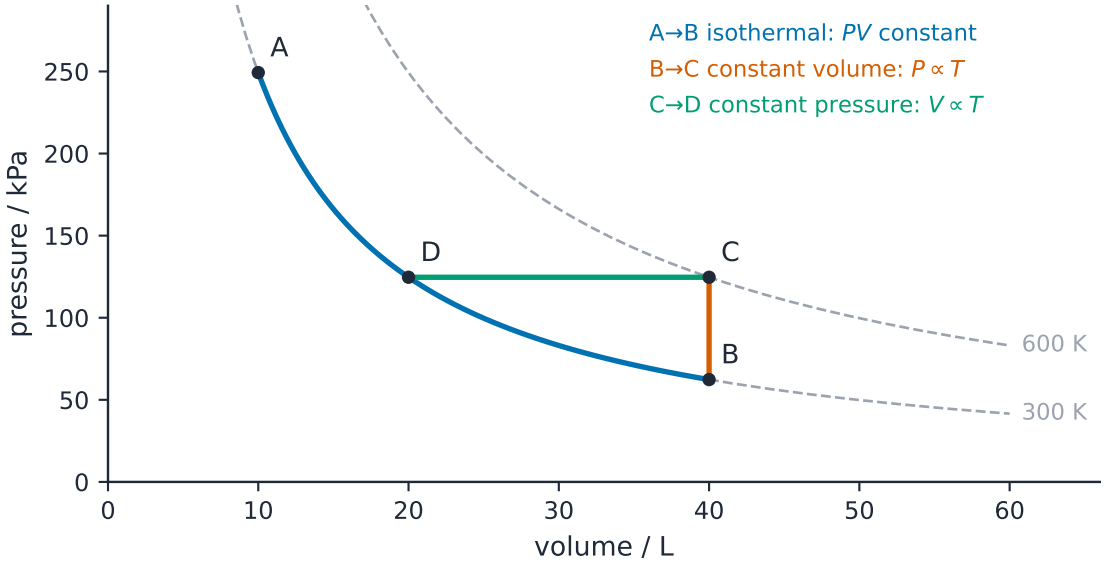
Gauge pressure is not the pressure in a gas law

An invented rigid, sealed vessel has gauge pressure 150 kPa when the gas is at 300 K. Atmospheric pressure stays at 100 kPa. Heating to 360 K gives initial absolute pressure $150 + 100 = 250$ kPa, then final absolute pressure $250(360/300) = 300$ kPa. The final gauge reading is $300 - 100 = 200$ kPa. Multiplying the original gauge reading by the temperature ratio would wrongly give 180 kPa. Hold amount and volume fixed, and convert back to the requested pressure reference at the end.

Pressure–volume diagrams

Changes of a gas are drawn on a **pressure–volume diagram** 压强体积图. Each **isotherm** 等温线 is a curve PV = constant; hotter isotherms lie further from the origin.

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Isothermal changes follow an isotherm, constant-volume changes are vertical lines and constant-pressure changes are horizontal lines

For 1.0 mol: at A (10 L, 300 K) $P = 249$ kPa; at B (40 L, 300 K) $P = 62$ kPa, a quarter of A; heating at constant volume to 600 K doubles the pressure to 125 kPa at C; cooling at constant pressure to 300 K halves the volume to 20 L at D.

The ideal gas law

■ PV = nRT

Experiments show that PV/T is proportional to the amount of gas. The constant of proportionality is the same for every gas at low pressure: the **molar gas constant** 摩尔气体常数 R. This gives the **ideal gas law** 理想气体定律:

$$PV = nRT \qquad \text{or} \qquad PV = Nk_BT$$

The two forms agree because $nR = Nk_B$, so $R = N_Ak_B$.

Worked example: how many moles and molecules of air are in a 1.0 m^3 box at 100 kPa and 300 K?

- $n = PV/RT = 1.0 \times 10^5 \times 1.0/(8.31 \times 300) = 40$ mol.
- $N = nN_A = 40 \times 6.02 \times 10^{23} = 2.4 \times 10^{25}$ molecules.

One mole of any ideal gas at 273 K and 101 kPa occupies $\frac{8.31 \times 273}{1.01 \times 10^5} = 0.0225$ m^3, 22.5 L, whatever the gas.

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Units

Use SI units throughout: P in Pa, V in m^3 ($1\ L = 10^{-3}\ m^3$, $1\ cm^3 = 10^{-6}\ m^3$), T in K and n in mol. A ratio question with the gas laws can keep any units for P and V, as long as both sides match, but T must always be in kelvin.

Kinetic theory of an ideal gas

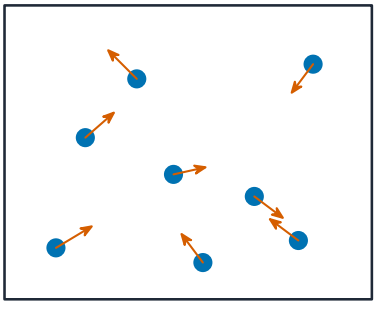
The model

An **ideal gas** 理想气体 is a model that explains the gas laws by the motion of its molecules. The **kinetic theory** 分子动理论 assumes:



An aneroid barometer: air pressure is the force per unit area from countless molecular collisions

- The gas has a very large number of identical molecules in random motion.
- The volume of the molecules themselves is negligible compared with the volume of the container.
- There are no forces between molecules except during collisions.
- Collisions with each other and with the walls are elastic, and last a negligible time.
- Between collisions, molecules move in straight lines at constant speed.



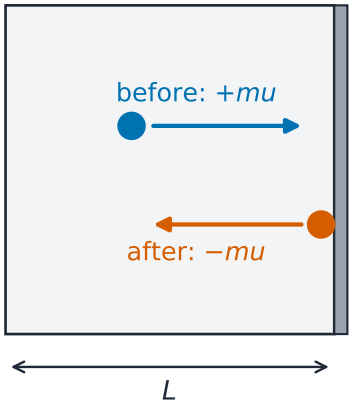
1. many identical molecules, in continuous **random motion**
2. their own volume is **tiny** next to the container
3. straight lines between hits - intermolecular forces **ignored**
4. collisions are **elastic**: no kinetic energy is lost

poor at very high pressure (volume matters) or very low temperature (forces matter)

The key assumptions of the kinetic theory of an ideal gas

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Pressure from molecular collisions



each collision: $\Delta p = -2mu$

time between hits on this wall: $2L/u$

force on the wall $= \dfrac{2mu}{2L/u} = \dfrac{mu^2}{L}$

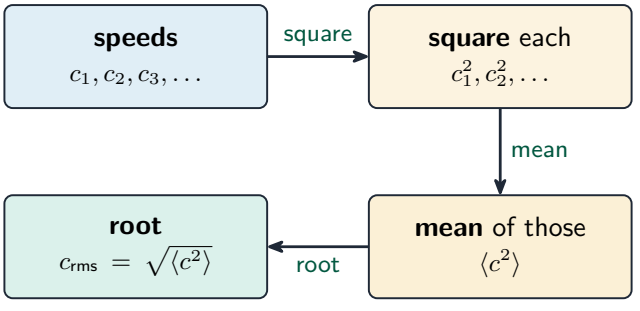
add all N molecules, average over 3 directions:
$P = \dfrac{1}{3}\rho\overline{v^2}$

Each collision changes the molecule's momentum by 2mu; many collisions each second make a steady force

In the simplified cube model, take u as the velocity component perpendicular to the wall. A molecule of mass m bounces back elastically, so its momentum changes by $-2mu$. By Newton's third law the wall receives an impulse $+2mu$. Ignoring intervening collisions for this one-molecule argument, the perpendicular round trip is $2L$, so it hits it every $2L/u$ seconds, and the average force it exerts is $\dfrac{2mu}{2L/u} = \dfrac{mu^2}{L}$. Adding up all the molecules, and averaging over the three directions of motion, gives:

$$P = \frac{1}{3}\rho\overline{v^2}$$

where ρ is the density of the gas and $\overline{v^2}$ is the mean of the squared speeds. The square root $\sqrt{\overline{v^2}}$ is the **root mean square speed** 方均根速率 v_{rms}.



read the name **backwards**: **r**oot of the **m**ean of the **s**quares

Root-mean-square speed: square each speed, take the mean, then the square root

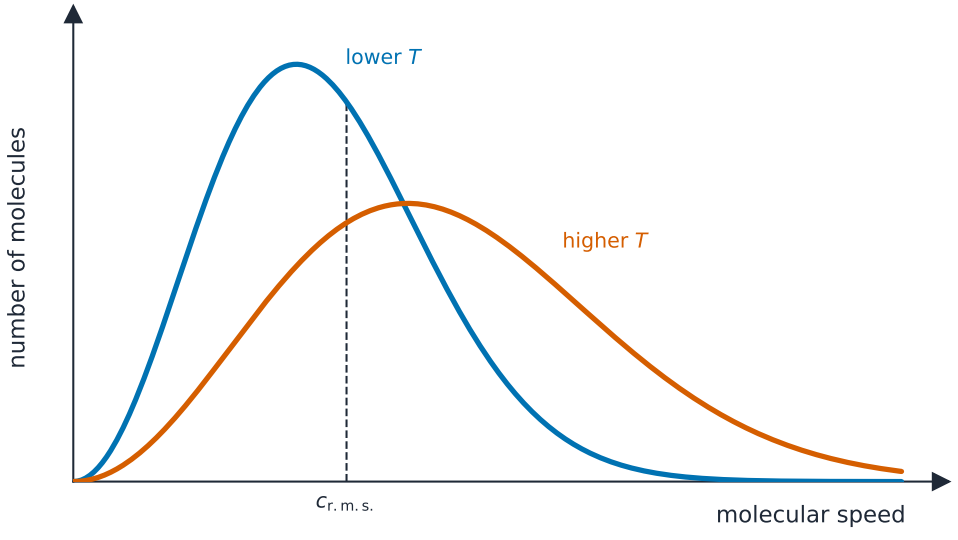
Worked example: an invented single-species ideal-gas sample at 100 kPa has density 1.2 kg m^{-3}. $v_{\text{rms}} = \sqrt{3P/\rho} = \sqrt{3 \times 1.0 \times 10^5 / 1.2} = 500$ m s^{-1} in this model. For a mixture such as air, species have different rms speeds; a one-species calculation is an effective approximation.

Explaining the gas laws

- **Boyle's law:** halving the volume at constant temperature keeps the mean translational kinetic energy unchanged and doubles the number density. The mean momentum transferred per unit wall area per second therefore doubles, so the pressure doubles. Individual wall-hit rates also depend on container geometry.
- **Pressure law:** heating at constant volume makes the molecules faster. They hit the walls more often and with a greater change of momentum each time, so the pressure rises.
- **Charles's law:** heating increases molecular speeds, and the gas expands against a movable boundary. The reduced number density balances the increased momentum transfer per particle until the pressure again matches the external constraint.

Temperature and molecular speed

Combining $PV = Nk_BT$ with $P = \frac{1}{3}\rho\overline{v^2} = \frac{1}{3}\frac{Nm}{V}\overline{v^2}$ gives $\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$: the average translational kinetic energy of a molecule depends only on the temperature. At the same temperature lighter molecules move faster. At 300 K, nitrogen (molar mass 28 g mol^{-1}) has $v_{\text{rms}} = \sqrt{3k_BT/m} = 520$ m s^{-1}, while helium (4 g mol^{-1}) has 1370 m s^{-1}. Doubling the kelvin temperature multiplies v_{rms} by $\sqrt{2}$.



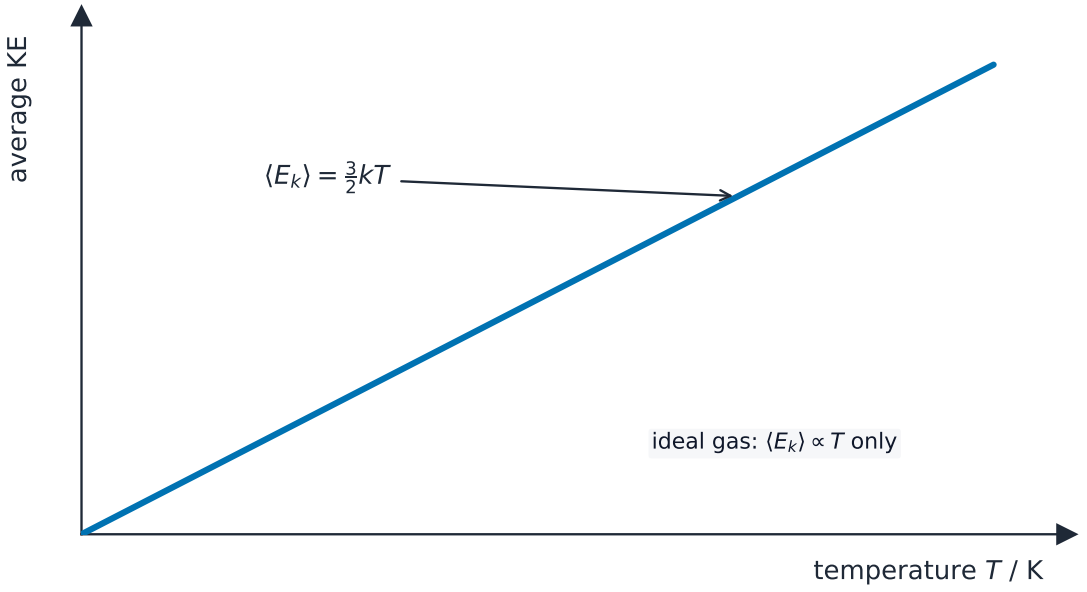
higher T: peak → higher speeds, flatter curve

The spread of molecular speeds: a higher temperature broadens the curve and shifts it to faster speeds

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Internal energy of an ideal gas

In the ideal-gas model, intermolecular interaction energy is neglected. For a classical **monatomic** 单原子 gas, such as the helium or argon models here, internal energy is translational kinetic energy:



Average molecular kinetic energy is proportional to kelvin temperature: $\overline{E}_k = \frac{3}{2}k_BT$

$$U = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$$

Polyatomic ideal gases can also have rotational and vibrational internal energy; neglecting intermolecular forces does not remove those contributions. The three-halves formula is not a formula for every ideal gas. For a fixed amount and composition in the ideal model, internal energy depends only on temperature. Worked example: 2.0 mol of helium at 400 K has $U = 1.5 \times 2.0 \times 8.31 \times 400 = 1.0 \times 10^4$ J. Heated to 800 K, its internal energy doubles.

Real gases and ideal gases

A **real gas** 实际气体 behaves like an ideal gas when its molecules are far apart and fast:

- **low pressure** and **low density**: the volume of the molecules is negligible, and molecules are rarely close enough to attract each other;
- **high temperature**, well above the temperature at which the gas liquefies: the molecules' kinetic energy is much larger than any intermolecular potential energy.

At high pressure the volume of the molecules is no longer negligible, and at suitable lower temperatures attractions can reduce pressure relative to the ideal prediction.

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Condensation can occur when temperature and pressure permit it; lowering temperature does not imply that every gas at every pressure must pass into a liquid phase. The ideal gas model cannot describe a liquid at all. Air at room temperature and atmospheric pressure is very close to ideal.



Compressed gas cylinders store a fixed mass of gas at high pressure.

Practice

1. A 1200 kg car rests on four tyres, each touching the road over 0.015 m^2. Find the pressure on the road.
2. Find the number of molecules in 0.50 mol of carbon dioxide, and the mass of this gas (molar mass 44 g mol^{-1}).
3. A diver at 10 m depth (200 kPa) breathes out a 4.0 cm^3 bubble. Find its volume at the surface (100 kPa), assuming constant temperature.
4. A rigid, sealed container of ideal gas at 20 °C and absolute pressure 150 kPa is heated to 120 °C. Find the new pressure.
5. Explain why the temperature in the gas laws must be in kelvin.
6. Find the volume of 0.25 mol of an ideal gas at 300 K and 120 kPa.
7. Use an ideal-gas estimate for a cylinder of volume 0.020 m^3 holding oxygen at absolute pressure 1.5×10^7 Pa and 290 K. Find the amount and mass of oxygen (molar mass 32 g mol^{-1}), and state why the high pressure makes checking this approximation important.
8. On a P–V diagram, sketch (a) an isothermal compression, (b) a heating at constant volume, (c) a cooling at constant pressure.
9. State four assumptions of the kinetic model of an ideal gas.

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10. Use the kinetic model to explain why the pressure of a gas rises when it is heated at constant volume.
11. A gas of density 0.18 kg m^{-3} is at 100 kPa. Find the root mean square speed of its molecules.
12. Find the internal energy of 3.0 mol of argon at 300 K, and the energy that must be added to warm it to 400 K at constant volume.
13. State and explain the conditions under which a real gas behaves most like an ideal gas.
14. An invented rigid, sealed ideal-gas vessel initially reads 150 kPa on a gauge at 300 K. Atmospheric pressure is 100 kPa and remains constant. Find the final absolute pressure and gauge reading at 360 K. Explain why scaling the gauge reading directly fails.

Answers.

1. $P = 1200 \times 9.8/(4 \times 0.015) = 2.0 \times 10^5$ Pa.
2. $N = 0.50 \times 6.02 \times 10^{23} = 3.0 \times 10^{23}$; mass $= 0.50 \times 44 = 22$ g.
3. $V = 4.0 \times 200/100 = 8.0$ cm^3.
4. $P = 150 \times 393/293 = 201$ kPa.
5. The gas laws are proportionalities that pass through zero at absolute zero. On the Celsius scale zero is arbitrary: doubling a Celsius temperature does not double the pressure or volume.
6. $V = nRT/P = \frac{0.25 \times 8.31 \times 300}{1.2 \times 10^5} = 5.2 \times 10^{-3}$ m^3 (5.2 L).
7. $n = PV/(RT) = 300\,000/(8.31 \times 290) = 124.49$ mol; $m = 124.49(0.032) \approx 4.0$ kg. This is an ideal estimate at 150 bar; a reliable real mass needs real-gas data or measurement.
8. (a) A curve along an isotherm towards smaller volume and higher pressure. (b) A vertical line upwards. (c) A horizontal line towards smaller volume.
9. Any four of: very many identical molecules in random motion; negligible molecular volume; no intermolecular forces except in collisions; elastic collisions; collisions take negligible time; straight-line motion between collisions.
10. The molecules gain kinetic energy and move faster. They hit the walls more often and each collision gives a larger change of momentum, so the force per unit area on the walls increases.
11. $v_{\text{rms}} = \sqrt{3P/\rho} = \sqrt{3 \times 1.0 \times 10^5/0.18} = 1.3 \times 10^3$ m s^{-1}.
12. $U = 1.5 \times 3.0 \times 8.31 \times 300 = 1.1 \times 10^4$ J; $\Delta U = 1.5 \times 3.0 \times 8.31 \times 100 = 3.7 \times 10^3$ J.
13. Low pressure (low density) and high temperature. The molecules are then far apart, so their own volume is negligible, and fast, so intermolecular forces have little effect.
14. Use absolute pressure: $P_1 = 150 + 100 = 250$ kPa, so $P_2 = 250(360/300) = 300$ kPa. Subtract 100 kPa to get 200 kPa gauge. Scaling the gauge reading gives the wrong 180 kPa because its reference is atmosphere, not vacuum.

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Exam tips

Use kelvin and absolute pressure; convert volume to m^3 with $PV = nRT$. Identify the fixed amount and constraint before selecting a gas law. Keep n, R together and N, k_B together. On P–V axes, fixed volume is vertical, fixed pressure horizontal, and fixed temperature an isotherm. Explain pressure through normal momentum change and collision rate. Reserve $U = \frac{3}{2}nRT$ for the classical monatomic ideal-gas model.

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Thermodynamics

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches B.4 Thermodynamics in full, for the course first assessed in 2025. The whole unit is additional higher level (AHL); there is no SL counterpart.

Prerequisites: internal energy and $\overline{E}_k = \frac{3}{2}k_BT$ from B.1; $PV = nRT$, P–V diagrams and $U = \frac{3}{2}nRT$ from B.3; work as energy transferred from A.3. Quantitative problems use monatomic ideal gases. Use $R = 8.31$ J K^{-1} mol^{-1} and $k_B = 1.38 \times 10^{-23}$ J K^{-1}.

AHL: Systems and the first law

Why can't a power station use all its fuel energy?

An invented power-station model receives fuel energy at 1300 MW but delivers only 500 MW of electricity; the rest warms a river or the air through its cooling towers. Better engineering helps, but no design, however perfect, could turn all of that heat into work. Ice melts in a warm drink, but a cold drink never spontaneously freezes into ice while warming the room. Thermodynamics explains both: the first law counts energy, and the second law says which way energy can flow.



A power station is a heat engine: it converts heat into useful work.

Closed and isolated systems

- A **closed system** 封闭系统 exchanges no mass with its surroundings but can exchange energy, as heat or as work. A gas sealed in a cylinder with a piston is a closed system.
- An **isolated system** 孤立系统 exchanges neither mass nor energy. The universe as a whole is an isolated system.

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The first law of thermodynamics

The **first law of thermodynamics** 热力学第一定律 is conservation of energy for a closed system:

$$Q = \Delta U + W$$

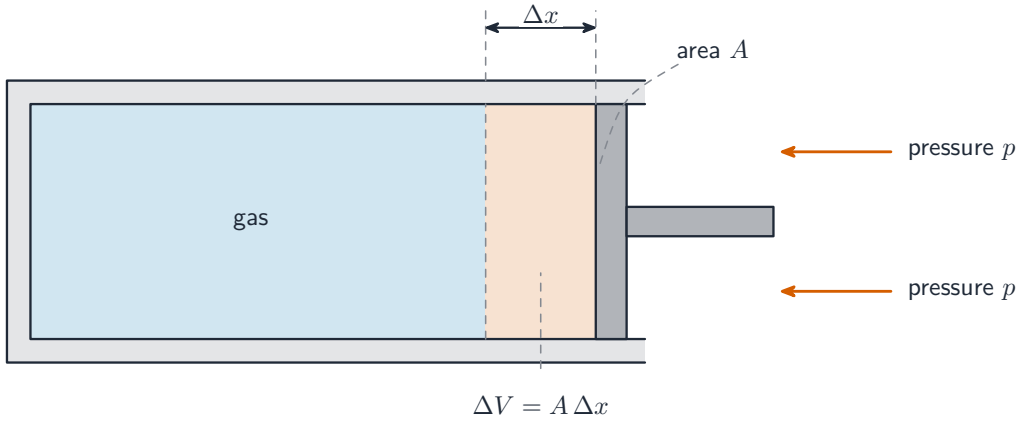
- Q is the thermal energy supplied **to** the system: positive if heat enters, negative if heat leaves.
- ΔU is the change in the system's internal energy.
- W is the work done **by** the system: positive when the gas expands, negative when it is compressed (work done on the system).

Worked example: 300 J of heat enters a gas and the gas does 120 J of work pushing a piston out. $\Delta U = Q - W = 300 - 120 = 180$ J, so a fixed amount of ideal gas warms. If instead 120 J of work were done **on** the gas, $W = -120$ J and $\Delta U = 420$ J.

AHL: Work and internal energy of a gas

Work done when the volume changes

A gas at pressure P pushes a piston of area A with force PA. If the piston moves a small distance Δx, the work done by the gas is $PA\Delta x$:

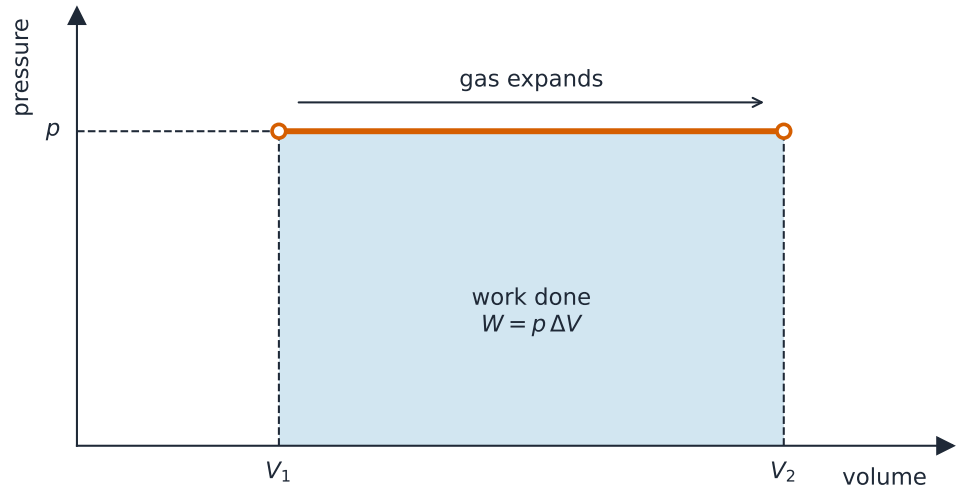


The piston moves out by Δx: the gas does work $W = PA\,\Delta x = P\Delta V$

$$W = P\Delta V$$

This holds for constant external resisting pressure. In a slow equilibrium piston process the gas pressure closely matches that pressure; when it changes, the work is the **area under the actual P–V path** between the initial and final volumes. A rapid irreversible change need not have a single uniform gas pressure throughout. Expansion ($\Delta V > 0$) gives positive W; compression gives negative W.

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work = area under p–V · $W = \int p\,dV$

On a pressure–volume graph, the work done at constant pressure is the area under the line: $W = p\,\Delta V$

Change in internal energy

For a monatomic ideal gas, the internal energy depends only on temperature, so

$$\Delta U = \frac{3}{2} N k_B \Delta T = \frac{3}{2} n R \Delta T$$

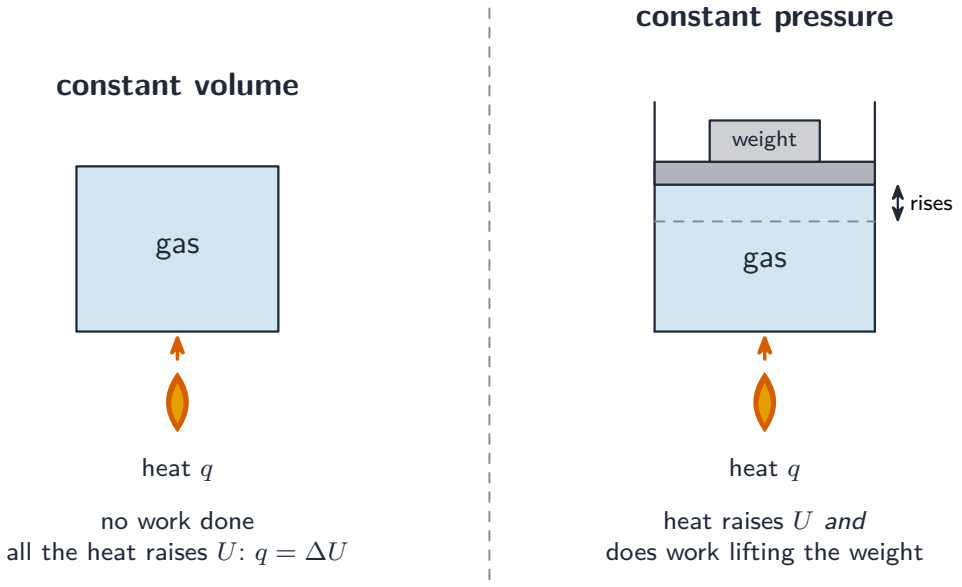
whatever process caused the change. Because $PV = nRT$, this is also $\Delta U = \frac{3}{2}\Delta(PV)$.

Worked example: 0.50 mol of helium at 100 kPa is heated from 300 K to 400 K at constant pressure.

- $\Delta V = \frac{nR\Delta T}{P} = \frac{0.50 \times 8.31 \times 100}{1.0 \times 10^5} = 4.2 \times 10^{-3}$ m^3, so $W = P\Delta V = 416$ J.
- $\Delta U = 1.5 \times 0.50 \times 8.31 \times 100 = 623$ J.
- $Q = \Delta U + W = 1040$ J.

Heating the same gas through 100 K at constant volume needs only $Q = \Delta U = 623$ J, because no work is done. More heat is needed at constant pressure because part of it leaves again as work.

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At constant volume all the heat raises U ($q = \Delta U$); at constant pressure the gas also does work

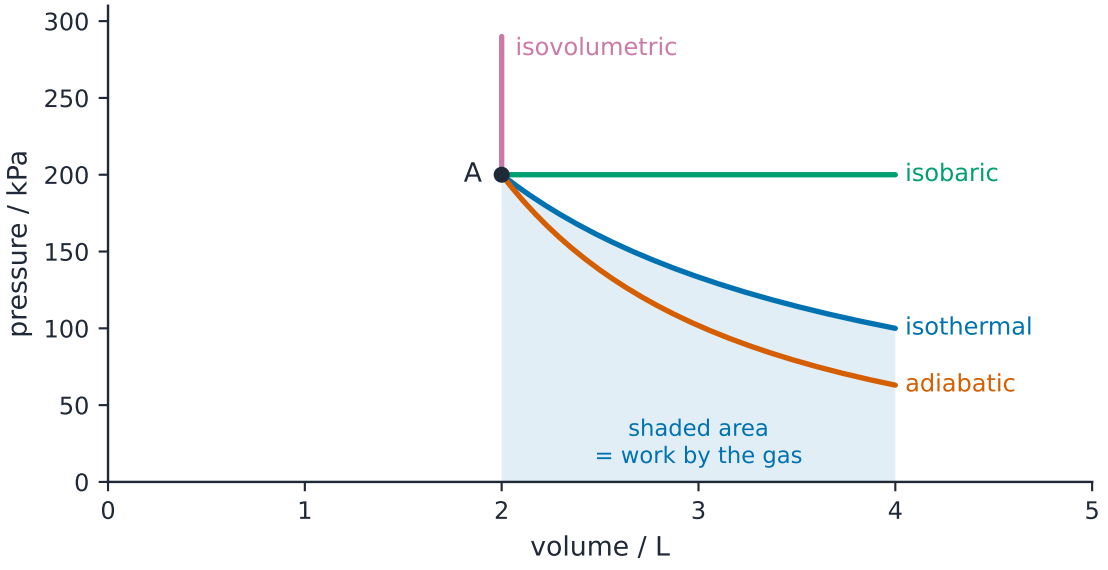
AHL: Four thermodynamic processes

Holding one quantity fixed

The named processes specify different constraints: an **isovolumetric process** 等容过程 keeps the volume fixed and an **isobaric process** 等压过程 keeps the pressure fixed.

Process	**Constant**	**First law becomes**	**On a P–V diagram**
isovolumetric	V	$W = 0$, so $Q = \Delta U$	vertical line
isobaric	P	$W = P\Delta V$; $Q = \Delta U + P\Delta V$	horizontal line
isothermal	T	$\Delta U = 0$, so $Q = W$	isotherm, PV constant
adiabatic	no heat flow, $Q = 0$	$\Delta U = -W$	reversible ideal curve is steeper than an isotherm

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The adiabatic curve falls more steeply than the isothermal one because the gas also cools as it does work

Isothermal processes

An **isothermal process** 等温过程 happens at constant temperature, so $\Delta U = 0$ and all the heat supplied leaves as work, $Q = W$. A slow equilibrium expansion in good thermal contact with a reservoir can maintain constant temperature as heat enters. This is one way to realise an isothermal path, rather than its definition.

Adiabatic processes

In an **adiabatic process** 绝热过程 no heat enters or leaves, $Q = 0$. It happens when the gas is insulated or the change is so fast that heat has no time to flow. Then $\Delta U = -W$: an ideal gas that expands **while doing positive work** cools; compression with work done on the gas warms it. In insulated free expansion into vacuum, however, $Q = W = 0$: ideal-gas temperature is unchanged, even though the process is adiabatic and irreversible. This is why a bicycle pump gets hot and why air cools as it rises and expands to form clouds.

For a monatomic ideal gas in a **reversible** adiabatic change:

$$PV^{5/3} = \text{constant}$$

Worked example: a monatomic gas at 100 kPa and 300 K is compressed reversibly and adiabatically to one eighth of its volume.

- $P_2 = 100 \times 8^{5/3} = 3200$ kPa.
- From $PV = nRT$, $T_2 = 300 \times \dfrac{3200}{100} \times \dfrac{1}{8} = 1200$ K.

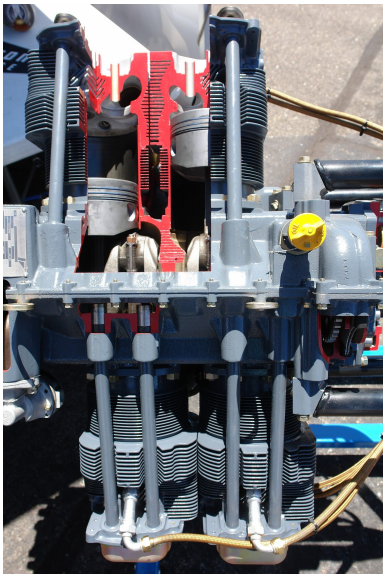
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Compression heating underlies diesel ignition. This invented reversible monatomic calculation is not a measurement of a real engine; rapid compression alone does not justify the reversible power law.

AHL: Heat engines and cycles

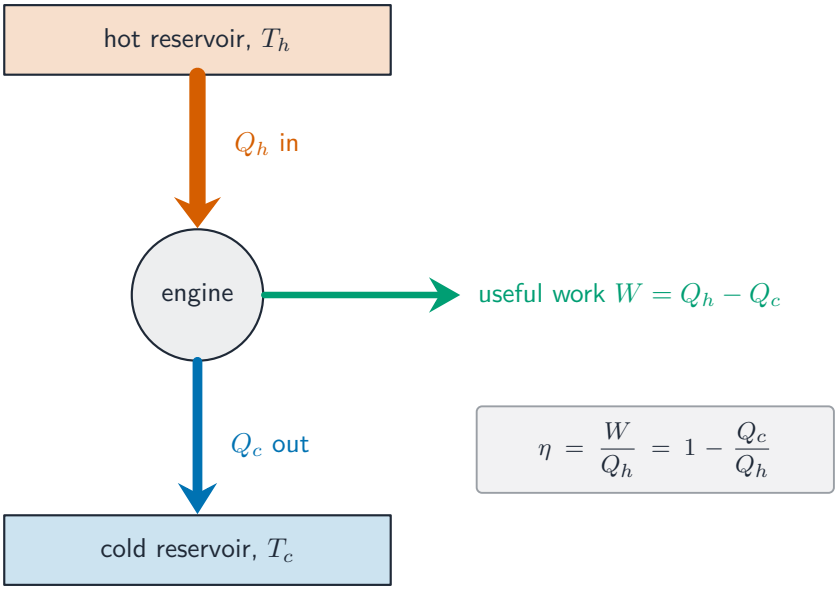
Engines run in cycles

A **heat engine** 热机 takes in heat Q_h from a hot reservoir, does useful work W, and gives out heat Q_c to a cold reservoir. To keep running, the gas goes through a **cycle** 循环 and returns to its starting state, so over one cycle $\Delta U = 0$ and



A cutaway piston engine: each cylinder runs a heat-engine cycle, taking in heat, doing work and rejecting the rest

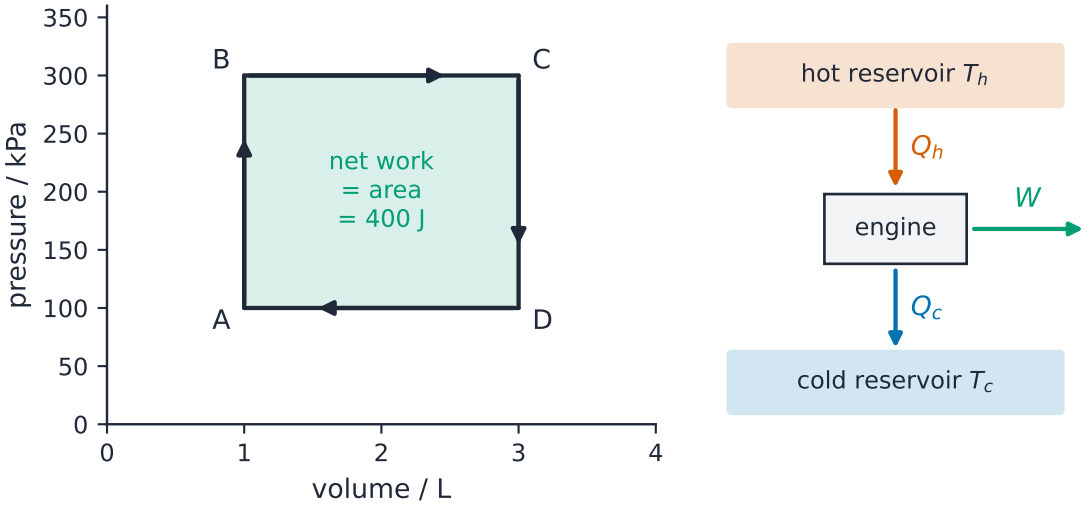
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Every heat engine must reject some heat Q_c to a cold reservoir

$$W = Q_h - Q_c$$

On a P–V diagram, the net work done in one cycle is the area enclosed by the loop. A clockwise loop is an engine: the gas does more work expanding at high pressure than is done on it compressing at low pressure.



The net work in a cycle is the enclosed area; the engine must reject heat to the cold reservoir

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Efficiency

The **efficiency** 效率 of a heat engine is

$$\eta = \frac{\text{useful work}}{\text{input energy}} = \frac{W}{Q_h}$$

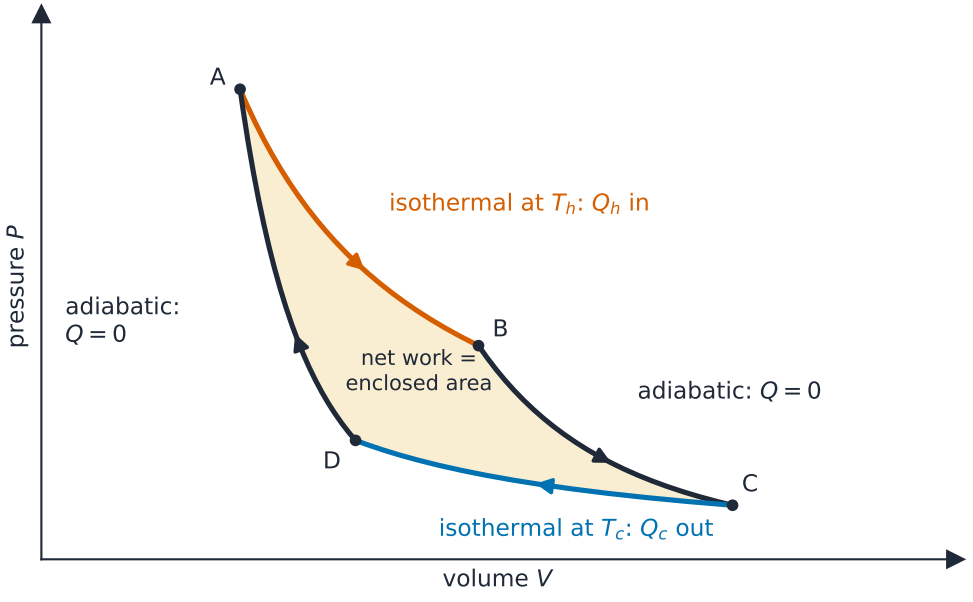
Worked example: the rectangular cycle above, for a monatomic ideal gas. In each stage $\Delta U = \frac{3}{2}\Delta(PV)$ and, at constant pressure, $W = P\Delta V$.

Stage	Process	W / J	ΔU / J	Q / J
A → B	constant volume	0	+300	+300
B → C	constant pressure	+600	+900	+1500
C → D	constant volume	0	−900	−900
D → A	constant pressure	−200	−300	−500

Net work = 600−200 = 400 J; heat in Q_h = 300+1500 = 1800 J; heat out Q_c = 1400 J. So $\eta = 400/1800 = 0.22$.

The Carnot cycle and the maximum efficiency

The **Carnot cycle** 卡诺循环 is an idealized reversible cycle of two isothermal and two adiabatic processes: isothermal expansion at T_h, adiabatic expansion to T_c, isothermal compression at T_c and adiabatic compression back to T_h. No engine working between the same two reservoirs can be more efficient:



Two isotherms joined by two adiabats: the most efficient cycle between two temperatures

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$$\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$$

with temperatures in kelvin. An invented two-reservoir engine model at 800 K and 300 K has maximum efficiency $1 - 300/800 = 0.625$, about 0.63; an actual irreversible engine between these reservoirs has lower efficiency. The only way to approach 100% would be a cold reservoir at 0 K. Efficiency improves with a hotter source or a colder sink.



A steam turbine does work as expanding steam pushes its blades around.

AHL: Entropy

Disorder and entropy

Entropy 熵 S is a thermodynamic state quantity, in J K^{-1}, related statistically to the number of accessible microstates. "Disorder" is a useful shorthand, not a definition based on visual untidiness. When thermal energy ΔQ is transferred reversibly to a system at temperature T, its entropy changes by

$$\Delta S = \frac{\Delta Q}{T}$$

Worked example: 1.0 kg of ice melts at 273 K, absorbing 3.34×10^5 J. $\Delta S = 3.34 \times 10^5/273 = 1.2 \times 10^3$ J K^{-1}: the ordered crystal becomes a disordered liquid.

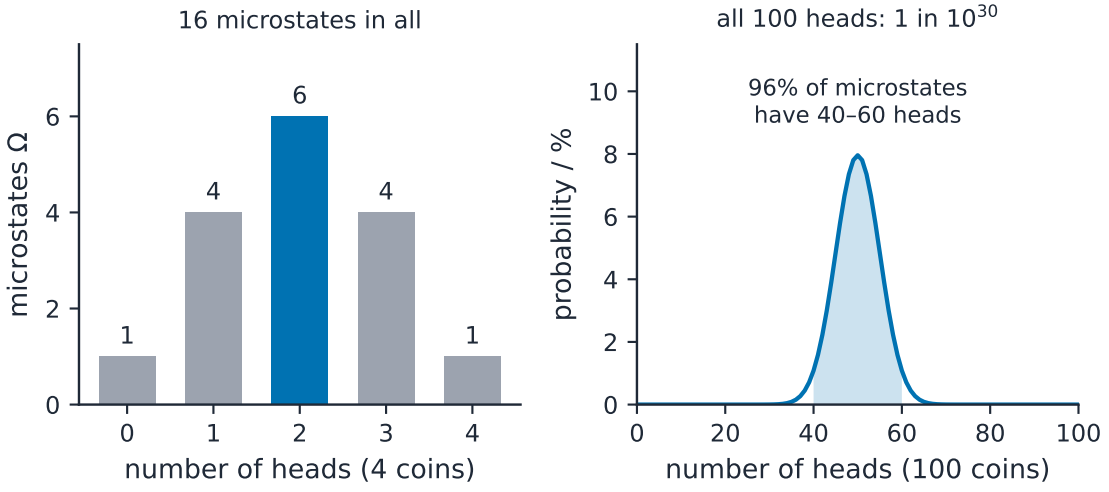
Microstates

A **microstate** 微观状态 is one particular arrangement of all the particles; a **macrostate** 宏观状态 is what we measure, such as the total energy or the number of heads. Accessible microstates are treated as equally likely under the equilibrium counting assumptions used here. For an analogy with 4 independent fair coins: the macrostate "2 heads" can happen in 6 ways (HHTT, HTHT, …) out of 16, but "4 heads" in only 1. Entropy counts microstates:

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$$S = k_B \ln \Omega$$

where Ω is the number of microstates for the macrostate. For "2 heads", $S = 1.38 \times 10^{-23} \ln 6 = 2.5 \times 10^{-23}$ J K^{-1}.



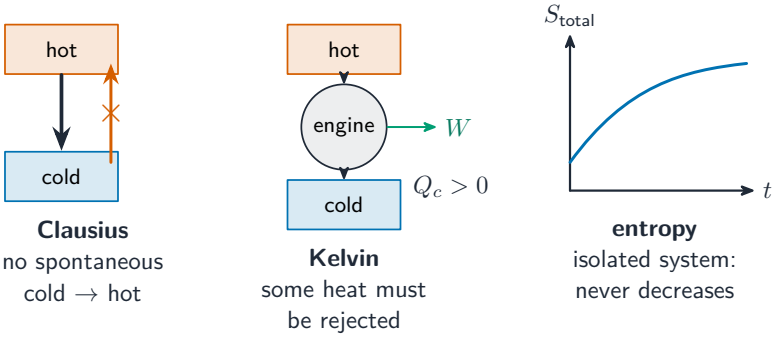
With more particles the most disordered macrostates become overwhelmingly the most probable

With 100 independent fair coins the chance of all heads is 1 in 10^{30}. A real gas has about 10^{23} particles, so a gas that spreads through a room never gathers back into one corner: not because it is forbidden, but because it is fantastically improbable.

AHL: The second law of thermodynamics

Three statements of one law

The **second law of thermodynamics** 热力学第二定律 can be stated in three equivalent ways:



One law, three statements

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- **Clausius form:** thermal energy cannot spontaneously flow from a colder body to a hotter body. A refrigerator moves heat from cold to hot only because work is done on it.
- **Kelvin form:** no heat engine working in a cycle can take heat from one reservoir and turn it completely into work; some heat must be given to a cold reservoir.
- **Entropy form:** the total entropy of an isolated system never decreases. It stays constant in a reversible process and increases in an irreversible one.

■ Irreversible processes increase entropy

Real processes, with friction, mixing or heat flowing across a temperature difference, are irreversible, so the entropy of a real isolated system always increases. Worked example: 1000 J flows between large reservoirs maintained at 400 K and 300 K.

- The hot body loses entropy: $-1000/400 = -2.5$ J K^{-1}.
- The cold body gains: $+1000/300 = +3.3$ J K^{-1}.
- Total change $= +0.8$ J K^{-1}, an increase. Heat flowing the other way would decrease the total entropy, so it never happens on its own: this is the Clausius form.

■ Local decreases

The entropy of a non-isolated system **can** decrease. Water freezing in a freezer becomes more ordered, and a living cell builds ordered molecules. A full account includes heat and any matter transferred to the surroundings; their entropy gain must at least balance the local decrease. Actual irreversible changes increase the total entropy. A refrigerator therefore always releases more heat into the kitchen than it removes from the food.

Practice

1. A gas absorbs 500 J of heat while 200 J of work is done on it. Find the change in its internal energy.
2. 2.0 mol of a monatomic ideal gas cools from 400 K to 250 K. Find ΔU.
3. A gas expands at a constant pressure of 150 kPa from 2.0 L to 5.0 L. Find the work done by the gas.
4. 0.20 mol of helium is heated at constant volume from 300 K to 450 K. Find Q, W and ΔU.
5. Explain why a fixed ideal-gas sample cools when it expands adiabatically while doing positive work.
6. A monatomic ideal gas at 200 kPa and 3.0 L expands reversibly and adiabatically to 6.0 L. Find the final pressure.
7. For a fixed amount of monatomic ideal gas doing only pressure–volume work, identify: (a) a process at constant temperature, (b) a process with no heat transfer, (c) a process at constant volume. State the corresponding first-law simplification.

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8. An engine takes in 2400 J of heat per cycle and rejects 1800 J. Find the work done per cycle and the efficiency.
9. Find the Carnot efficiency of an engine working between 600 K and 300 K. Explain why a real engine between these temperatures is less efficient.
10. 2.0 kg of water at 100 °C boils into steam ($L_v = 2.26 \times 10^6$ J kg^{-1}). Find the entropy change of the water.
11. Six coins are tossed. Find the number of microstates with 3 heads and the entropy of this macrostate.
12. 600 J of heat flows between large reservoirs maintained at 500 K and 300 K. Find the total entropy change and explain its sign.
13. State the Kelvin and Clausius forms of the second law.
14. A refrigerator lowers the entropy of the food inside it. Explain why this does not break the second law.

Answers.

1. $Q = +500$ J, $W = -200$ J: $\Delta U = Q - W = 700$ J.
2. $\Delta U = 1.5 \times 2.0 \times 8.31 \times (-150) = -3.7 \times 10^3$ J.
3. $W = P\Delta V = 1.5 \times 10^5 \times 3.0 \times 10^{-3} = 450$ J.
4. $W = 0$; $\Delta U = 1.5 \times 0.20 \times 8.31 \times 150 = 374$ J; $Q = 374$ J.
5. $Q = 0$, so $\Delta U = -W$. The gas does positive work as it expands, so its internal energy, and so its temperature, falls.
6. $P_2 = 200 \times (3.0/6.0)^{5/3} = 63$ kPa.
7. (a) Isothermal: $\Delta U = 0$, $Q = W$. (b) Adiabatic: $Q = 0$, $\Delta U = -W$. (c) Isovolumetric: $W = 0$, $Q = \Delta U$. A zero net work value alone does not uniquely identify a constant-volume path.
8. $W = 2400 - 1800 = 600$ J; $\eta = 600/2400 = 0.25$.
9. $\eta = 1 - 300/600 = 0.50$. Real engines have friction, heat leaks and processes that are fast and irreversible, all of which increase entropy and waste energy.
10. $\Delta S = mL/T = 2.0 \times 2.26 \times 10^6/373 = 1.2 \times 10^4$ J K^{-1}.
11. $\Omega = \binom{6}{3} = 20$; $S = 1.38 \times 10^{-23} \ln 20 = 4.1 \times 10^{-23}$ J K^{-1}.
12. $\Delta S = -600/500 + 600/300 = -1.2 + 2.0 = +0.8$ J K^{-1}. Positive, because heat flowing from hot to cold is irreversible.
13. Kelvin: no cyclic engine can convert heat from a single reservoir completely into work. Clausius: heat cannot spontaneously flow from a colder to a hotter body.
14. The refrigerator is not isolated. It releases heat (the heat removed plus the work done by the motor) into the room, raising the room's entropy by more than the food's entropy falls, so the total entropy increases.

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Exam tips

- Use the IB sign convention: Q is heat **to** the system, W is work done **by** the system; work done on the gas is negative.
- For a monatomic ideal gas $\Delta U = \frac{3}{2}nR\Delta T$ in **every** process, not only at constant volume.
- Work is the area under the P–V graph; in a cycle the net work is the enclosed area.
- In $\eta_{\text{Carnot}} = 1 - T_c/T_h$ and $\Delta S = \Delta Q/T$, temperatures must be in kelvin.
- When explaining a local entropy decrease, say where the heat goes and that the surroundings gain at least as much entropy.

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Current and circuits

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches B.5 Current and circuits in full, for the course first assessed in 2025. There is no additional higher level content in B.5, so SL and HL students study the same material.

Prerequisites: work, energy and power from A.3; area of a circle; rearranging equations. The charge on an electron is -1.60×10^{-19} C. Circuit symbols follow the IB data booklet. Meters are ideal unless stated, and only direct current (dc) circuits are required.

Cells and sources of emf

Why does a phone battery go flat?

An invented battery model stores 50 kJ, roughly enough in an ideal loss-free lift to raise a 50 kg mass by 100 m. As the phone runs, charge flows round its circuits and the stored chemical energy is transferred to light, sound and heating. When you use a phone hard, the battery even warms up: some energy is wasted inside the battery itself. This unit explains where the energy goes in a circuit and why every real source loses some of it.

Electromotive force

A source of electrical energy does work on the charges passing through it. Its **electromotive force** 电动势 (emf) ε is the energy transferred to each coulomb of charge that passes through the source:

$$\varepsilon = \frac{W}{q}$$

in volts, $1\ \text{V} = 1\ \text{J C}^{-1}$. A 1.5 V cell gives 1.5 J to every coulomb that passes through it.

Chemical cells and solar cells

- A **chemical cell** 化学电池 transfers chemical energy into electrical energy. Primary cells (such as alkaline cells) are used once; secondary cells (such as lithium-ion batteries) can be recharged. Advantages: portable and ready at any time. Disadvantages: limited energy store, they wear out, and they can contain toxic materials.
- A **solar cell** 太阳能电池 transfers the energy of light into electrical energy. A single cell gives about 0.6 V, so cells are joined into panels. Advantages: no fuel and no emissions while running. Disadvantages: output depends on the light, so output needs illumination, and a large area is needed.

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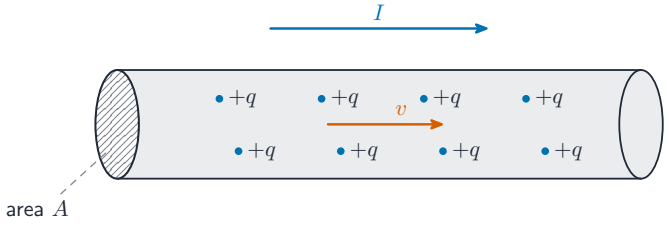


Solar cells use the photoelectric effect to turn light into electricity.

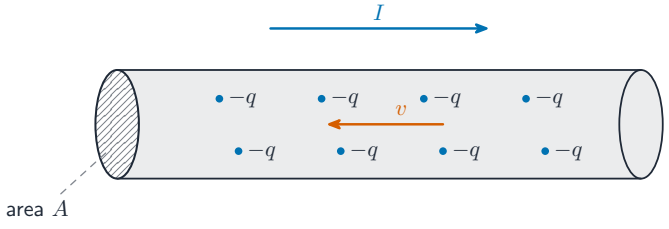
Current and potential difference

Electric current

An **electric current** 电流 is a flow of **charge carriers** 载流子. In a metal they are free electrons; in an electrolyte they are ions. The current is the rate of flow of charge:



(a) positive carriers drift with I



(b) electrons drift against I

Charge carriers drifting inside a conductor: positive carriers drift with I, electrons against it

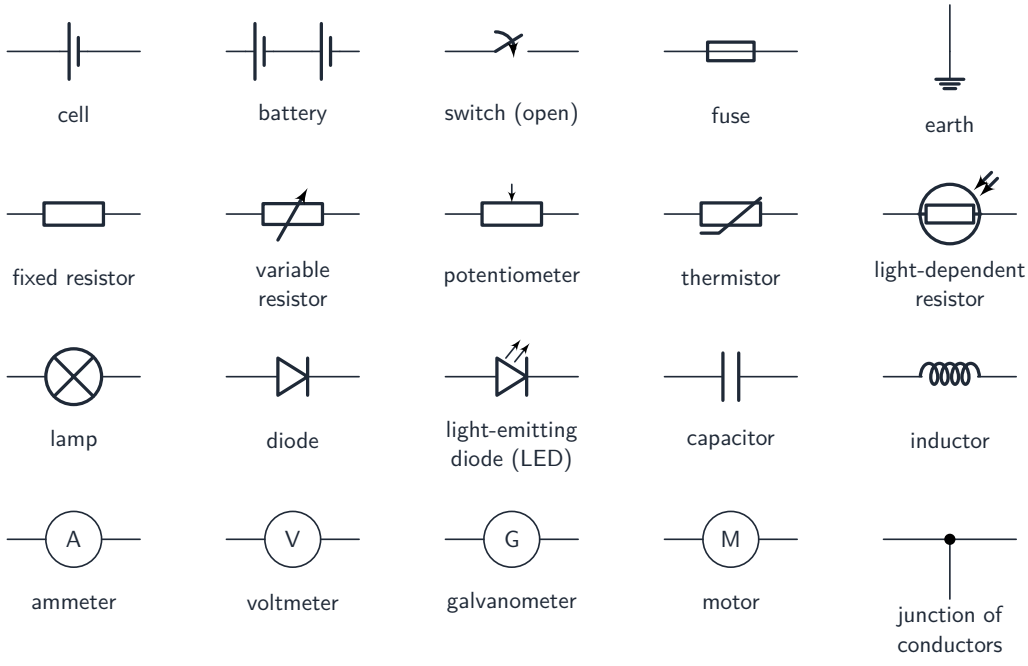
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$$I = \frac{\Delta q}{\Delta t}$$

in amperes, 1 A = 1 C s^{-1}. If 3.0 C passes a point in 1.5 s, I = 2.0 A. That is $\frac{2.0}{1.60\times10^{-19}} = 1.25 \times 10^{19}$ electrons every second. By convention the current direction is the direction a positive charge would move: from the + terminal round the circuit to the − terminal, opposite to the electrons.

■ Conductors and insulators

A **conductor** 导体 has charge carriers that can move freely: metals have about 10^{28} free electrons per cubic metre. An **insulator** 绝缘体 has almost no mobile charge carriers, because its electrons are tightly bound to their atoms. Semiconductors such as silicon lie in between.



The standard circuit symbols you need to recognise and draw

■ Potential difference

The **potential difference** 电势差 (pd) V between two points is the work done per unit charge in moving a positive charge between them:

$$V = \frac{W}{q}$$

If 12 J of energy is transferred as 2.0 C passes through a lamp, the pd across it is 6.0 V. Emf and pd have the same unit, but emf is energy given **to** charges by a source

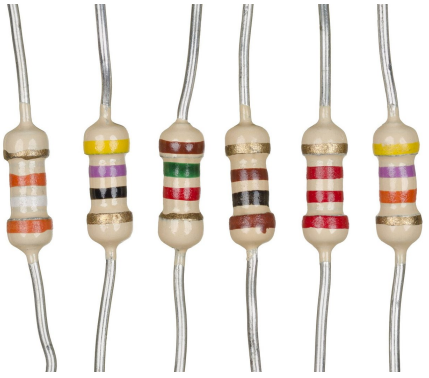
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and pd is energy transferred **from** charges in a component. A voltmeter is connected in parallel with a component; an ammeter is connected in series.

Resistance and resistivity

Resistance

The **resistance** 电阻 of a component is



Real fixed resistors – the coloured bands code the resistance in ohms

$$R = \frac{V}{I}$$

in ohms, $1\ \Omega = 1\ \text{V}\ \text{A}^{-1}$. Resistance arises because the moving electrons collide with the vibrating ions of the metal lattice. Each collision transfers energy to the lattice, which is why a resistor warms up.

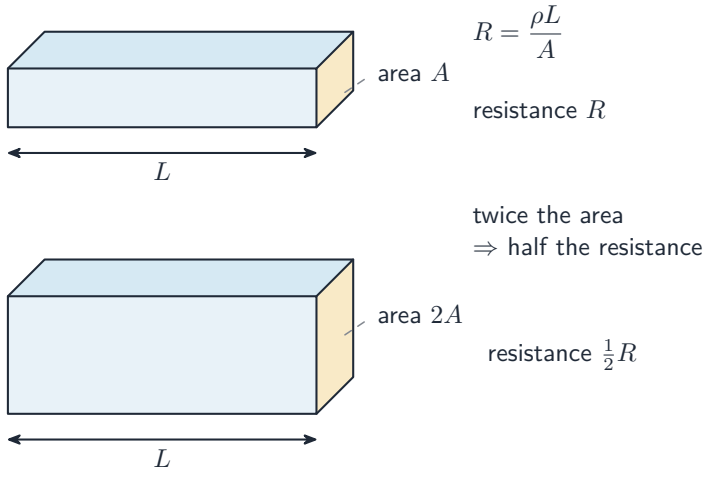
Resistivity

The resistance of a wire depends on its material, length L and cross-sectional area A. The **resistivity** 电阻率 ρ is a property of the material at specified physical conditions, including temperature:

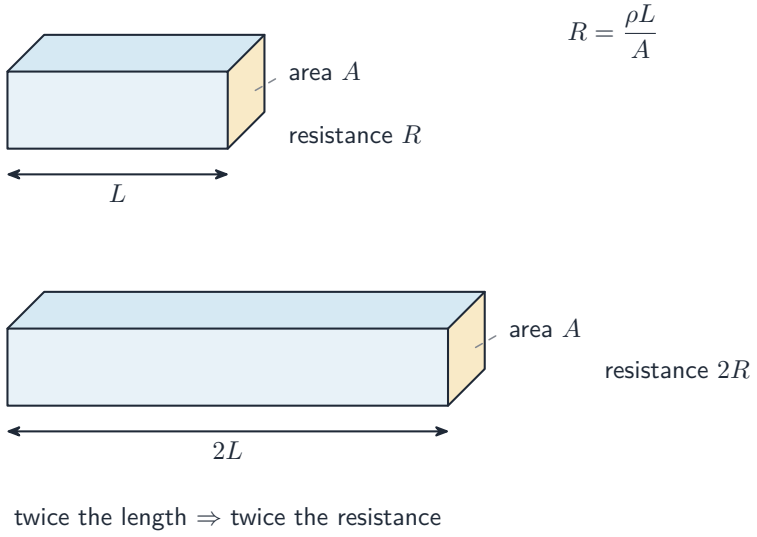
$$\rho = \frac{RA}{L} \qquad \text{so} \qquad R = \frac{\rho L}{A}$$

in Ω m. Doubling the length doubles R; doubling the area halves R.

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A wider conductor has less resistance – doubling A halves R



A longer conductor has more resistance – doubling L doubles R

Worked example: a copper wire ($\rho = 1.7 \times 10^{-8}$ Ω m) is 2.0 m long with diameter 0.50 mm.

- $A = \pi r^2 = \pi(0.25 \times 10^{-3})^2 = 1.96 \times 10^{-7}$ m^2.
- $R = \frac{1.7 \times 10^{-8} \times 2.0}{1.96 \times 10^{-7}} = 0.17$ Ω.

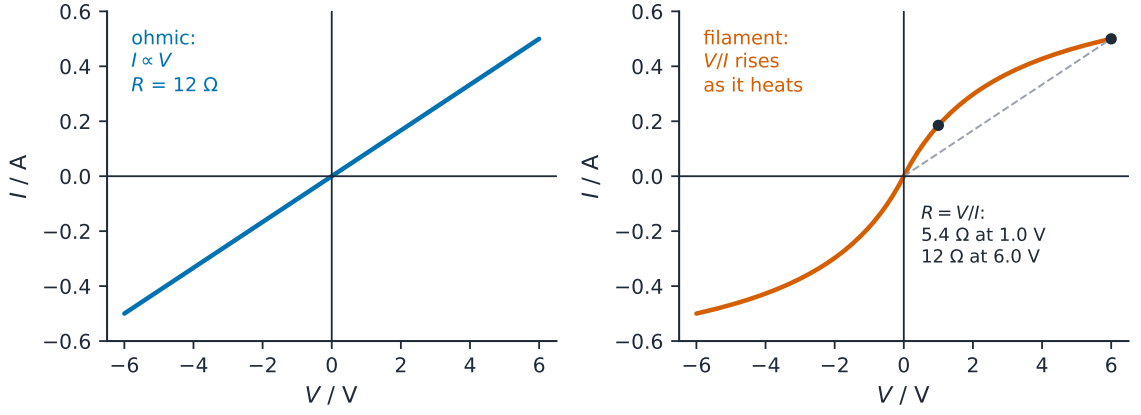
The same wire made of nichrome ($\rho = 1.1 \times 10^{-6}$ Ω m) has resistance 11 Ω, which is why heater elements are made of nichrome.

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Ohmic and non-ohmic behaviour

Ohm's law

Ohm's law 欧姆定律: the current in a conductor is proportional to the potential difference across it, provided the temperature and other physical conditions stay constant. A component that obeys Ohm's law is **ohmic** 欧姆的: its I–V graph is a straight line through the origin and its resistance is constant. A metal wire at constant temperature is ohmic.



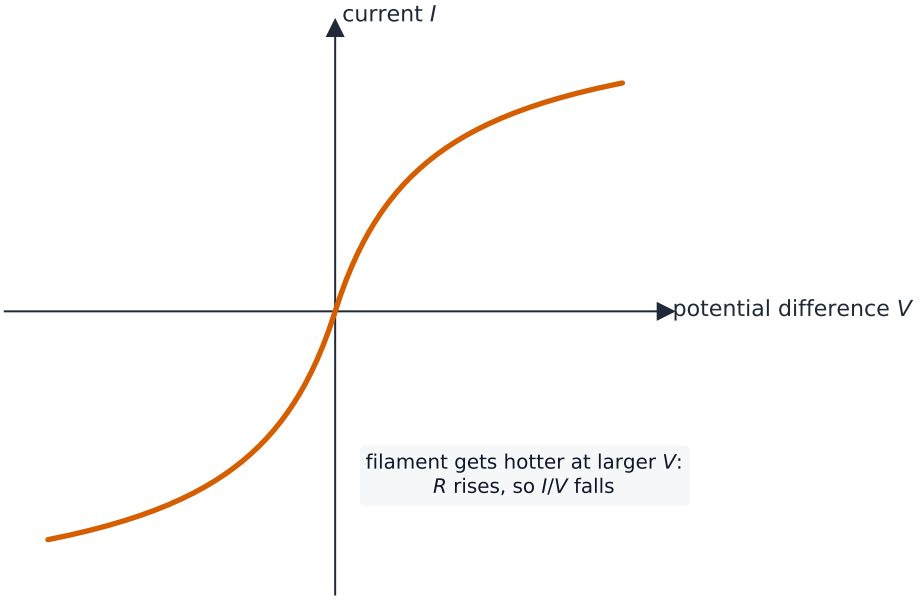
For a non-ohmic component the resistance is V/I at each point, not the gradient of the graph

In the lab: connect an ammeter in series with the component and a voltmeter in parallel across it. Use a variable low-voltage dc supply to change the pd in steps, recording both V and I at each setting. Reverse the supply polarity to measure negative values when the component permits it, and plot I against V. Limit the current and switch off between readings to reduce unwanted heating; for an ohmic wire, keep its temperature constant.

Non-ohmic components and heating

A **filament lamp** 白炽灯 is **non-ohmic** 非欧姆的. As the current rises the filament heats up, its ions vibrate more strongly, electrons collide more often, and the resistance rises: the I–V graph curves towards the V axis. Always find resistance from $R = V/I$ at a point, never from the gradient.

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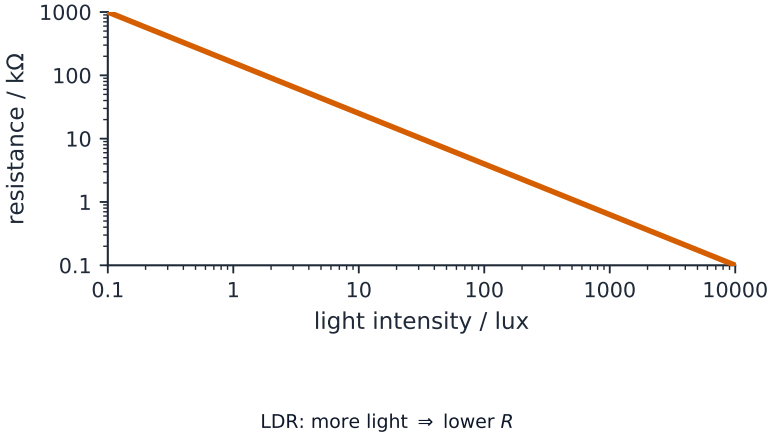


I–V characteristic of a filament lamp

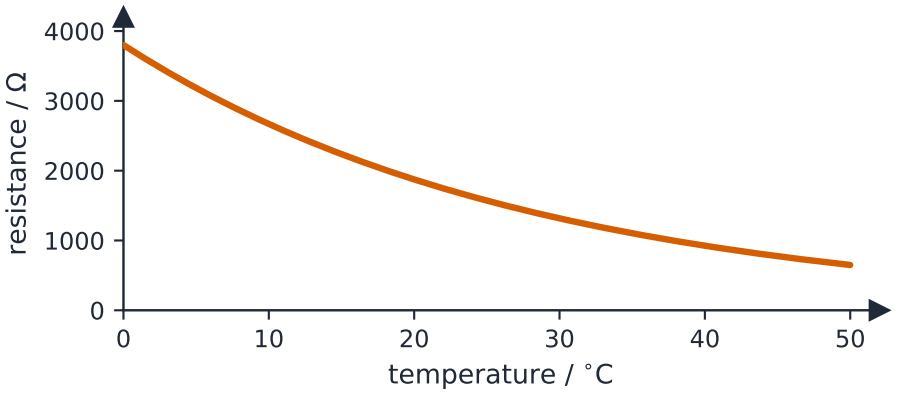
Variable resistors

Some resistors are designed to change their resistance.

- A **thermistor** 热敏电阻 (negative temperature coefficient): its resistance **falls** as its temperature rises, because more charge carriers are released. Used in thermostats and electronic thermometers.
- A **light-dependent resistor** 光敏电阻 (LDR): its resistance **falls** as the light intensity rises. Used in automatic street lights.
- A **potentiometer** 电位器: a resistor with a sliding contact. Used as a volume control or to give a variable output pd.



Resistance of an LDR decreases as light intensity increases



Resistance of a thermistor falls as temperature rises

Electrical power

The rate of energy transfer in a component is

$$P = IV = I^2R = \frac{V^2}{R}$$

The three forms follow from $P = IV$ and $V = IR$. Use whichever form contains the quantities you know.

Worked example: a 2.3 kW kettle runs on a 230 V supply.

- $I = P/V = 2300/230 = 10$ A.
- $R = V^2/P = 230^2/2300 = 23\ \Omega$.
- In 3.0 minutes it transfers $2300 \times 180 = 4.1 \times 10^5$ J.

$P = I^2R$ shows why power lines carry small currents: halving the current quarters the energy wasted heating the cables.



High-voltage power lines carry electrical energy across the country.

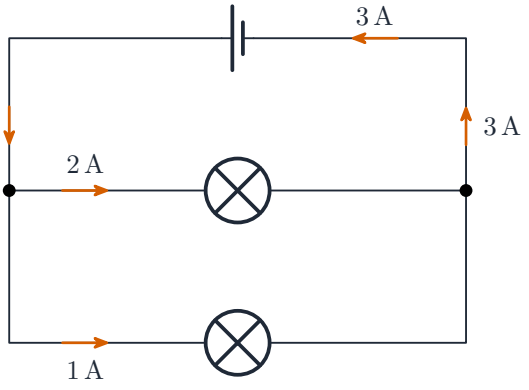
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Series and parallel circuits

The rules

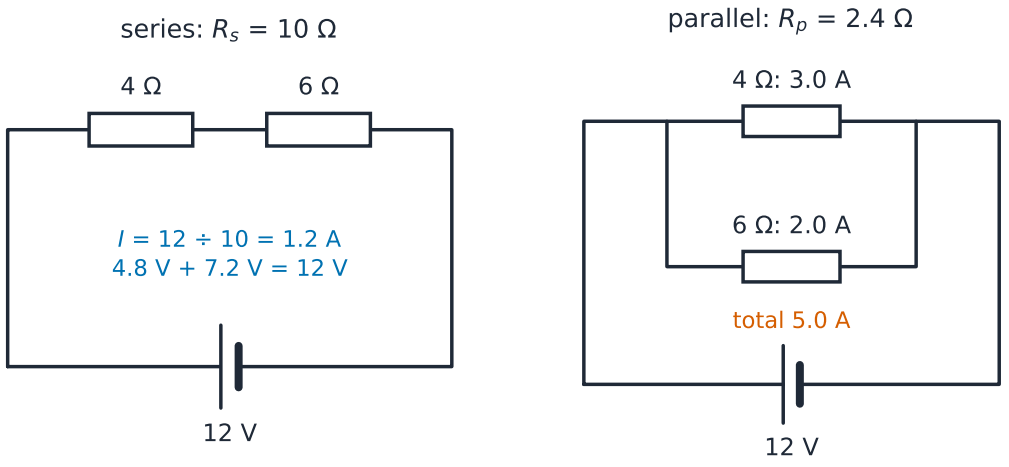
	Series	Parallel
current	same through each: $I = I_1 = I_2$	shared: $I = I_1 + I_2$
potential difference	shared: $V = V_1 + V_2$	same across each: $V = V_1 = V_2$
total resistance	$R_s = R_1 + R_2 + \ldots$	$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$

The current rule is **conservation of charge**: charge cannot pile up at a junction. The pd rule is **conservation of energy**: the energy given to each coulomb by the source is all transferred round any complete loop.



Current divides at a junction in a parallel circuit (3 A in equals 2 A plus 1 A)

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In series the pd is shared; in parallel the current is shared

Adding a resistor in series always increases the total resistance; adding one in parallel always decreases it, because it gives the charge another path. The total resistance of a parallel combination is smaller than the smallest resistor in it.

A combined circuit

Worked example: a 2.0 Ω resistor is in series with a parallel pair of 6.0 Ω and 3.0 Ω, across a 12 V supply.

- Parallel pair: $\frac{1}{R_p} = \frac{1}{6.0} + \frac{1}{3.0}$, so $R_p = 2.0\ \Omega$. Total $R = 4.0\ \Omega$.
- Current from the supply: $I = 12/4.0 = 3.0$ A.
- pd across the 2.0 Ω resistor = 6.0 V, so 6.0 V across the pair.
- Currents: $6.0/6.0 = 1.0$ A and $6.0/3.0 = 2.0$ A, which add to 3.0 A.

Internal resistance

Lost volts

Every real cell has an **internal resistance** 内阻 r: the materials inside it resist the current. Some of the emf is used driving current through the cell itself, so the **terminal potential difference** 端电压 V across an external resistor R is less than the emf:

$$\varepsilon = I(R + r) = V + Ir$$

Ir is the energy per coulomb wasted inside the cell, the "lost volts". It warms the cell.

Worked example: a cell of emf 6.0 V and internal resistance 0.50 Ω drives current through a 2.5 Ω resistor.

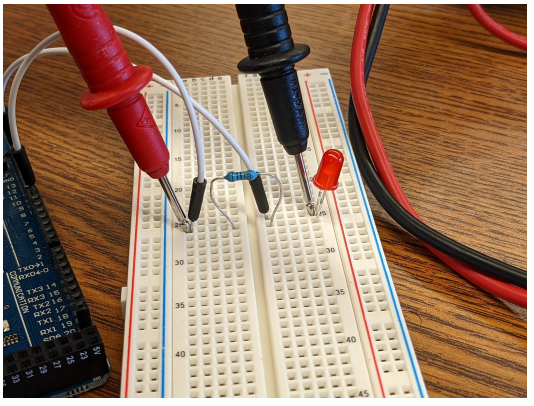
- $I = \varepsilon/(R + r) = 6.0/3.0 = 2.0$ A.

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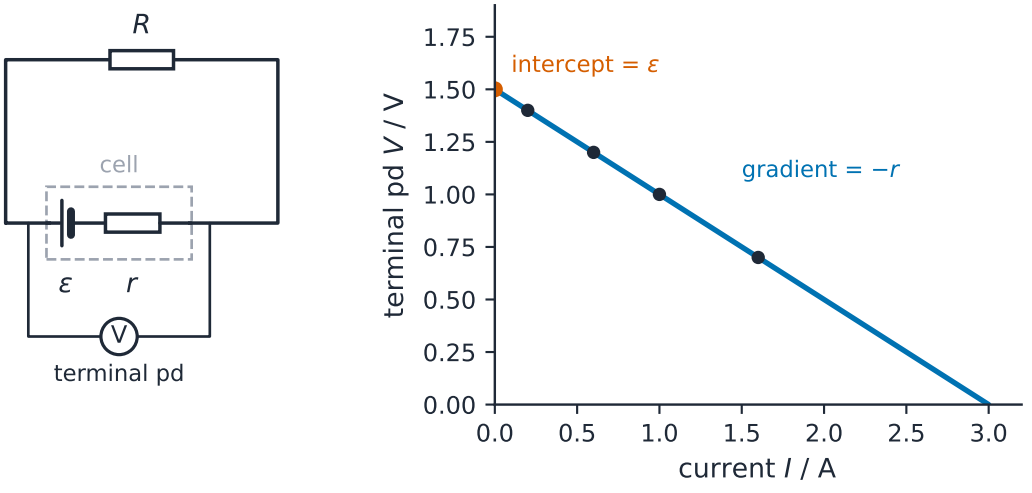
- Terminal pd $V = IR = 5.0$ V; lost volts $Ir = 1.0$ V.

Measuring emf and internal resistance

In the lab: connect the cell to a variable resistor with an ammeter in series and a voltmeter across the cell's terminals. Record V and I for several settings and plot V against I. Since $V = \varepsilon - rI$, the graph is a straight line with intercept ε and gradient $-r$.



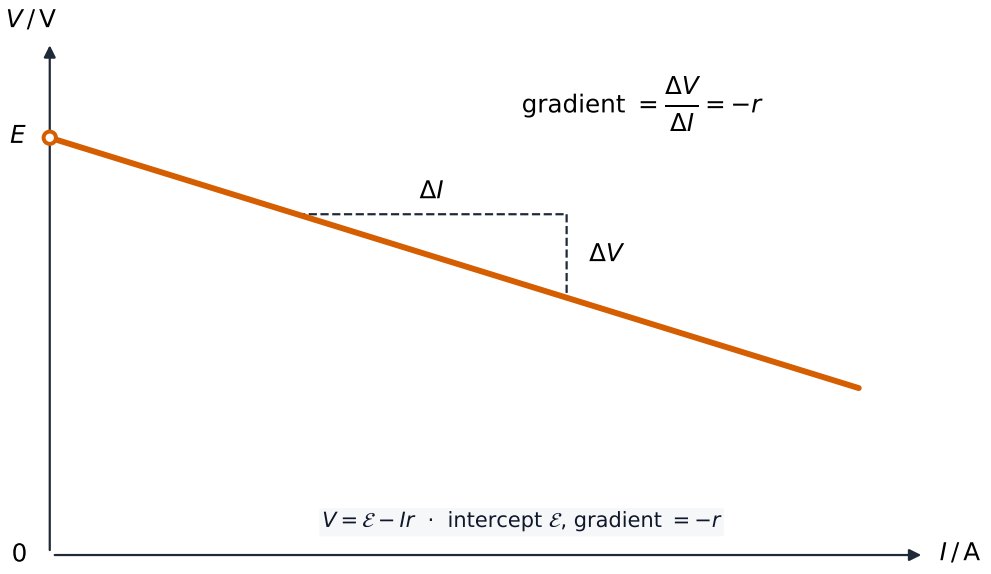
A real circuit on a breadboard, being measured with a multimeter



The more current the cell supplies, the more of its emf is lost inside it

A car battery needs a very small internal resistance: with $\varepsilon = 12.6$ V and $r = 0.020\ \Omega$, a 150 A starter motor still receives $12.6 - 150 \times 0.020 = 9.6$ V.

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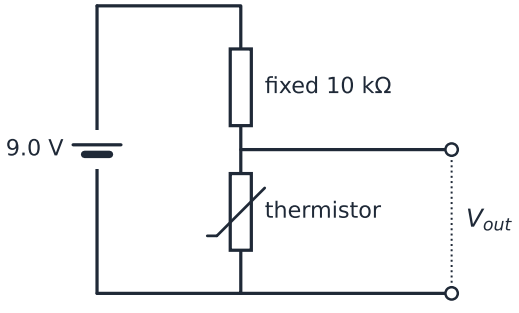
Terminal p.d. against current – the intercept is the e.m.f. and the gradient is minus the internal resistance

Potential dividers

Two resistors in series across a supply share its pd in the ratio of their resistances. This **potential divider** 分压器 gives an output pd across one of them:

$$V_{\text{out}} = V_{\text{in}} \times \frac{R_2}{R_1 + R_2}$$

where R_2 is the resistor the output is taken across and the output draws negligible current. A load in parallel with R_2 changes its effective resistance, so recalculate the divider. With a thermistor or an LDR as R_2, the output changes with temperature or light: a sensor circuit.



$$V_{out} = 9.0 \times \frac{R_{th}}{R_{th} + 10\ \text{k}\Omega}$$

cold: R_{th} = 20 kΩ → V_{out} = 6.0 V

warm: R_{th} = 5 kΩ → V_{out} = 3.0 V

As the thermistor warms its resistance falls, so it takes a smaller share of the 9.0 V

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Practice

1. A charge of 240 C flows through a lamp in 2.0 minutes. Find the current and the number of electrons passing each second.
2. A 9.0 V battery transfers 54 J to the charge passing through it. Find the charge.
3. A wire of length 1.5 m and cross-sectional area 2.0×10^{-7} m^2 has resistance 0.13 Ω. Find the resistivity of its metal.
4. A wire is replaced by one of the same material, twice as long and with twice the diameter. Find the ratio of the new resistance to the old.
5. Sketch the I–V graph of a filament lamp and explain its shape.
6. A filament lamp carries 0.25 A at 3.0 V. Find its resistance at this pd.
7. A 1000 W heater runs on 230 V. Find the current and the resistance of its element.
8. Resistors of 3.0 Ω and 6.0 Ω are connected (a) in series, (b) in parallel, to a 9.0 V supply of negligible internal resistance. Find the total resistance and the current from the supply in each case.
9. Three 12 Ω resistors are connected so that two are in parallel and the third is in series with the pair. Find the total resistance.
10. A cell of emf 1.5 V and internal resistance 0.40 Ω is connected to a 2.6 Ω resistor. Find the current, the terminal pd and the power wasted inside the cell.
11. A student's V–I graph for a cell gives an intercept of 4.5 V and a gradient of −1.5 V A^{-1}. State the emf and internal resistance.
12. In the potential divider above, the thermistor's resistance is 10 kΩ. Find V_{out}.
13. Explain why the output of the potential divider above rises when the room cools.
14. Give one advantage and one disadvantage of solar cells compared with chemical cells.

Answers.

1. $I = 240/120 = 2.0$ A; $\frac{2.0}{1.60\times10^{-19}} = 1.25 \times 10^{19}$ electrons per second.
2. $q = W/\varepsilon = 54/9.0 = 6.0$ C.
3. $\rho = RA/L = 0.13 \times 2.0 \times 10^{-7}/1.5 = 1.7 \times 10^{-8}$ Ω m (consistent with the model copper value).
4. Length × 2 and area × 4, so $R \times 2/4 = 0.5$: the new resistance is half the old.
5. A curve through the origin that becomes less steep as V rises (in both directions). The filament heats, its ions vibrate more, electrons collide more often and the resistance increases.
6. $R = 3.0/0.25 = 12$ Ω.
7. $I = 1000/230 = 4.3$ A; $R = 230^2/1000 = 53$ Ω.
8. (a) $R = 9.0$ Ω, $I = 1.0$ A. (b) $1/R = 1/3.0 + 1/6.0$, $R = 2.0$ Ω, $I = 4.5$ A.
9. $6.0 + 12 = 18$ Ω.
10. $I = 1.5/3.0 = 0.50$ A; $V = 0.50 \times 2.6 = 1.3$ V; $P = I^2r = 0.25 \times 0.40 = 0.10$ W.
11. $\varepsilon = 4.5$ V; $r = 1.5$ Ω.

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12. $V_{\text{out}} = 9.0 \times 10/20 = 4.5$ V.
13. When the room cools the thermistor's resistance rises, so it takes a larger share of the supply pd and V_{out} rises.
14. Advantage: no fuel or emissions, and there is no fuel consumption while operating; solar cells can still age and need replacement. Disadvantage: no output in the dark, and a large area is needed for useful power.

Exam tips

- Emf is energy **given to** each coulomb by a source; pd is energy **transferred from** each coulomb in a component.
- Find resistance from $R = V/I$ at a point; the gradient of an I–V curve is not $1/R$ for a non-ohmic component.
- Convert diameters to radii and mm^2 to m^2 before using $R = \rho L/A$.
- For a combined circuit, reduce the parallel parts first, find the supply current, then work back through each part.
- With internal resistance, the terminal pd falls as the current rises: $V = \varepsilon - Ir$.

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Simple harmonic motion

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches C.1 Simple harmonic motion in full, for the course first assessed in 2025. Sections marked **AHL** are for Higher Level only.

Prerequisites: displacement, velocity and acceleration from A.1; Newton's second law, Hooke's law and the forces on a pendulum from A.2; kinetic, gravitational and elastic potential energy from A.3; angular velocity ω and radians from circular motion in A.2. Use $g = 9.8\ \text{m s}^{-2}$.

Oscillations and the conditions for SHM

Why does a tall building sway at its own rhythm?

An invented tower model sways back and forth after a disturbance, with an approximately steady period for small movements. A child on a swing, a guitar string, a car bouncing on its springs and the atoms in a crystal all do the same: they move back and forth about a fixed point, with a steady rhythm. This unit builds one model, **simple harmonic motion** 简谐运动 (SHM), that can approximate small oscillations when its restoring-force conditions hold.

Oscillations

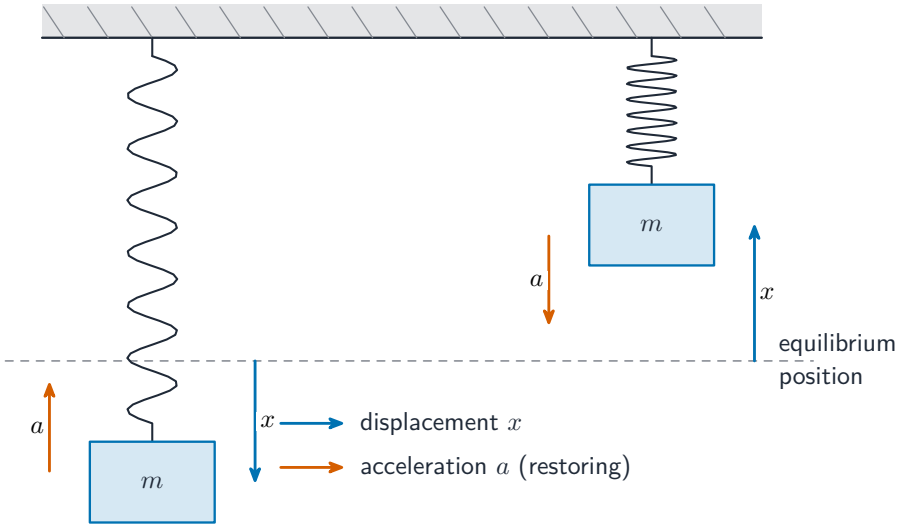
An **oscillation** 振动 is a repeated back-and-forth motion about a fixed position. The **equilibrium position** 平衡位置 is where the object would rest if it were not moving: the resultant force on it there is zero. When the object is pulled away from equilibrium, a force pulls it back. This force is called the **restoring force** 回复力.

Conditions for SHM

For one-dimensional motion, SHM requires:

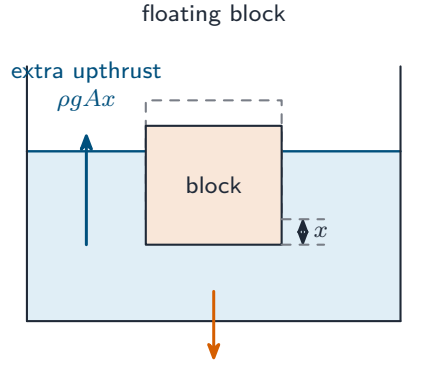
- the resultant restoring force (and so the acceleration) is **proportional** to the displacement from the equilibrium position;
- the resultant restoring force always acts **towards** the equilibrium position, opposite to the displacement.

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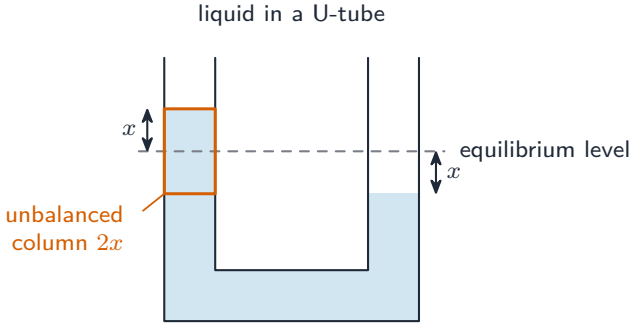
Acceleration always points back towards equilibrium, opposite to the displacement

A mass on a spring obeys both conditions, because Hooke's law gives $F = -kx$. A pendulum's tangential motion obeys them closely when the angle is small. A ball bouncing on the floor does **not** perform SHM: while it is in the air its acceleration is g, the same at every height, not proportional to its distance from any point.



extra volume submerged $= Ax$

restoring force $= \rho g A x \Rightarrow a = -\dfrac{\rho g A}{m}\, x$



restoring force $= 2x\rho g A$ on mass $2L\rho A$

$\Rightarrow a = -\dfrac{g}{L}\, x$

Two "show that it is SHM" set-ups: find the extra force when the system is displaced by x, and show it is proportional to x and directed back

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The defining equation of SHM

■ $\mathbf{a = -\omega^2 x}$

The two conditions can be written as one equation:

$$a = -\omega^2 x$$

where a is the acceleration, x is the **displacement** 位移 from equilibrium and ω is a constant called the **angular frequency** 角频率, in rad s^{-1}.

- The acceleration is proportional to x: double the displacement and the acceleration doubles.
- The **minus sign** means the acceleration is always in the opposite direction to the displacement. When the object is to the right of equilibrium ($x > 0$), it accelerates to the left ($a < 0$), and the other way round. The acceleration always points back towards equilibrium.

A graph of a against x is a straight line through the origin with gradient $-\omega^2$ (right-hand graph below). The acceleration magnitude is greatest at the ends and zero at equilibrium; the signed acceleration is positive at the negative extreme and negative at the positive extreme.

Worked example: an object oscillates with $\omega = 12.6$ rad s^{-1}. When its displacement is $+0.020$ m, $a = -12.6^2 \times 0.020 = -3.2$ m s^{-2}, directed back towards equilibrium.

Describing an oscillation

■ Amplitude, period and frequency

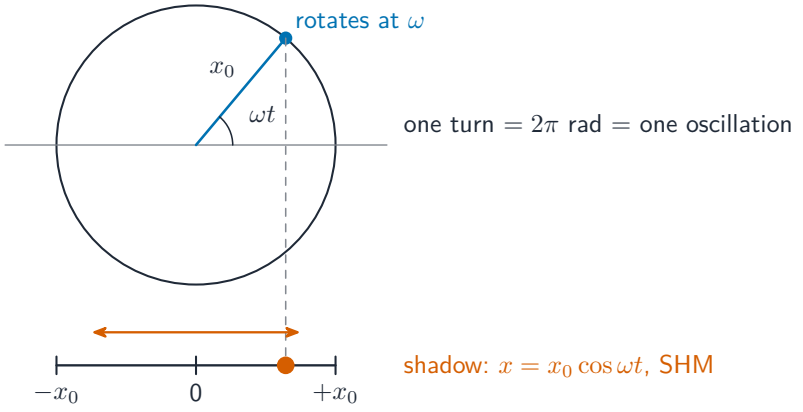
Quantity	Symbol	Meaning	Unit
displacement	x	distance and direction from the equilibrium position	m
amplitude	x_0	the maximum displacement	m
period	T	the time for one complete oscillation	s
frequency	f	the number of oscillations per second	Hz
angular frequency	ω	2π times the frequency	rad s^{-1}

The **amplitude** 振幅 x_0 is the largest displacement, measured from equilibrium (not from one end to the other). The **period** 周期 T and the **frequency** 频率 f are linked to ω by:

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

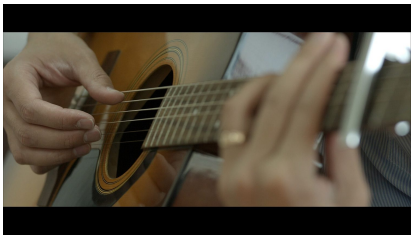
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One oscillation corresponds to 2π radians, just as one turn of a circle does. This is why the same ω appears in circular motion: the shadow of an object moving in a circle at constant speed moves with SHM.



The shadow of uniform circular motion is SHM: that is why ω appears in both

Worked example: a guitar string vibrates at 440 Hz. Its period is $T = 1/440 = 2.3 \times 10^{-3}$ s and its angular frequency is $\omega = 2\pi \times 440 = 2.8 \times 10^{3}$ rad s^{-1}.

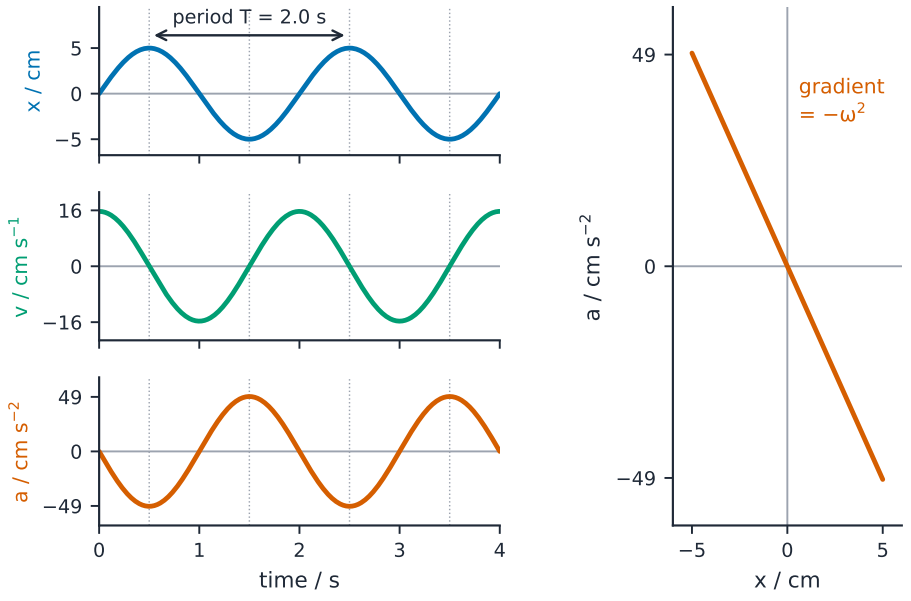


A plucked guitar string vibrates at its resonant frequencies.

SHM is **isochronous** 等时的: the period does not depend on the amplitude. A pendulum swinging through a small arc and one swinging through a larger (but still small) arc take the same time per swing. This is why pendulums were used to keep time in clocks.

■ Graphs of SHM

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Displacement, velocity and acceleration over two periods, and the defining graph $a = -\omega^2 x$

Read the graphs together, at the same time:

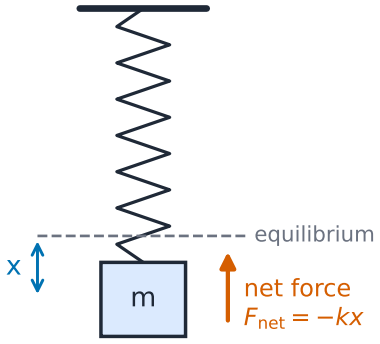
- At the **ends** of the motion ($x = \pm x_0$) the object stops for an instant: velocity is zero and the acceleration magnitude is greatest, pointing back towards equilibrium.
- At **equilibrium** ($x = 0$) the speed is greatest and the acceleration is zero.
- The velocity graph is the gradient of the displacement graph; the acceleration graph is the gradient of the velocity graph.
- The acceleration graph is the displacement graph turned upside down, because $a = -\omega^2 x$.

Worked example: for the oscillation above, $T = 2.0$ s, so $\omega = 2\pi/2.0 = 3.14$ rad s^{-1}. The greatest acceleration magnitude is $\omega^2 x_0 = 3.14^2 \times 5.0 = 49$ cm s^{-2}, as the graph shows.

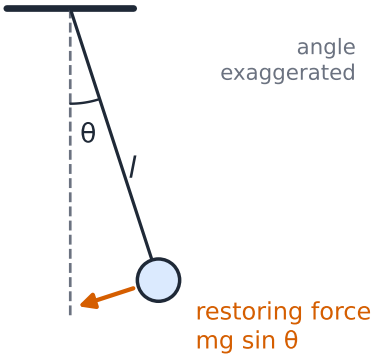
The mass–spring system

Take a constant mass m, an ideal linear spring of **spring constant** 劲度系数 k, negligible spring mass and no driving or dissipative forces. Measure x from equilibrium. The resultant restoring force is $F_{\text{net}} = -kx$, so $ma = -kx$ and $a = -\frac{k}{m}x$. For a horizontal frictionless model this can be the spring force alone. For a vertical model include weight as well. Comparing with $a = -\omega^2 x$ gives $\omega^2 = k/m$, so:

$$T = 2\pi\sqrt{\frac{m}{k}}$$



$$T = 2\pi\sqrt{\frac{m}{k}}$$



$$T = 2\pi\sqrt{\frac{l}{g}}$$

The spring arrow is net force; the pendulum arrow is tangential weight. The drawn pendulum angle is exaggerated; its period formula uses the small-angle approximation.

- A **heavier** mass oscillates more slowly: it has more inertia.
- A **stiffer** spring (larger k) oscillates faster: the restoring force is larger.
- The period does **not** depend on the amplitude, or on g. A vertical spring has the same period as a horizontal one, because gravity only moves the equilibrium position.

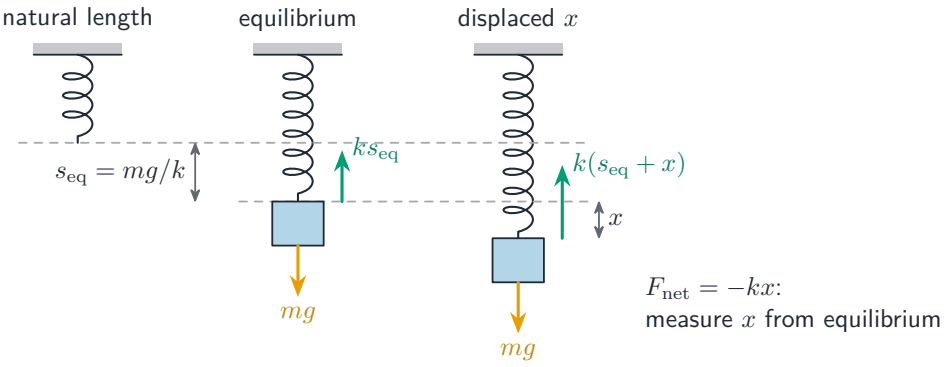
Worked examples:

- $m = 0.40$ kg on $k = 40$ N m^{-1}: $T = 2\pi\sqrt{0.40/40} = 0.63$ s and $f = 1.6$ Hz.
- A 1200 kg car rests on springs with a combined spring constant of 6.0×10^4 N m^{-1}. After a bump it bounces with $T = 2\pi\sqrt{\frac{1200}{6.0\times10^4}} = 0.89$ s. Shock absorbers stop this oscillation quickly.
- To double the period you must multiply the mass by 4, because $T \propto \sqrt{m}$.

■ A hanging spring is already stretched at equilibrium

Let s be extension from natural length and take downward as positive. At equilibrium, $ks_{eq} = mg$. A further downward displacement x makes $s = s_{eq} + x$, so

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Measure x from equilibrium, not from natural length

$$F_{net} = mg - k(s_{eq} + x) = -kx.$$

The spring force itself is $-k(s_{eq}+x)$, not $-kx$. Gravity shifts the equilibrium position; for this constant-k model it does not change the period.

Invented worked model: $m = 0.40$ kg, $k = 40$ N/m and $g = 9.8$ m/s^2. The equilibrium extension is $s_{eq} = 0.40(9.8)/40 = 0.098$ m. At $x = +0.025$ m below equilibrium, total extension is 0.123 m. The spring pulls upward with $40(0.123) = 4.92$ N while weight is 3.92 N downward. Thus $F_{net} = 3.92 - 4.92 = -1.00$ N and $a = -1.00/0.40 = -2.5$ m/s^2, upward towards equilibrium. Substituting total extension into $-kx$ would incorrectly omit weight.

The simple pendulum

A **simple pendulum** 单摆 is a small, heavy bob on a light, inextensible string of length l. When the string makes an angle θ with the vertical, the component of the weight along the arc, $mg\sin\theta$, pulls the bob back towards equilibrium. For small angles (less than about 10°), $\sin\theta \approx \theta$ in radians and the arc displacement $x = l\theta$, so the acceleration is $a = -\frac{g}{l}x$. This is SHM with $\omega^2 = g/l$:



A pendulum clock keeps time using simple harmonic motion.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

- The period depends only on the length and on g, not on the mass of the bob or (for small angles) on the amplitude.
- A longer pendulum swings more slowly: $T \propto \sqrt{l}$.
- At large angles $\sin\theta$ is less than θ, so the motion is no longer exactly SHM, and the period becomes a little longer.

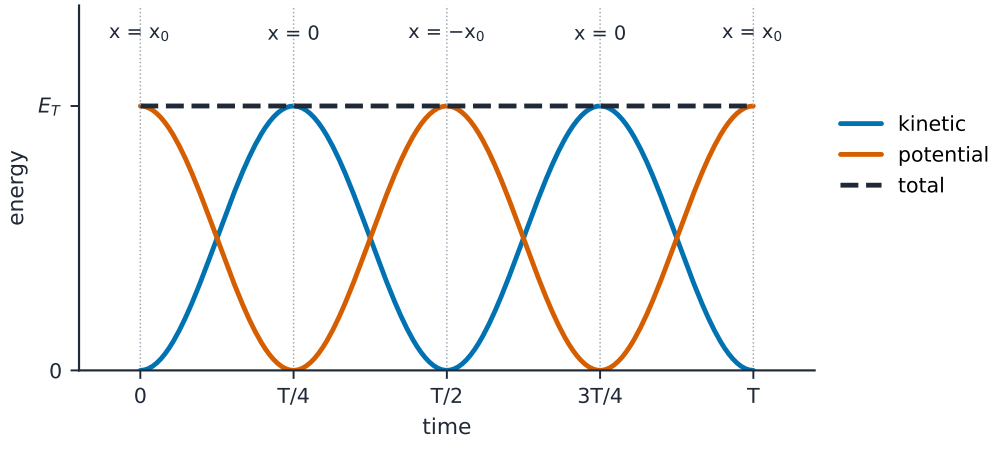
Here a is the tangential acceleration along the arc. At equilibrium that component is zero; the bob can still have radial acceleration towards the pivot.

Worked examples:

- $l = 1.0$ m: $T = 2\pi\sqrt{1.0/9.8} = 2.0$ s. A pendulum of length 0.99 m has a half-period of about one second on its way across: a "seconds pendulum".
- The same 1.0 m pendulum on the Moon ($g = 1.6$ m s^{-2}) has $T = 2\pi\sqrt{1.0/1.6} = 5.0$ s.
- A pendulum is used to measure g: plot T^2 against l. The graph is a straight line through the origin with gradient $4\pi^2/g$. Time 10 or 20 oscillations and divide, to reduce the effect of reaction time.

Energy in SHM

In an ideal undriven oscillator with no dissipative forces, energy changes between kinetic energy and potential energy (elastic potential energy for a spring, gravitational potential energy for a pendulum). The **total energy** 总能量 of these mechanical stores stays constant. For the vertical spring, include both elastic and gravitational potential energy; the combined potential is least at equilibrium, although spring energy alone need not be.



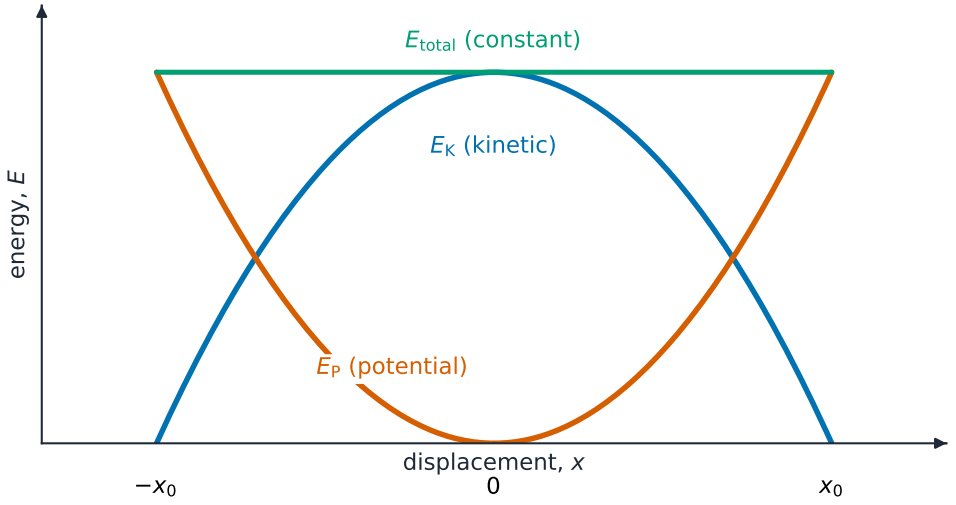
Kinetic and potential energy swap twice in each period; their sum is constant

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Follow a pendulum released from one end of its swing:

- At the **end** ($x = x_0$): the bob is momentarily at rest, so its kinetic energy is zero and its potential energy is the greatest.
- Moving **towards equilibrium**: potential energy changes to kinetic energy and the bob speeds up.
- At **equilibrium** ($x = 0$): the speed is greatest, so kinetic energy is the greatest and potential energy is the least.
- Moving **away** from equilibrium on the other side: kinetic energy changes back to potential energy, and the bob slows down to rest at $x = -x_0$.

Kinetic energy reaches its maximum twice in each period (once in each direction), so the energy graphs have half the period of the displacement graph. In a real oscillator friction and air resistance slowly transfer energy to the surroundings, the total energy falls and the amplitude decreases: this is called **damping** 阻尼. Periodic driving near a natural frequency can produce an enhanced response, or resonance; damping limits the response. Standing waves and resonance (C.4) develops the driven-motion model.



SHM: $E_k + E_p =$ const · max E_k at centre

Kinetic and potential energy swap over a cycle while the total energy stays constant

AHL: Phase angle

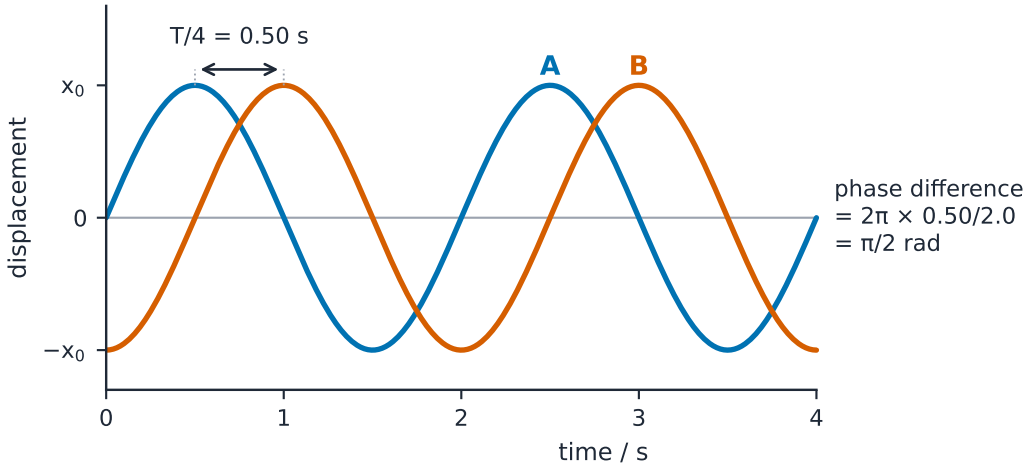
Using the sine convention $x = x_0 \sin(\omega t + \phi)$, the **phase angle** 相位角 ϕ describes where an oscillation is in its cycle at $t = 0$. It is measured in radians: one complete cycle is 2π rad.

- An oscillator that starts at equilibrium, moving in the positive direction, has $\phi = 0$.
- One that starts at its positive amplitude, at rest, has $\phi = \pi/2$.

Two oscillators with the same period can be out of step. The **phase difference** 相位差 between them is

$$\Delta\phi = 2\pi\frac{\Delta t}{T}$$

where Δt is the time by which one reaches a given point (for example, its maximum) after the other.



B lags behind A by a quarter period: a phase difference of $\pi/2$ rad

- A phase difference of 0 (or 2π) means the oscillators are **in phase**: they move together.
- A phase difference of π means they are in **antiphase** 反相: one is at $+x_0$ when the other is at $-x_0$.
- Displacement and velocity in SHM are $\pi/2$ out of phase; displacement and acceleration are in antiphase.

AHL: The equations of SHM

■ Displacement and velocity against time

The solutions of $a = -\omega^2 x$ are:

$$x = x_0 \sin(\omega t + \phi) \qquad v = \omega x_0 \cos(\omega t + \phi)$$

Set your calculator to **radians** before using them. Starting at equilibrium ($\phi = 0$) gives $x = x_0 \sin \omega t$; starting at the amplitude ($\phi = \pi/2$) gives $x = x_0 \cos \omega t$.

The maximum speed is $v_{\max} = \omega x_0$, at equilibrium, and the maximum acceleration is $a_{\max} = \omega^2 x_0$, at the ends.

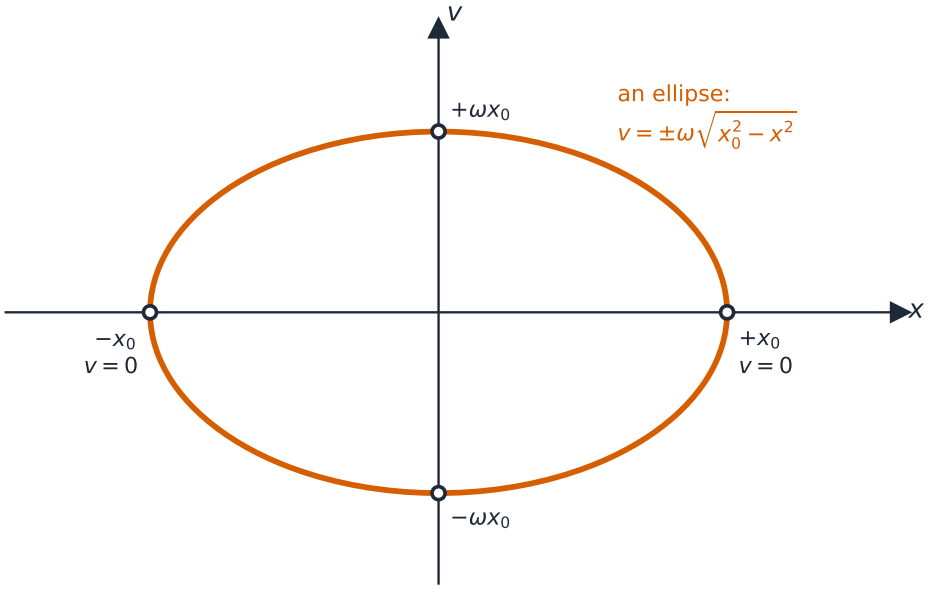
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Velocity at a given displacement

Eliminating t gives the speed at any displacement:

$$v = \pm\omega\sqrt{x_0^2 - x^2}$$

The $\pm$ shows that at each position the object can be moving either way.



Velocity against displacement is an ellipse: $v = 0$ at the extremes and $v = \pm\omega x_0$ through equilibrium – read x_0 and v_0 from where it cuts the axes, then $\omega = v_0/x_0$

Worked example: a 0.40 kg mass on a spring of $k = 40$ N m^{-1} has $\omega = \sqrt{40/0.40} = 10$ rad s^{-1}. It passes equilibrium in the positive direction at $t = 0$ with amplitude 0.050 m, so $\phi = 0$.

- At $t = 0.10$ s: $\omega t = 1.0$ rad, $x = 0.050\sin(1.0) = 0.042$ m and $v = 10 \times 0.050 \times \cos(1.0) = 0.27$ m s^{-1}.
- Check with the other equation: $v = 10\sqrt{0.050^2 - 0.042^2} = 0.27$ m s^{-1}.
- Maximum speed: $\omega x_0 = 0.50$ m s^{-1}; maximum acceleration: $\omega^2 x_0 = 5.0$ m s^{-2}.

Worked example: an oscillator of period 2.0 s starts at equilibrium moving in the positive direction. How long does it take to reach half its amplitude? $\frac{1}{2}x_0 = x_0\sin\omega t$, so $\omega t = \sin^{-1}(0.5) = \pi/6$ rad and $t = (\pi/6)/\pi = 0.17$ s. This is $T/12$, not $T/8$: the object is fastest near equilibrium, so it covers the first half of the amplitude in less time than the second half.

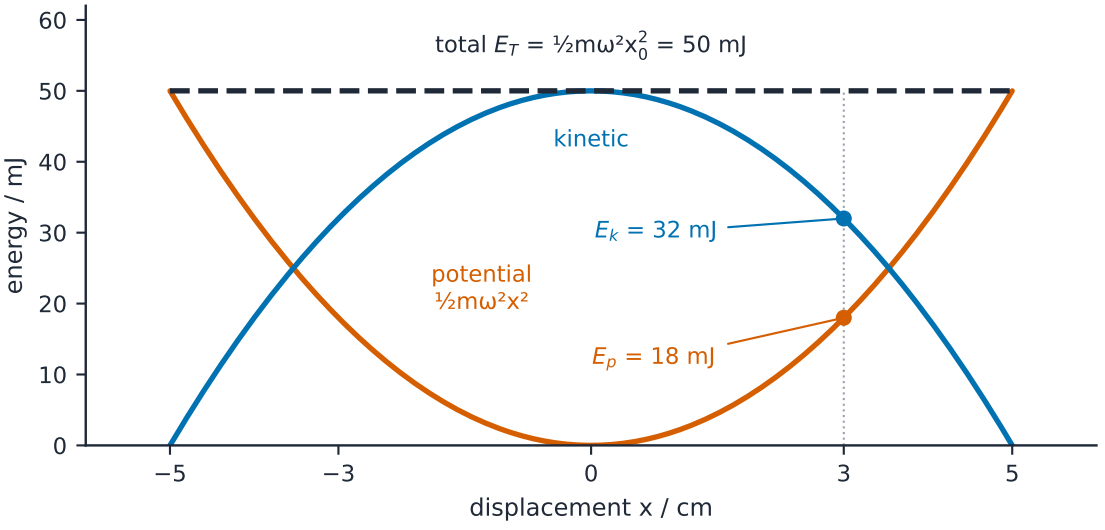
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AHL: Energy equations

For a mass m in SHM, the total energy and the potential energy at displacement x are:

$$E_T = \frac{1}{2}m\omega^2 x_0^2 \qquad E_p = \frac{1}{2}m\omega^2 x^2$$

so the kinetic energy is $E_k = E_T - E_p = \frac{1}{2}m\omega^2(x_0^2 - x^2)$, which agrees with $v = \pm\omega\sqrt{x_0^2 - x^2}$. The total energy is proportional to the **square** of the amplitude: double the amplitude and the energy is four times as great.



Energy against displacement for the 0.40 kg mass: the two parabolas always add to the total

Worked example, for the same mass ($m = 0.40$ kg, $\omega = 10$ rad s^{-1}, $x_0 = 0.050$ m):

- $E_T = \frac{1}{2} \times 0.40 \times 10^2 \times 0.050^2 = 0.050$ J.
- At $x = 0.030$ m: $E_p = \frac{1}{2} \times 0.40 \times 10^2 \times 0.030^2 = 0.018$ J, so $E_k = 0.032$ J.
- Speed there: $v = \sqrt{2 \times 0.032/0.40} = 0.40$ m s^{-1}, the same as $10\sqrt{0.050^2 - 0.030^2}$.

For a horizontal spring, $m\omega^2 = k$ and $E_p = \frac{1}{2}kx^2$ is elastic potential energy measured from its unstretched equilibrium. For a vertical spring, x is measured from its gravity-shifted equilibrium: this quadratic describes the combined elastic and gravitational potential energy relative to its minimum, not the spring energy alone.

Practice

1. State the two conditions needed for an object to perform simple harmonic motion.
2. Explain the meaning of the minus sign in $a = -\omega^2 x$.

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3. An object moves with $a = -64x$, where a is in m s^{-2} and x is in m. Find ω, the period and the frequency.
4. A 0.50 kg mass hangs on a spring of spring constant 200 N m^{-1}. Find the period and frequency of its oscillations.
5. Find the length of a simple pendulum with a period of 1.0 s.
6. A mass on a spring has a period of 0.80 s. Find the period when (a) the mass is made four times as large, (b) the amplitude is doubled.
7. Describe the energy changes of a pendulum bob during one complete oscillation, starting from rest at one end of its swing.
8. For an object in SHM, state where in the motion (a) the speed is greatest, (b) the acceleration magnitude is greatest, (c) the acceleration is zero.
9. A student measures 20 oscillations of a pendulum of length 0.60 m in 31.0 s. Find g.
10. **AHL** An object oscillates with amplitude 0.080 m and $\omega = 5.0$ rad s^{-1}. Find its maximum speed, its maximum acceleration and its speed when $x = 0.040$ m.
11. **AHL** An oscillator of period 2.0 s passes through equilibrium in the positive direction at $t = 0$. Find the time at which it first reaches half its amplitude, and its displacement at $t = 0.25$ s if the amplitude is 6.0 cm.
12. **AHL** A 0.20 kg mass on a spring of $k = 80$ N m^{-1} oscillates with amplitude 0.050 m. Find the total energy, the maximum speed and the kinetic energy when $x = 0.030$ m.
13. **AHL** Two pendulums have the same period of 2.0 s. B reaches its maximum displacement 0.25 s after A. Find the phase difference in radians.
14. An ideal vertical spring has stiffness 40 N/m and supports a 0.40 kg mass ($g = 9.8$ m/s^2). Find its equilibrium extension. At a point 0.025 m below equilibrium, find the total spring extension, spring force, resultant force and acceleration. Take downward as positive.

Answers.

1. The acceleration (restoring force) is proportional to the displacement from equilibrium, and it is always directed towards the equilibrium position.
2. The acceleration is always in the opposite direction to the displacement, so it always points back towards the equilibrium position.
3. $\omega^2 = 64$, so $\omega = 8.0$ rad s^{-1}; $T = 2\pi/8.0 = 0.79$ s; $f = 1.3$ Hz.
4. $T = 2\pi\sqrt{0.50/200} = 0.31$ s; $f = 3.2$ Hz.
5. $l = \frac{gT^2}{4\pi^2} = 9.8 \times 1.0^2/39.5 = 0.25$ m.
6. (a) $T \propto \sqrt{m}$, so the period doubles to 1.6 s. (b) The period does not depend on amplitude: it stays 0.80 s.
7. At the end: all potential energy, zero kinetic energy. Towards equilibrium: potential changes to kinetic. At equilibrium: maximum kinetic, minimum potential. To the other end: kinetic changes back to potential, zero kinetic at the end. The

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same happens on the way back. Without friction the total stays constant.

8. (a) At the equilibrium position. (b) At the ends, where the displacement is greatest. (c) At the equilibrium position.
9. $T = 31.0/20 = 1.55$ s; $g = 4\pi^2 l/T^2 = 39.5 \times 0.60/1.55^2 = 9.9$ m s^{-2}.
10. **AHL** $v_{\max} = 5.0 \times 0.080 = 0.40$ m s^{-1}; $a_{\max} = 25 \times 0.080 = 2.0$ m s^{-2}; $v = 5.0\sqrt{0.080^2 - 0.040^2} = 0.35$ m s^{-1}.
11. **AHL** $\omega = \pi$ rad s^{-1}. $\sin\omega t = 0.5$ gives $\omega t = \pi/6$, so $t = 0.17$ s. At 0.25 s: $x = 6.0\sin(\pi/4) = 4.2$ cm.
12. **AHL** $\omega = \sqrt{80/0.20} = 20$ rad s^{-1}; $E_T = \frac{1}{2} \times 0.20 \times 400 \times 0.050^2 = 0.10$ J; $v_{\max} = 20 \times 0.050 = 1.0$ m s^{-1}; $E_p = \frac{1}{2} \times 0.20 \times 400 \times 0.030^2 = 0.036$ J, so $E_k = 0.064$ J.
13. **AHL** $\Delta\phi = 2\pi \times 0.25/2.0 = \pi/4$ rad (0.79 rad).
14. Equilibrium extension is $mg/k = 0.098$ m. Total extension is $0.098 + 0.025 = 0.123$ m; spring force is $-40(0.123) = -4.92$ N. Including weight, $F_{\text{net}} = 3.92 - 4.92 = -1.00$ N and $a = -1.00/0.40 = -2.5$ m/s^2. Negative means upward. The check $-kx = -40(0.025) = -1.00$ N uses displacement from equilibrium, not extension from natural length.

Exam tips

- To show that a motion is SHM, show that $a \propto -x$: proportional to the displacement and opposite in direction.
- Measure amplitude from equilibrium, never from one end to the other.
- The periods of a mass–spring system and a small-angle pendulum do not depend on the amplitude; a pendulum's period does not depend on its mass either.
- Speed is greatest at equilibrium; acceleration magnitude is greatest at the ends.
- **AHL** Set your calculator to radians for $x = x_0\sin(\omega t + \phi)$ and phase calculations.
- **AHL** Energy is proportional to the amplitude squared: $E_T = \frac{1}{2}m\omega^2 x_0^2$.

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Wave model

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

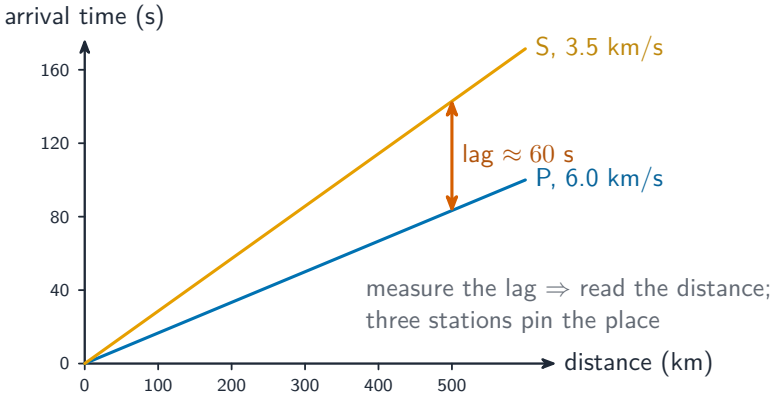
This handout teaches C.2 Wave model in full, for the course first assessed in 2025. There is no additional higher level content in C.2, so SL and HL students study the same material.

Prerequisites: displacement, period, frequency and amplitude from C.1 Simple harmonic motion; speed = distance/time from A.1. Use $c = 3.00 \times 10^8$ m s^{-1} for the speed of light in a vacuum and about 340 m s^{-1} for the speed of sound in air.

Travelling waves

■ How does a seismometer 500 km away know where an earthquake was?

Earthquake body waves include longitudinal P-waves and transverse S-waves. In an invented uniform-rock model, let their speeds be 6.0 and 3.5 km/s. Over a shared 500 km path their arrival-time difference is about 60 s, although the rock does not travel 500 km with the disturbance. The British Geological Survey explains their different motions and the absence of S-wave transmission through the liquid outer core. Real location calculations need an Earth-velocity model and multiple stations, fitting depth and origin time as well as position; see the USGS explanation. The constant speeds here are teaching assumptions, not measured universal speeds.



The P–S lag grows with distance

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Energy, not matter

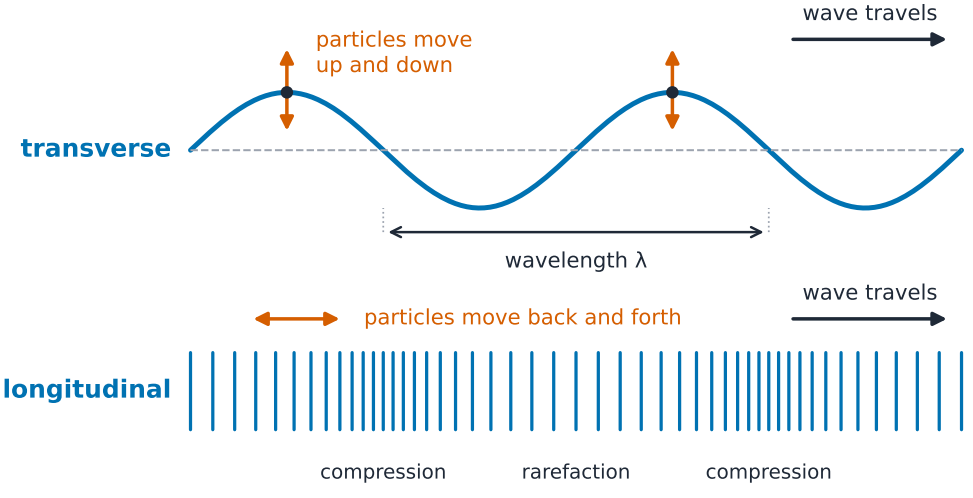
A **travelling wave** 行波 is a disturbance that moves through a medium (or through space), transferring energy from one place to another. The particles of the **medium** 介质 oscillate about their equilibrium positions, each one a little later than the one before, but they do not travel with the wave. In the ideal small-amplitude model there is no net transport of medium with the wave. Real water waves can also produce drift, and currents can carry a cork: those effects are not described by the simple oscillation-only model.



Ripples spreading on water are progressive waves that carry energy outward.

Transverse and longitudinal waves

- In a **transverse wave** 横波 the oscillations are **perpendicular** to the direction in which the wave travels. Examples: waves on a string or rope, S-waves, ripples on water (approximately), and all electromagnetic waves.
- In a **longitudinal wave** 纵波 the particles oscillate **parallel** to the direction in which the wave travels. Examples: sound, P-waves, and pushes along a stretched spring.



Transverse and longitudinal waves differ in the direction the particles move

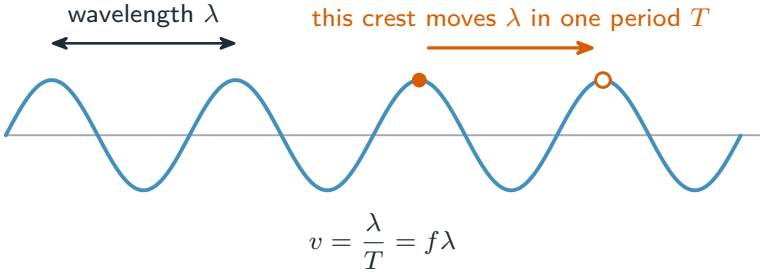
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A longitudinal wave has regions where the particles are closer together than normal, called **compressions** 密部, and regions where they are further apart, called **rarefactions** 疏部. The pressure is higher in a compression and lower in a rarefaction.

Wave speed, frequency and wavelength

Describing a wave

Quantity	Symbol	Meaning	Unit
displacement	x	distance of a particle from its equilibrium position	m
amplitude	x_0	maximum displacement of a particle	m
wavelength	λ	shortest distance between two particles oscillating in phase	m
period	T	time for one particle to make one complete oscillation	s
frequency	f	number of oscillations per second	Hz
wave speed	v	distance the wave travels per second	m s^{-1}



In one period a crest moves forward one wavelength, so $v = \lambda/T = f\lambda$.

The **wavelength** 波长 is, for example, the distance from one crest to the next, or from one compression to the next. The **wave speed** 波速 is the speed at which the energy (and the wave shape) moves, not the speed of the particles.

The wave equation

In one period T, each particle makes one oscillation and the wave moves forward by one wavelength. So:

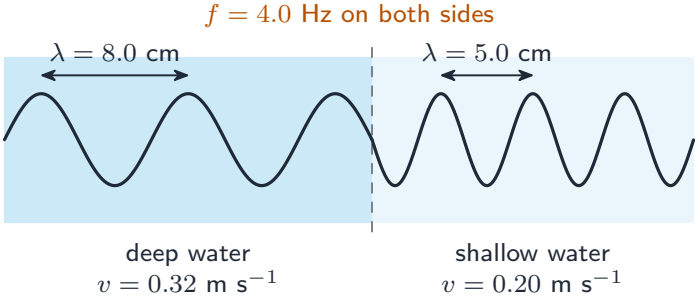
$$v = \frac{\lambda}{T} = f\lambda$$

Worked examples:

- A wave on a string has $f = 5.0$ Hz and $\lambda = 0.40$ m: $v = 5.0 \times 0.40 = 2.0$ m s^{-1} and $T = 1/5.0 = 0.20$ s.
- Middle C on a piano is 262 Hz. In air, $\lambda = 340/262 = 1.3$ m.
- A radio station broadcasts at 100 MHz: $\lambda = \frac{3.00\times10^8}{1.00\times10^8} = 3.0$ m.

Changing the frequency or the medium

- The **frequency** is set by the source. It stays unchanged across a stationary boundary in the linear wave model used here.
- The **speed** depends on the medium and its physical conditions; in dispersive media it also depends on frequency. Sound travels at about 340 m s^{-1} in air, 1500 m s^{-1} in water and 5000 m s^{-1} in steel.
- So the **wavelength** adjusts: $\lambda = v/f$. A 500 Hz sound has $\lambda = 0.68$ m in air but $\lambda = 3.0$ m in water.
- In a nondispersive approximation for one medium, speed is fixed and a higher frequency gives a shorter wavelength. Some waves, including many water waves, are dispersive; do not assume fixed speed across every frequency range.



Crossing into a slower medium: the frequency stays the same, so the wavelength shortens.

Worked example: water waves of frequency 4.0 Hz travel at 0.32 m s^{-1} in deep water ($\lambda = 8.0$ cm). In shallow water they slow to 0.20 m s^{-1}; the frequency is still 4.0 Hz, so $\lambda = 0.20/4.0 = 5.0$ cm.

In the lab: use a ripple-tank snapshot and a ruler to measure the distance across several complete crest spacings, then divide by the number of spacings to find the wavelength. This reduces the fractional reading uncertainty compared with measuring one spacing. For light, trace a narrow ray-box beam into a glass block, draw the normal at the entry point and measure the incident and refracted angles from it. Keep the beam away from eyes. The ray bends towards the normal when it slows; its frequency stays fixed and its wavelength becomes shorter. C.3 develops this refraction model.

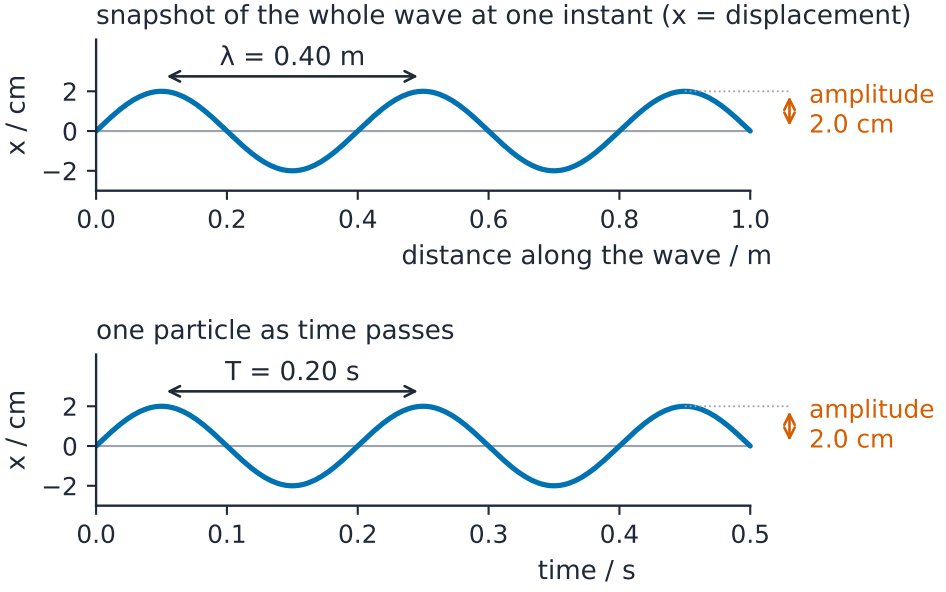
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Graphs of wave motion

Two kinds of graph

A wave can be drawn in two ways, and exam questions use both. Always read the horizontal axis first.

- A **displacement–distance graph** is a snapshot of the whole wave at one instant. The repeat length on this graph is the **wavelength**.
- A **displacement–time graph** shows how **one particle** moves as time passes. The repeat length on this graph is the **period**.



The same 5.0 Hz wave, as a snapshot in space and as one particle in time

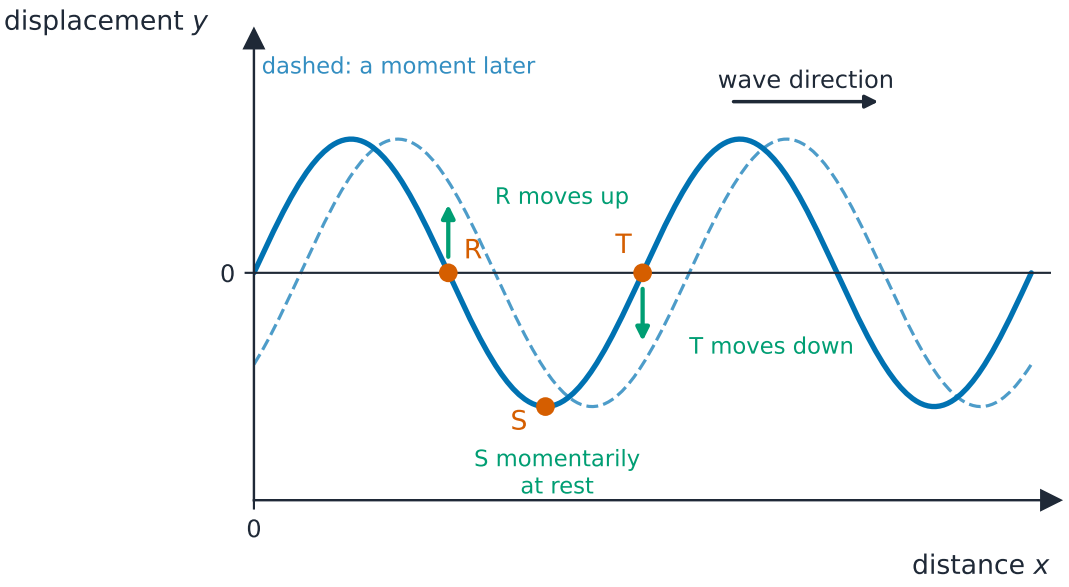
From the two graphs above: $\lambda = 0.40$ m and $T = 0.20$ s, so $v = \lambda/T = 2.0$ m s^{-1}.

Which way is a particle moving?

On a displacement–distance graph of a transverse wave travelling to the right, each particle will next copy the displacement of the particle just to its left (the one the wave has already reached). So:

- a particle on the front slope of a crest (the side facing the direction of travel) is moving **up**;
- a particle on the back slope is moving **down**;
- particles at a crest or a trough are momentarily at rest.

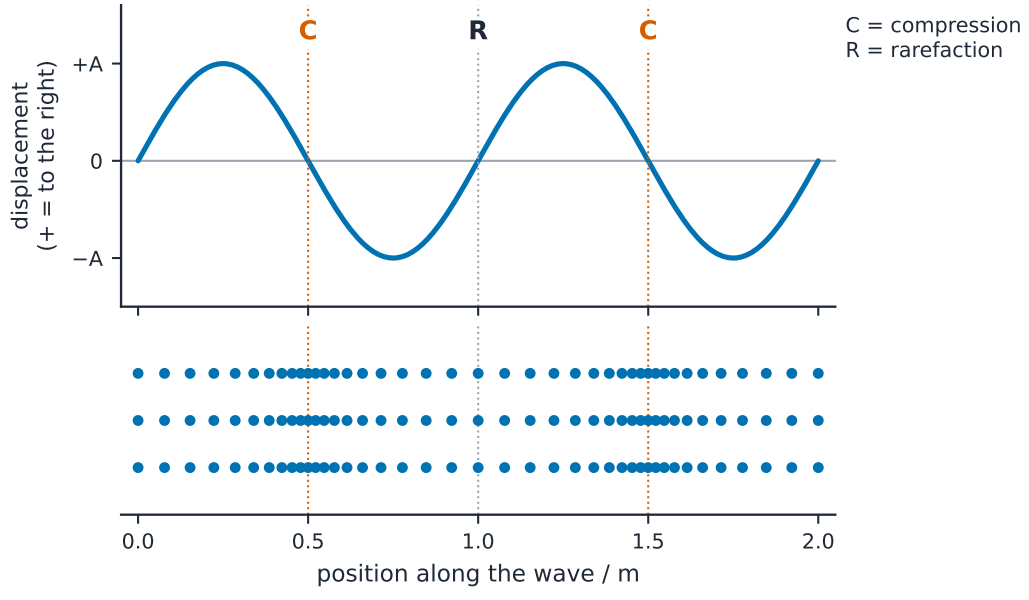
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Which way each point moves: towards the displacement its neighbour on the incoming side has now; a crest or trough is momentarily at rest

Graphs of longitudinal waves

The displacement of a particle in a longitudinal wave is along the direction of travel, but it is still drawn on the vertical axis of the graph. Take displacement to the right as positive.



On the graph, a compression is where particles on both sides are displaced towards it

At 0.50 m, the particles just to the left have positive displacement (pushed right) and

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those just to the right have negative displacement (pushed left): they crowd together, so this is the centre of a **compression**. At 1.00 m the particles on either side move away, giving a **rarefaction**. Compressions and rarefactions are where the displacement is **zero**, not where it is largest.

Sound waves

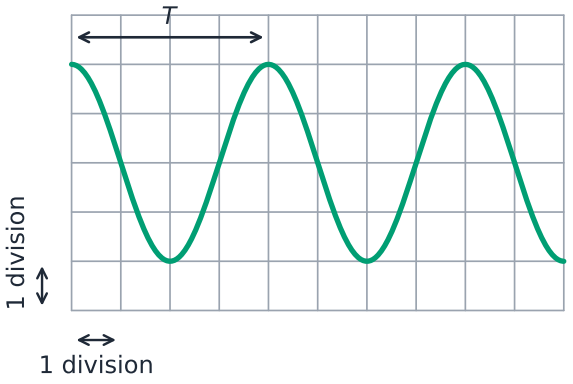
Sound 声音 in air is a longitudinal mechanical wave. A vibrating object, such as a loudspeaker cone, pushes the air molecules next to it together (a compression) and then pulls back, leaving them spread out (a rarefaction). Each layer of air pushes on the next, and the pattern travels outwards.



Tuning forks: a longer, heavier fork vibrates more slowly, giving a lower frequency

- Sound needs a medium. A ringing bell inside a jar goes silent as the air is pumped out, although it can still be seen ringing.
- Sound speed depends on elastic response and inertia, not particle spacing alone. The listed air, water and steel values illustrate common conditions; they do not prove a universal speed ordering for every gas, liquid and solid.
- Humans hear frequencies from about 20 Hz to 20 kHz: wavelengths in air from 17 m down to 17 mm. Sound above 20 kHz is called **ultrasound** 超声波; doctors use it for scans and ships use it to measure depth.
- A louder sound has a larger amplitude; a higher-pitched sound has a higher frequency.

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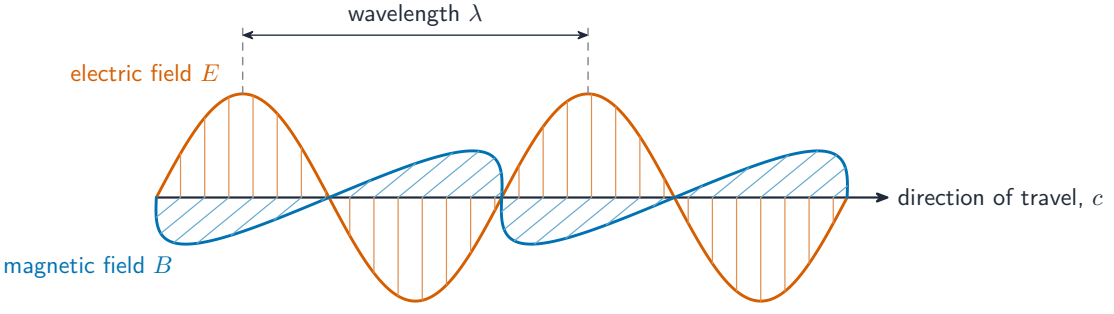
period $= n \times$ timebase · peak voltage $= n \times$ Y-gain

Reading the period T from a CRO trace using the grid and time-base

Worked example: thunder is heard 3.0 s after lightning is seen. Light arrives almost at once, so the storm is about $340 \times 3.0 = 1.0$ km away.

Electromagnetic waves

An **electromagnetic wave** 电磁波 is an oscillating electric field and an oscillating magnetic field, at right angles to each other and to the direction of travel. Electromagnetic waves are therefore **transverse**. They are produced by accelerating electric charges, such as electrons oscillating up and down a radio aerial.

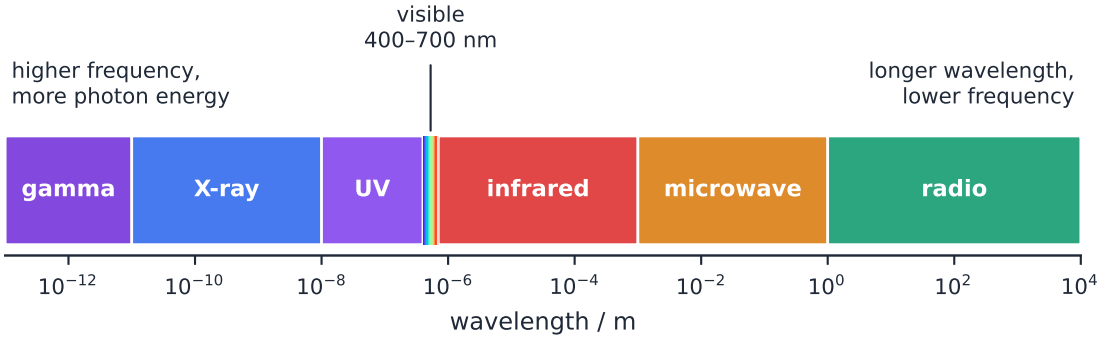


The electric and magnetic fields oscillate in phase, at right angles to each other and to the direction of travel

- They need **no medium**: they travel through a vacuum, which is how light and heat reach us from the Sun.

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- All electromagnetic waves travel at the same speed in a vacuum, $c = 3.00 \times 10^8$ m s^{-1}; they slow down in materials such as glass and water.
- They form the **electromagnetic spectrum** 电磁波谱, from radio waves with wavelengths of kilometres to gamma rays shorter than an atomic nucleus.



Each step on the scale is a factor of 100 in wavelength

Region	Order of magnitude of wavelength / m	Example use
radio	10^1 and longer	broadcasting, communication
microwave	10^{-2}	mobile phones, cooking, radar
infrared	10^{-5}	thermal imaging, remote controls
visible	10^{-7} (400–700 nm)	seeing, optical fibres
ultraviolet	10^{-8}	sterilising, fluorescent lamps
X-ray	10^{-10}	medical imaging
gamma ray	10^{-12}	cancer treatment

Worked examples:

- A microwave oven uses 2.45 GHz: $\lambda = \frac{3.00\times10^8}{2.45\times10^9} = 0.12$ m.
- Green light has $\lambda = 550$ nm: $f = \frac{3.00\times10^8}{5.5\times10^{-7}} = 5.5 \times 10^{14}$ Hz.

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Mechanical and electromagnetic waves compared

	Mechanical waves (sound, water, seismic)	**Electromagnetic waves**
What oscillates	particles of the medium	electric and magnetic fields
Medium	needed	not needed; travel through a vacuum
Type	transverse or longitudinal	always transverse
Speed	depends on the medium; hundreds or thousands of m s^{-1}	3.00×10^8 m s^{-1} in a vacuum, slower in materials

Both kinds obey $v = f\lambda$, both transfer energy without transferring matter, and both show the wave behaviours of C.3: reflection, refraction, diffraction and interference.

Practice

1. Distinguish between transverse and longitudinal waves, giving one example of each.
2. A wave on a string has a frequency of 5.0 Hz and a wavelength of 0.40 m. Find its speed and period.
3. Find the wavelength of a 512 Hz sound (a) in air, (b) in water, where sound travels at 1500 m s^{-1}. State what happens to the frequency.
4. Find the wavelength of a radio station broadcasting at 98 MHz.
5. Thunder is heard 4.5 s after lightning is seen. Estimate the distance to the storm.
6. A displacement–distance graph shows exactly four complete waves in 2.4 m. The wave travels at 1.2 m s^{-1}. Find the wavelength and the frequency.
7. State three differences between sound waves and electromagnetic waves.
8. Put these in order of increasing wavelength: microwaves, X-rays, radio waves, visible light, ultraviolet.
9. Explain why astronauts on the Moon cannot hear each other without radios, although they can see each other.
10. Explain how a displacement–distance graph and a displacement–time graph of the same wave differ, and which quantity each gives directly.
11. On a displacement–position graph of a sound wave, state where the compressions are.
12. P-waves travel at 6.0 km s^{-1} and S-waves at 3.5 km s^{-1}. A station records the S-waves 30 s after the P-waves. Find its distance from the earthquake.
13. A wave passes from deep to shallow water. Its speed falls from 0.32 m s^{-1} to 0.20 m s^{-1} and its frequency is 4.0 Hz. Find the wavelength in each region.

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Answers.

1. Transverse: oscillations are perpendicular to propagation (electric and magnetic fields for light; particles for a string wave). Longitudinal: particles oscillate parallel to the direction of travel (sound, P-waves).
2. $v = 5.0 \times 0.40 = 2.0$ m s^{-1}; $T = 1/5.0 = 0.20$ s.
3. (a) $340/512 = 0.66$ m. (b) $1500/512 = 2.9$ m. The frequency stays 512 Hz: it is set by the source.
4. $\lambda = \frac{3.00\times10^8}{98\times10^6} = 3.1$ m.
5. $340 \times 4.5 = 1.5 \times 10^3$ m, about 1.5 km.
6. $\lambda = 2.4/4 = 0.60$ m; $f = 1.2/0.60 = 2.0$ Hz.
7. Any three: sound needs a medium, EM waves do not; sound is longitudinal, EM waves are transverse; sound is particles oscillating, EM is electric and magnetic fields oscillating; EM waves are about a million times faster.
8. X-rays, ultraviolet, visible light, microwaves, radio waves.
9. Sound is a mechanical wave and needs a medium; there is no air on the Moon. Light is electromagnetic and travels through a vacuum.
10. The displacement–distance graph is a snapshot of all the particles at one time; its repeat length is the wavelength. The displacement–time graph shows one particle over time; its repeat length is the period.
11. At the points of zero displacement where the graph slopes downwards (displacement changes from positive to negative), because the particles on both sides are displaced towards that point.
12. $d/3.5 - d/6.0 = 30$, so $d(0.286 - 0.167) = 30$ and $d = 2.5 \times 10^2$ km.
13. Deep: $0.32/4.0 = 0.080$ m (8.0 cm). Shallow: $0.20/4.0 = 0.050$ m (5.0 cm).

Exam tips

- Waves transfer energy, not matter: the particles oscillate about fixed positions.
- Read the horizontal axis before using a wave graph: distance gives λ, time gives T.
- When a wave enters a new medium, the frequency stays the same; the speed and wavelength change together.
- Compressions and rarefactions are at points of **zero** displacement on a displacement–position graph.
- Convert MHz, GHz and nm to SI units before using $v = f\lambda$.
- Learn the order of the electromagnetic spectrum and an order of magnitude for each wavelength.

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Wave phenomena

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches C.3 Wave phenomena in full, for the course first assessed in 2025, but HL students must also study the sections marked AHL: single-slit diffraction, the single-slit envelope on double-slit fringes, and multiple slits and diffraction gratings.

Prerequisites: wavelength, frequency, wave speed and $v = f\lambda$ from C.2; phase and phase difference from C.1; sine and inverse sine. Use $c = 3.00 \times 10^{8}$ m s^{-1}.

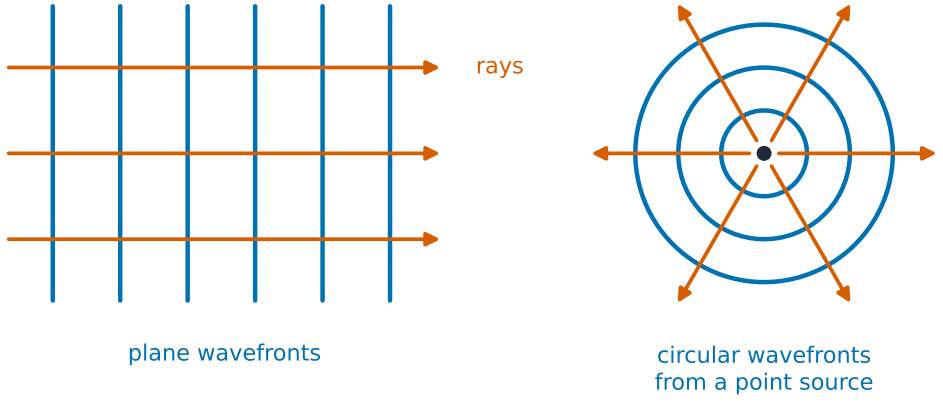
Wavefronts and rays

Why can you hear someone round a corner but not see them?

Stand outside an open door and you can hear a conversation in the room, even when nobody is in your line of sight. Sound waves bend round the edges of the doorway; light waves hardly do. The same wave model explains this, along with mirrors, lenses, rainbows, optical fibres that carry the internet under the oceans, and the coloured bands on a CD. This unit follows waves as they meet boundaries, pass through gaps and overlap each other.

Describing waves in two and three dimensions

A **wavefront** 波前 is a line (or surface) joining points that are in phase, for example all the points on one crest. A **ray** 波线 is a line showing the direction in which the wave travels; it is perpendicular to wavefronts in the ordinary isotropic wave model used here. The distance between neighbouring wavefronts drawn on crests is one wavelength.



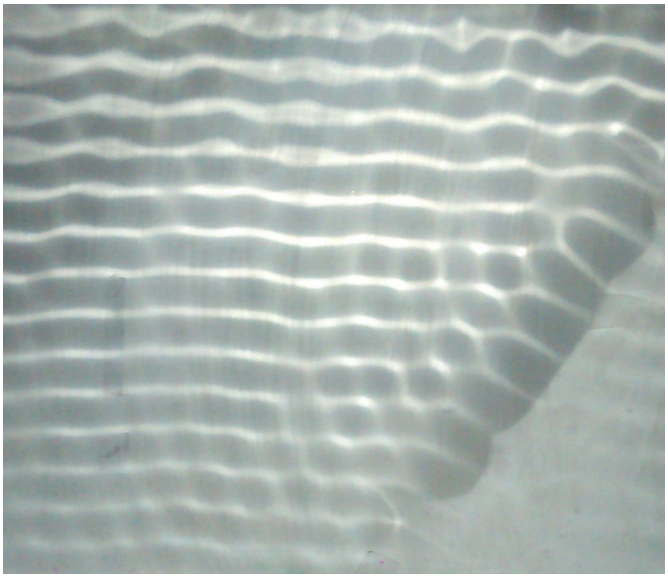
Rays are perpendicular to wavefronts in the isotropic model

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- Far from a source, or from a long straight source, the wavefronts are straight: these are **plane waves**.
- Near a small source the wavefronts are circles (in two dimensions) or spheres (in three), and the rays spread out radially.

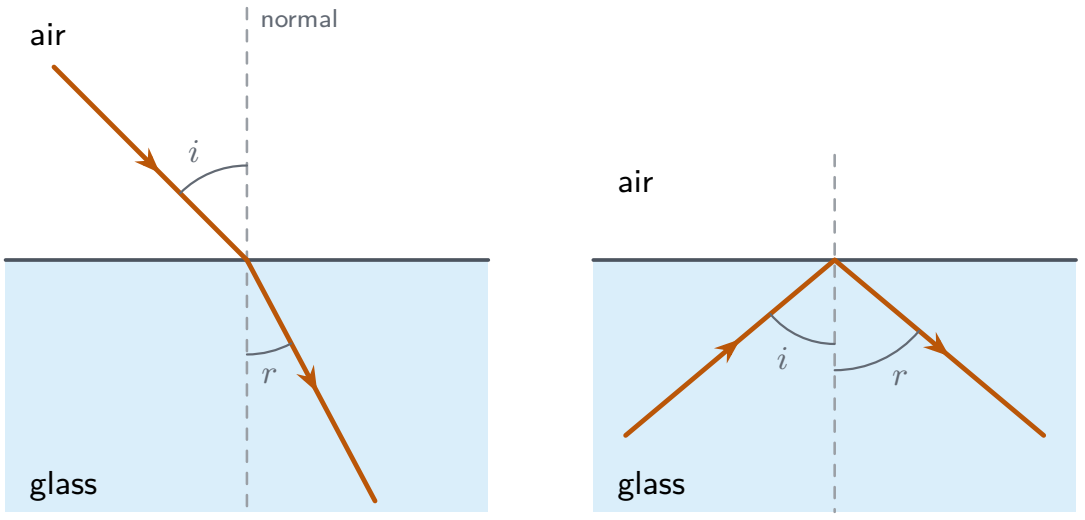
Reflection, refraction and transmission

When a wave reaches a **boundary** 界面 between two media, three things can happen, usually all at once:



Plane waves in a ripple tank reflect off a straight barrier: the reflected wavefronts leave at the same angle they arrived

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entering glass the ray slows and bends *towards* the normal: $r < i$

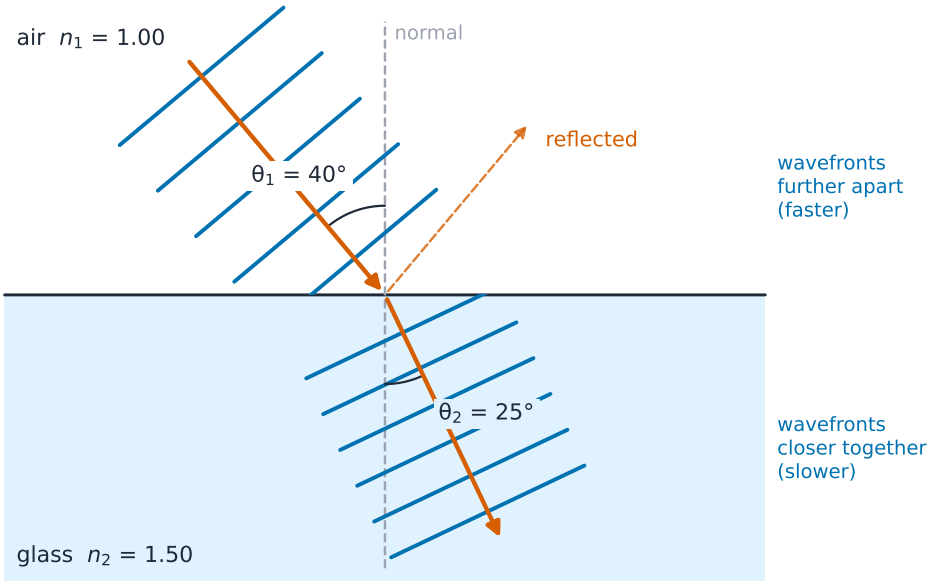
inside the glass, i greater than the critical angle: *all* the light reflects, $r = i$

Refraction towards the normal entering glass; total internal reflection beyond the critical angle.

- **Reflection** 反射: part of the wave bounces back into the first medium. The angle of incidence equals the angle of reflection, both measured from the **normal** 法线, the line at right angles to the boundary.
- **Transmission** 透射: part of the wave passes into the second medium.
- **Refraction** 折射: the transmitted wave changes speed and, unless it meets the boundary along the normal, changes direction.

At a stationary boundary in this linear model, frequency remains fixed by the source. If the wave slows down, the wavelength gets shorter, so the wavefronts crowd closer together. The end of each wavefront that enters the slow medium first slows first, so the wavefront swings round and the ray bends **towards the normal**. A wave that speeds up bends **away from the normal**.

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Slower in glass: shorter wavelength, and the ray bends towards the normal

Water waves show the same thing: they slow down in shallow water, so waves approaching a beach at an angle swing round until their crests are almost parallel to the shore.

In the lab: place a glass block on paper, trace its outline and send a narrow ray-box beam into one face. Mark the incident and refracted paths, then draw the normal at the entry point. Measure both angles from this normal, repeat for several incident angles and compare $\sin\theta_1/\sin\theta_2$. Keep the beam away from eyes. In a ripple-tank image, measure across several complete wavelengths and divide by their number to reduce fractional reading uncertainty; compare wavelengths on each side of the boundary while the source frequency stays fixed.

Snell's law and refractive index

The **refractive index** 折射率 n of a medium compares the speed of light in a vacuum with its speed v in the medium: $n = c/v$. Air has $n = 1.00$, water 1.33, glass about 1.50 and diamond 2.42. Light in water travels at $3.00 \times 10^8/1.33 = 2.26 \times 10^8$ m s^{-1}.

Snell's law 斯涅尔定律 links the angles, measured from the normal:

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

which is often written as $n_1 \sin\theta_1 = n_2 \sin\theta_2$. A larger refractive index means a slower speed and a smaller angle to the normal.

Worked examples:

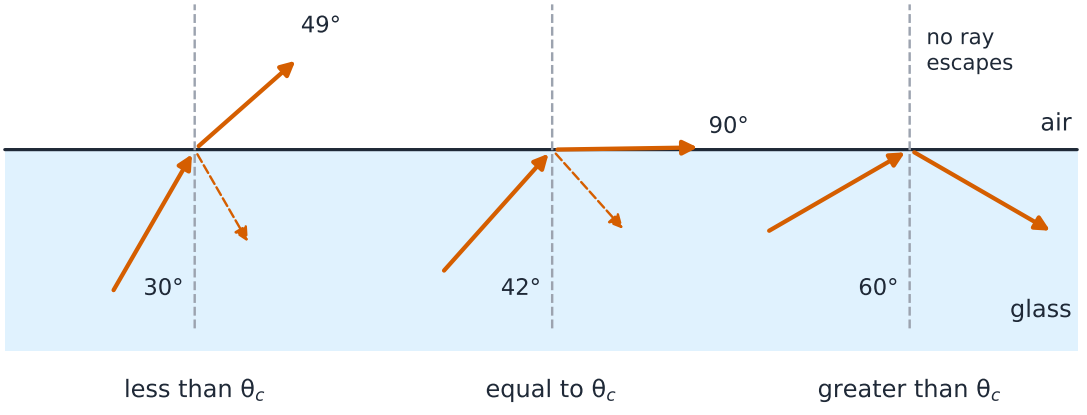
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- Light passes from air into glass at 40°: $\sin\theta_2 = 1.00 \times \sin 40°/1.50 = 0.429$, so $\theta_2 = 25°$.
- Light passes from water into glass at 50°: $\sin\theta_2 = 1.33\sin 50°/1.50 = 0.679$, so $\theta_2 = 43°$. It still bends towards the normal, but less, because the speeds are closer.
- A ray in air at 30° to the normal refracts to 19.5° in a plastic block: $n = \sin 30°/\sin 19.5° = 1.50$.

The same refraction relationship applies to the ordinary sound and water-wave models used here. For sound or water waves, use the ratio of speeds v_2/v_1.

Critical angle and total internal reflection

When light travels from a slower medium into a faster one (glass into air, for example) it bends away from the normal. As the angle of incidence increases, the refracted ray gets closer to the boundary. At the **critical angle** 临界角 θ_c the refracted ray travels along the boundary, at 90° to the normal. For larger angles no light escapes: all of it is reflected back. This is **total internal reflection** 全内反射.



Below, at and above the critical angle for glass of refractive index 1.50

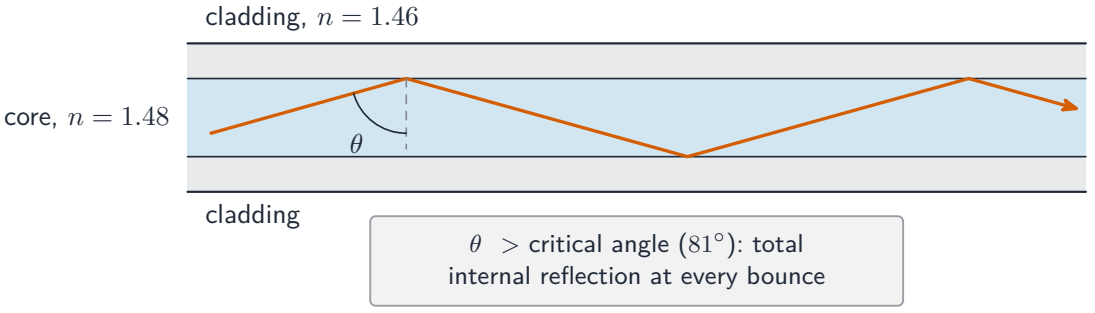
Setting $\theta_2 = 90°$ in Snell's law gives:

$$\sin\theta_c = \frac{n_2}{n_1}$$

for light going from medium 1 into medium 2 with $n_1 > n_2$. Two conditions are needed for total internal reflection: the light must be travelling into a medium with a **lower** refractive index, and the angle of incidence must be **greater** than the critical angle.

- Glass to air: $\sin\theta_c = 1/1.50$, so $\theta_c = 42°$.
- Water to air: $\theta_c = 49°$. Diamond to air: $\theta_c = 24°$, so light is trapped and reflected many times inside a cut diamond, which makes it sparkle.

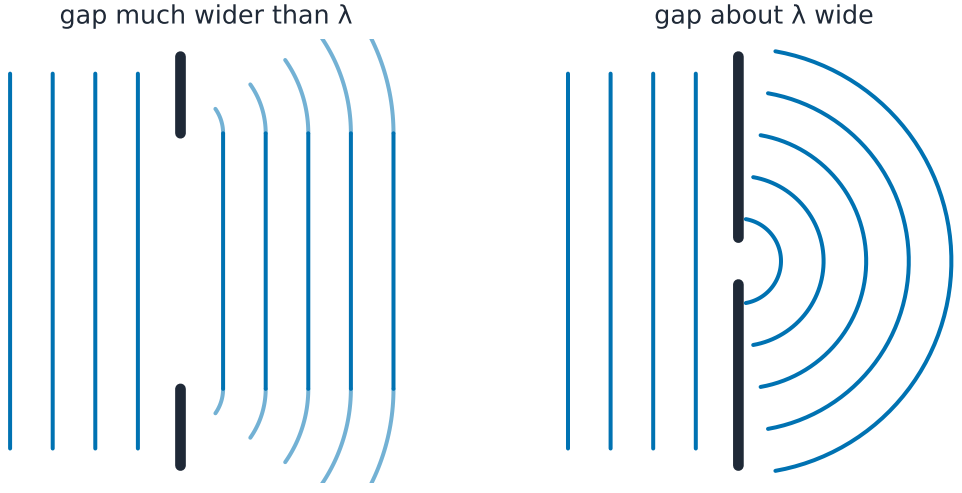
- An **optical fibre** 光纤 has a glass core ($n = 1.48$) inside a cladding ($n = 1.46$). The critical angle at the core edge is $\sin^{-1}(1.46/1.48) = 81°$, so light that travels nearly along the fibre is totally internally reflected again and again, carrying signals for many kilometres.



Light that meets the core edge above the critical angle is trapped in the core (angles not to scale)

Diffraction

Diffraction 衍射 is the spreading of a wave as it passes through a gap (an **aperture** 孔径) or round the edge of an obstacle. Diffraction does not change the wavelength, the frequency or the speed.



Diffraction is strong when the gap is comparable to the wavelength or smaller

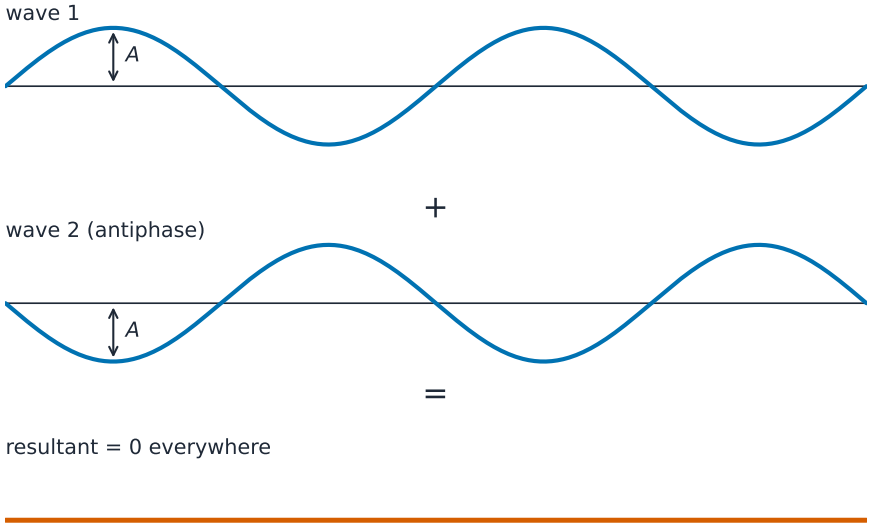
- The spreading is significant when the gap or obstacle is about the same size as the wavelength, or smaller.
- A gap much wider than the wavelength lets the wave through with little spreading.

This is why you can hear round a door but not see round it. Speech has wavelengths

of about 1 m, similar to a doorway, so it diffracts strongly. Light has wavelengths of about 5×10^{-7} m, a million times smaller than the doorway, so its diffraction is too small to notice. Longer radio wavelengths can diffract more broadly around a hill than much shorter wavelengths. Actual reception also depends on source power, terrain and other propagation effects; diffraction alone does not guarantee reception.

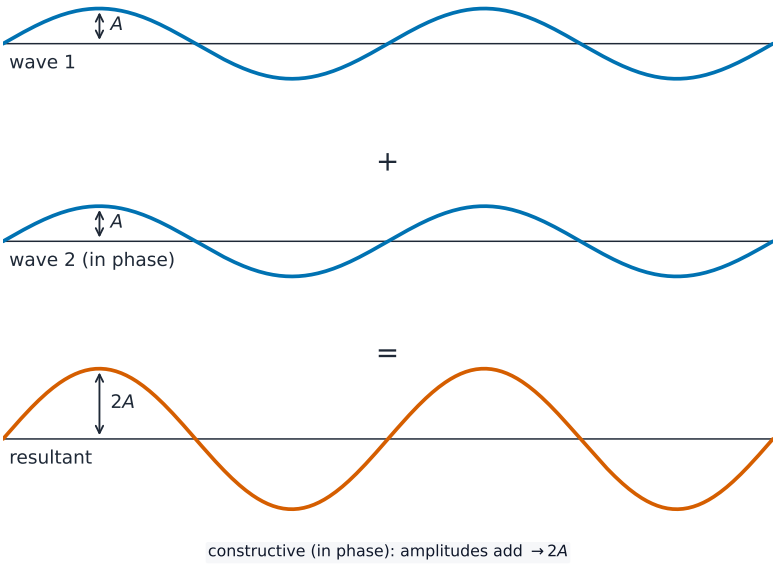
Superposition

In the linear wave model, when two waves meet, the resultant displacement at each point is the sum of the displacements of the individual waves. This is the **principle of superposition** 叠加原理. After they have passed through each other, each wave continues as if the other had never been there.



Two waves arriving exactly out of phase cancel to zero (destructive)

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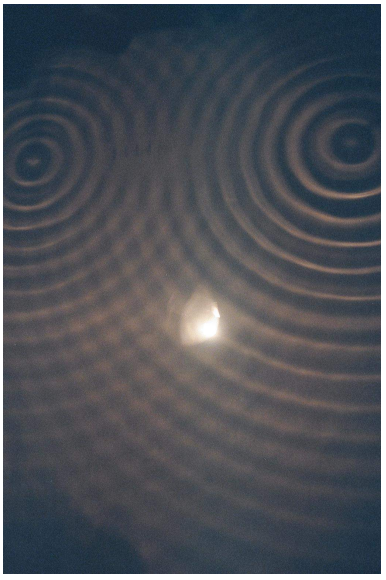
Two waves arriving in phase add to give double the amplitude (constructive)

- Two wave **pulses** 脉冲 that meet crest-to-crest add up to a pulse twice as tall for an instant.
- A crest meeting an equal trough cancels it for an instant: the string is momentarily flat.
- Add the displacements with their signs. A displacement of +3 mm and one of −1 mm give +2 mm.

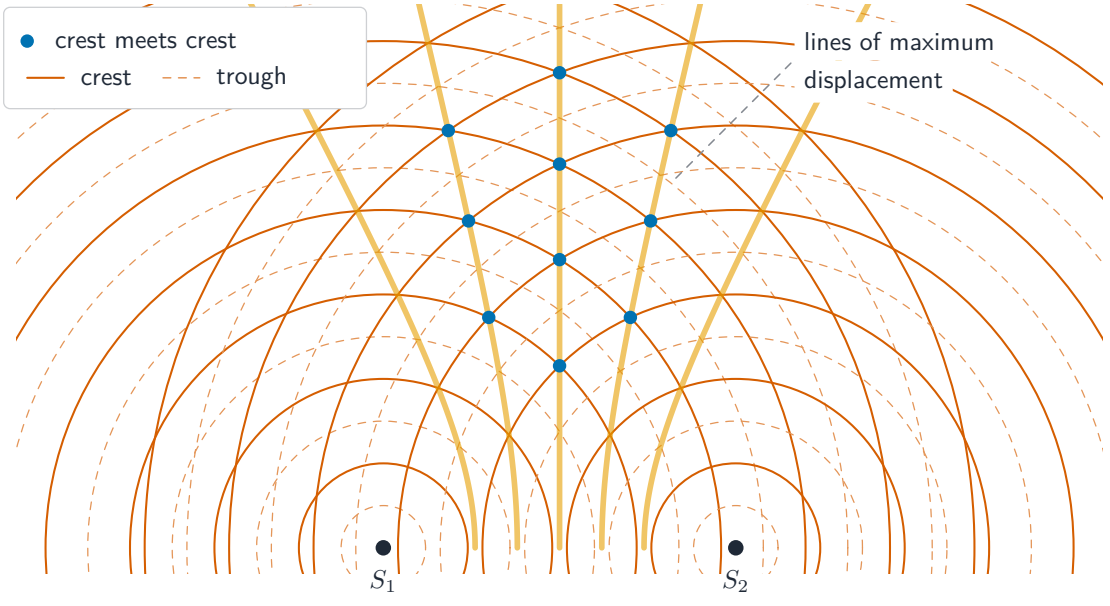
Interference

Coherent sources

When waves from two sources overlap, they can produce **interference** 干涉. With coherent sources this can give a steady pattern of maxima and minima. A steady pattern needs **coherent** 相干的 sources: sources with the same frequency and a constant phase difference. Two loudspeakers driven by the same signal generator are coherent. Two separate light bulbs are not, because each emits light in short random bursts, so their phase difference keeps changing. That is why light from one source is split into two, using two slits.



The same effect in a real ripple tank – two coherent sources give a steady interference pattern

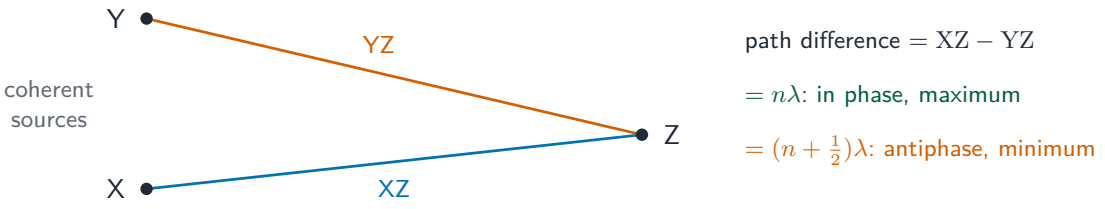


Two coherent sources give lines of maximum displacement where crests meet crests

Path difference

Each point in the pattern is a distance r_1 from one source and r_2 from the other. The **path difference** 程差 is $|r_1 - r_2|$. For two sources in phase:

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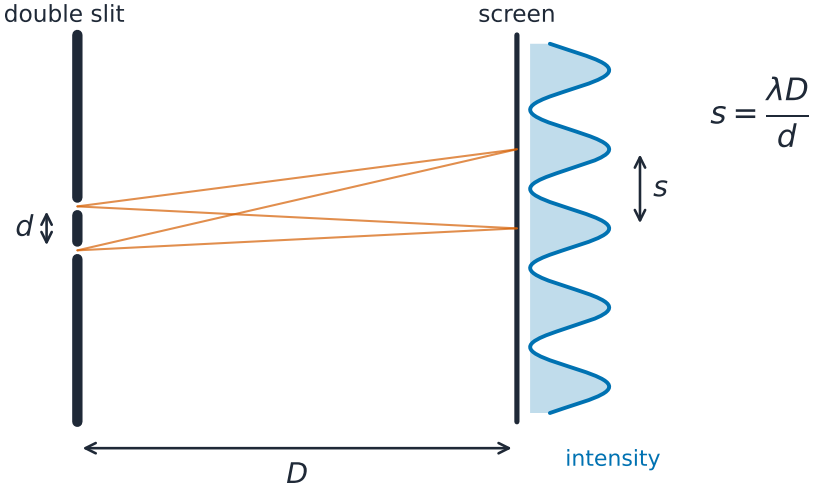
What happens at Z depends on the path difference XZ − YZ, measured in wavelengths

Path difference	Waves arrive	Result
$n\lambda$ (0, λ, 2λ, …)	in phase, crest on crest	**constructive interference** 相长干涉: maximum amplitude
$(n+\frac{1}{2})\lambda$ ($\lambda/2$, $3\lambda/2$, …)	in antiphase, crest on trough	**destructive interference** 相消干涉: minimum amplitude

where n is a whole number. Worked example: two speakers emit sound in phase with $\lambda = 0.50$ m. A listener 4.0 m from one speaker and 5.0 m from the other has a path difference of 1.0 m $= 2\lambda$: a loud point. At 4.0 m and 4.75 m, the path difference is 0.75 m $= 1.5\lambda$: a quiet point.

Young's double-slit experiment

In Young's double-slit arrangement, coherent monochromatic light illuminates two narrow, closely spaced slits. The light diffracts at each slit, the two diffracted waves overlap, and a pattern of equally spaced bright and dark **fringes** 条纹 appears on a screen. This interference was strong evidence that light is a wave.



Young's double slit: bright fringes where the path difference is a whole number of wavelengths

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The **fringe separation** 条纹间距 s, the distance between the centres of neighbouring bright fringes, is:

$$s = \frac{\lambda D}{d}$$

where d is the slit separation and D is the distance from the slits to the screen (D is much larger than d, and the observed angles are small).

- The central bright fringe is where the path difference is zero.
- Longer wavelength (red light), a more distant screen or closer slits all give more widely spaced fringes.

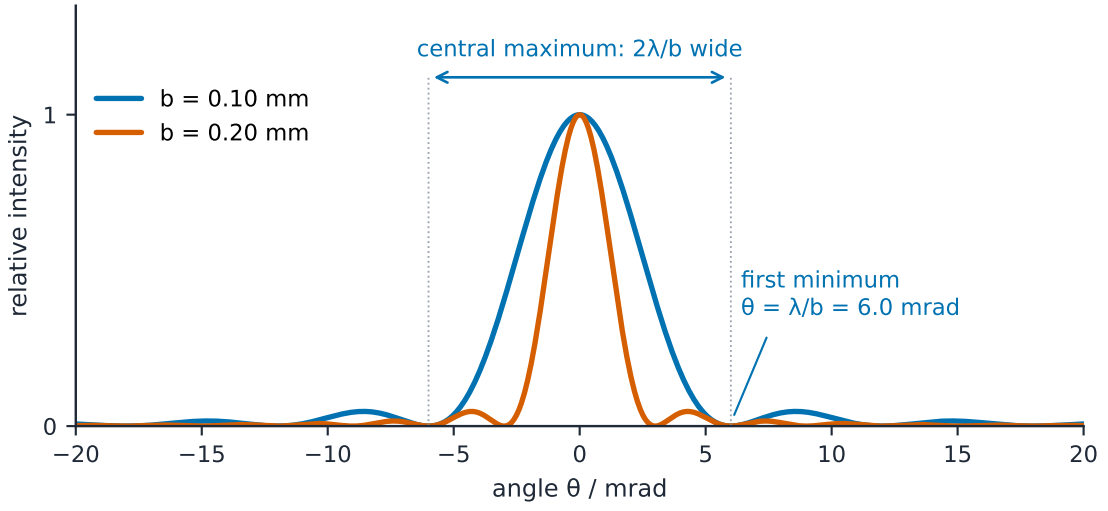
Worked example: a 633 nm laser shines on slits 0.50 mm apart, with a screen 2.0 m away. $s = \frac{633\times10^{-9}\times2.0}{0.50\times10^{-3}} = 2.5 \times 10^{-3}$ m (2.5 mm). To measure s accurately, measure across several fringes and divide: 10 fringe spacings span 25 mm.

AHL: Single-slit diffraction

Light passing through a single slit of width b also makes a pattern: a bright, wide **central maximum** with much fainter maxima on each side. Waves from different parts of the same slit interfere. The first minimum is at the angle:

$$\theta = \frac{\lambda}{b}$$

with θ in radians (the angle is small).



A narrower slit gives a wider central maximum; the plotted peaks are independently normalized

- The central maximum is twice as wide as each side maximum: it runs from $-\lambda/b$ to $+\lambda/b$.

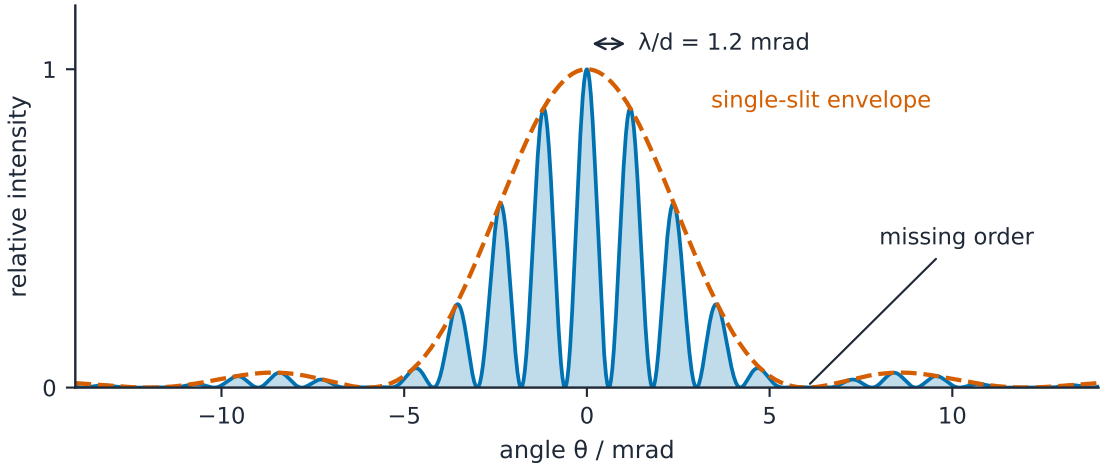
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- A **narrower** slit spreads the light more: the central maximum gets wider and dimmer, because less light gets through.
- A **wider** slit gives a narrower, brighter central maximum.
- A longer wavelength gives a wider pattern.

Worked example: 600 nm light through a 0.10 mm slit gives $\theta = \frac{600\times10^{-9}}{1.0\times10^{-4}} = 6.0 \times 10^{-3}$ rad. On a screen 2.0 m away the first minima are $2.0 \times 6.0 \times 10^{-3} = 12$ mm from the centre, so the central maximum is 24 mm wide.

AHL: The double-slit pattern and its envelope

Finite slit width gives a single-slit diffraction **envelope** 包络线 that **modulates** 调制 the double-slit fringes. This describes relative intensity versus angle; absolute peak height also depends on coherent addition and illumination.



Fine fringes (set by d) under a wide envelope (set by b)

- The fringe spacing is set by the slit **separation** d: angle λ/d.
- The envelope is set by the slit **width** b: zero at λ/b.
- Where a bright fringe falls at a minimum of the envelope, it is missing.

Worked example: with $\lambda = 600$ nm, $d = 0.50$ mm and $b = 0.10$ mm, the fringes are 1.2×10^{-3} rad apart and the envelope reaches zero at 6.0×10^{-3} rad, exactly five fringe spacings. So the central maximum of the envelope holds 9 bright fringes (orders 0, ±1, ±2, ±3, ±4), and order 5 is missing.

AHL: Multiple slits and diffraction gratings

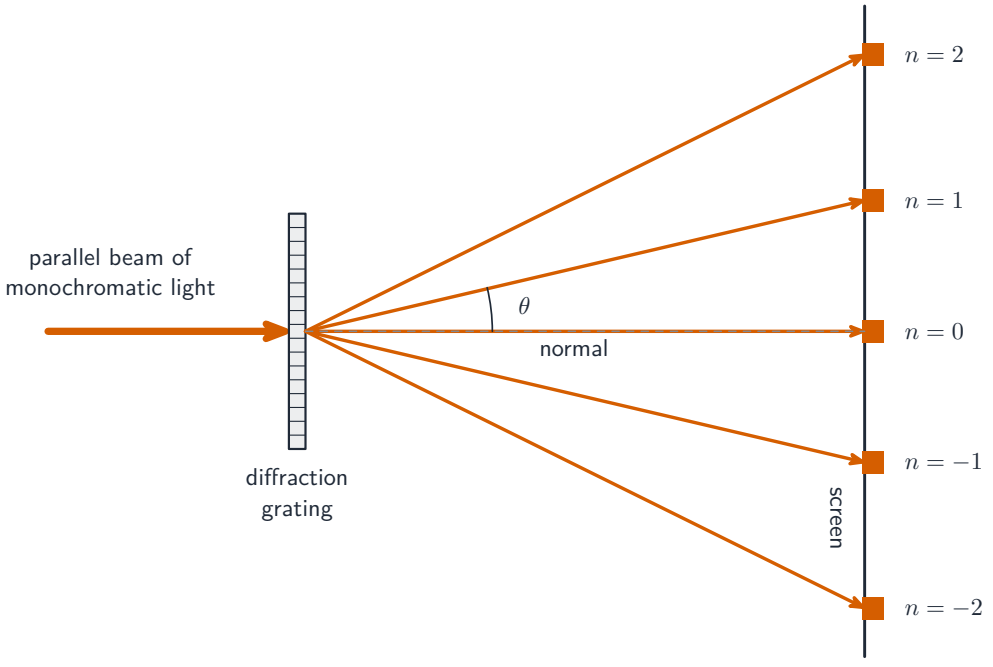
More slits, sharper maxima

A **diffraction grating** 衍射光栅 has a very large number of equally spaced slits, often hundreds per millimetre. The bright maxima are at the angles where waves from

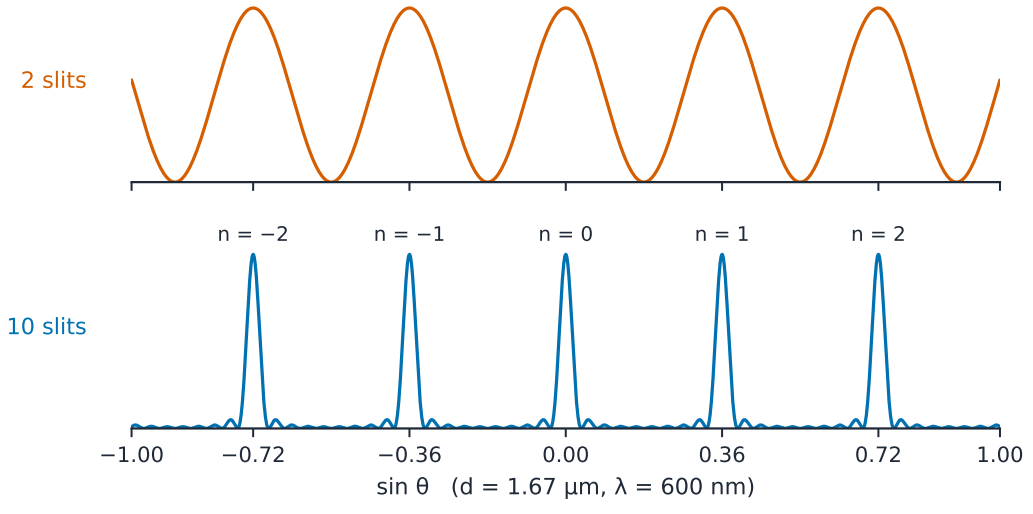
neighbouring slits have a path difference of a whole number of wavelengths:

$$n\lambda = d\sin\theta$$

where d is the slit spacing and n is the **order** of the maximum.



A diffraction grating splits monochromatic light into sharp maxima on a screen



The normalized patterns show sharper principal maxima at the same angles; they do not compare absolute brightness

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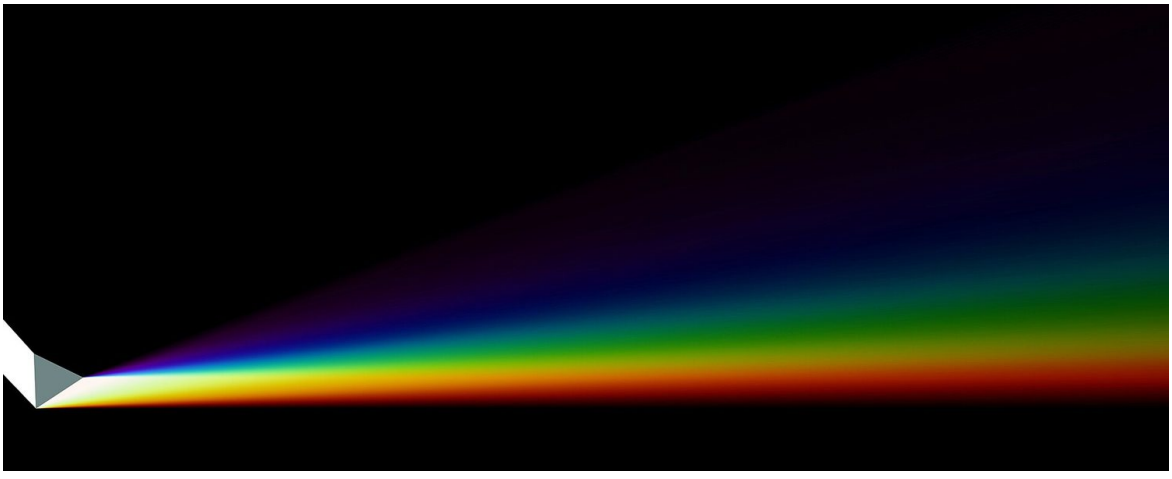
- With more slits, the principal maxima stay at the same angles but become much **sharper** and, with unchanged illumination per slit, **brighter**, with almost total darkness between them. This lets a grating measure wavelengths very precisely.
- The angles are large, so use $\sin\theta$, not the small-angle approximation.
- $\sin\theta$ cannot exceed 1, so the highest possible order is the largest whole number $n \leq d/\lambda$.

Worked example: a grating has 600 lines per mm, so $d = 1/600$ mm $= 1.67 \times 10^{-6}$ m. For 600 nm light:

- first order: $\sin\theta = \frac{600\times10^{-9}}{1.67\times10^{-6}} = 0.36$, $\theta = 21°$;
- second order: $\sin\theta = 0.72$, $\theta = 46°$;
- third order would need $\sin\theta = 1.08$, which is impossible, so only orders 0, 1 and 2 appear on each side.

White light and spectra

With white light, every wavelength makes its own maxima. The zero order is white, because all wavelengths have zero path difference there. In each other order the light is spread into a **spectrum** 光谱, violet (shortest wavelength, smallest angle) nearest the centre and red furthest out.



Refraction bends each colour by a different amount, so a prism spreads white light into a rainbow

For the 600 lines per mm grating:

Order	Violet, 400 nm	Red, 700 nm
1	14°	25°
2	29°	57°
3	46°	not formed

The second- and third-order spectra overlap: the third-order violet (46°) appears inside the second-order spectrum, which runs to 57°. Higher orders are spread out more

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widely, so they separate close wavelengths better, but they overlap more.

Practice

1. Draw a wavefront–ray diagram for plane waves entering a medium where they travel more slowly, at an angle to the boundary. Label the normal and show how the wavelength changes.
2. Light enters glass ($n = 1.50$) from air at an angle of incidence of 55°. Find the angle of refraction and the speed of light in the glass.
3. A ray in air at 30.0° to the normal is refracted to 22.0° in a liquid. Find the refractive index of the liquid.
4. Find the critical angle for light going from water ($n = 1.33$) into air, and state the two conditions for total internal reflection.
5. Explain why an optical fibre can carry light round gentle bends.
6. In a diffraction-only model, a hill has a characteristic size comparable to 1500 m. Compare spreading of waves of wavelength 1500 m and 3 m around it.
7. Two pulses travel towards each other on a rope: one is a crest of height 4 mm, the other a trough of depth 4 mm. Describe the rope when they overlap exactly, and after they have passed.
8. Two loudspeakers in phase emit sound of wavelength 0.40 m. Point P is 3.0 m from one and 3.6 m from the other. State and explain whether P is loud or quiet.
9. Explain why two car headlamps do not produce an interference pattern.
10. In a double-slit experiment, fringes are 1.8 mm apart on a screen 1.5 m from slits 0.40 mm apart. Find the wavelength. State the effect on the fringes of moving the screen to 3.0 m.
11. **HL.** Light of wavelength 500 nm passes through a slit 0.050 mm wide. Find the angle to the first minimum and the width of the central maximum on a screen 1.2 m away.
12. **HL.** In a double-slit experiment the slits are 0.30 mm apart and 0.060 mm wide. Which order of bright fringe is missing? How many bright fringes are inside the central envelope?
13. **HL.** A grating with 300 lines per mm gives a second-order maximum at 20.0°. Find the wavelength, and the highest order that can be seen.
14. **HL.** Describe how the pattern from a diffraction grating differs from a double-slit pattern with the same slit spacing, and describe what is seen when white light is used.

Answers.

1. Wavefronts get closer together in the slow medium (shorter wavelength); the ray bends towards the normal; the frequency is unchanged.
2. $\sin\theta_2 = \sin 55°/1.50 = 0.546$, so $\theta_2 = 33°$; $v = 3.00\times10^8/1.50 = 2.00\times10^8$ m s^{-1}.
3. $n = \sin 30.0°/\sin 22.0° = 0.500/0.375 = 1.33$.

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4. $\sin\theta_c = 1/1.33$, so $\theta_c = 49°$. TIR needs a lower destination index and incidence greater than θ_c.
5. Light meets the core–cladding boundary at angles greater than the critical angle, so it is totally internally reflected each time and stays inside the core.
6. The 1500 m waves diffract broadly round the comparable-sized hill; the 3 m waves spread less. Diffraction alone does not guarantee reception.
7. When they overlap, the displacements cancel and the rope is momentarily flat. Afterwards both pulses continue unchanged.
8. Path difference = 0.6 m = 1.5λ, a half-integer number of wavelengths: the waves arrive in antiphase, so P is quiet.
9. The lamps are not coherent: their changing phase difference prevents a steady pattern.
10. $\lambda = sd/D = 1.8 \times 10^{-3} \times 0.40 \times 10^{-3}/1.5 = 4.8 \times 10^{-7}$ m (480 nm). Doubling D doubles the fringe separation, to 3.6 mm.
11. **HL.** $\theta = \frac{500\times10^{-9}}{5.0\times10^{-5}} = 0.010$ rad. First minimum $1.2 \times 0.010 = 12$ mm from the centre, so the central maximum is 24 mm wide.
12. **HL.** $d/b = 5$: the envelope reaches zero at five fringe spacings, so order 5 is missing; 9 bright fringes (orders 0 to ±4) lie inside the central envelope.
13. **HL.** $d = 1/300$ mm $= 3.33 \times 10^{-6}$ m; $\lambda = d\sin 20.0°/2 = 5.7 \times 10^{-7}$ m (570 nm). $d/\lambda = 5.8$, so the highest order is 5.
14. **HL.** At fixed slit spacing, principal maxima keep their angles and narrow; fixed illumination per slit makes peaks brighter. White light gives a white zero order and spectra in other orders, violet nearer the centre. Higher orders spread farther and can overlap.

Exam tips

- Measure angles from the **normal**. Total internal reflection needs a lower destination index and incidence greater than the critical angle.
- Use **coherent** sources and include their initial phase difference. Convert mm and nm to metres before using $s = \lambda D/d$.

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Standing waves and resonance

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches C.4 Standing waves and resonance in full, for the course first assessed in 2025. There is no additional higher level content in C.4, so SL and HL students study the same material.

Prerequisites: superposition, wavelength, frequency and $v = f\lambda$ from C.2 and C.3; natural oscillation, phase and energy in simple harmonic motion from C.1. Use about 340 m s^{-1} for the speed of sound in air.

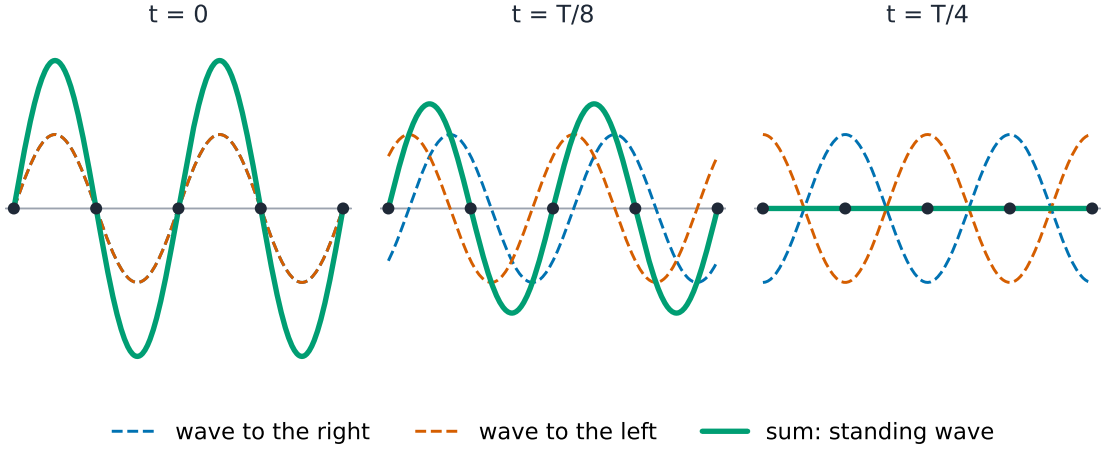
How standing waves form

Why does a guitar string play only certain notes?

A plucked guitar string produces a pitched sound containing several harmonics. Touch its midpoint lightly and pluck again: in the ideal string model this suppresses modes that move the midpoint, leaving the second harmonic as the lowest surviving mode, one octave above the first. Strings and air columns select modes through reflection and boundary conditions. A bottle blown across its neck can instead behave as a Helmholtz resonator; do not apply the uniform-pipe formulas to every cavity.

Two waves travelling in opposite directions

A **standing wave** 驻波 forms when two waves of the same frequency, wavelength and amplitude travel in **opposite directions** through the same medium and superpose. Usually one is a wave and the other is its own reflection from the end of a string or pipe.



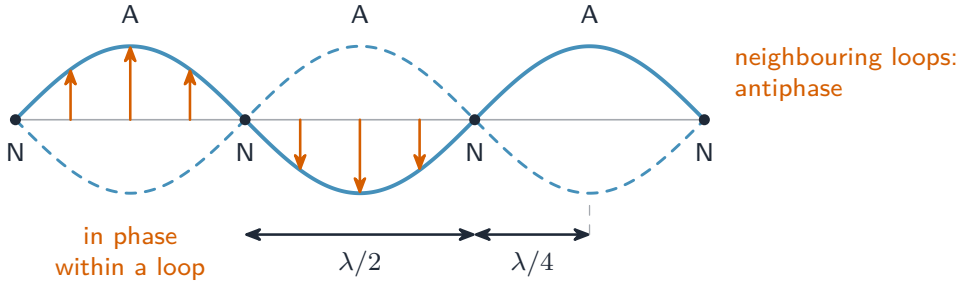
The sum of two opposite travelling waves: the nodes never move

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At some points the two waves always cancel: these points never move. At others the waves reinforce, and the medium oscillates with twice the amplitude of each wave. The pattern does not travel along the medium. At $t = T/4$ in this drawing, the waves cancel everywhere: the string is flat, while its non-node points are moving. The nodes remain at rest.

Nodes and antinodes

- A **node** 波节 is a point where the displacement is always zero.
- An **antinode** 波腹 is a point where the amplitude is greatest.
- Neighbouring nodes are **half a wavelength** apart; so are neighbouring antinodes. A node and the nearest antinode are a quarter of a wavelength apart.



Nodes $\lambda/2$ apart, node to antinode $\lambda/4$; one loop moves in phase, neighbouring loops in antiphase.

Amplitude and phase along a standing wave

- **Amplitude** varies with position: zero at the nodes, greatest at the antinodes.
- **Phase**: all points between two neighbouring nodes move **in phase**: they reach their maximum displacement at the same moment. Points in neighbouring loops, on opposite sides of a node, move in **antiphase** (phase difference π).

Worked example: microwaves reflected from a metal plate form a standing wave. A detector finds nodes 1.5 cm apart, so $\lambda = 3.0$ cm and $f = c/\lambda = 3.00 \times 10^8/0.030 = 1.0 \times 10^{10}$ Hz.

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Standing and travelling waves compared

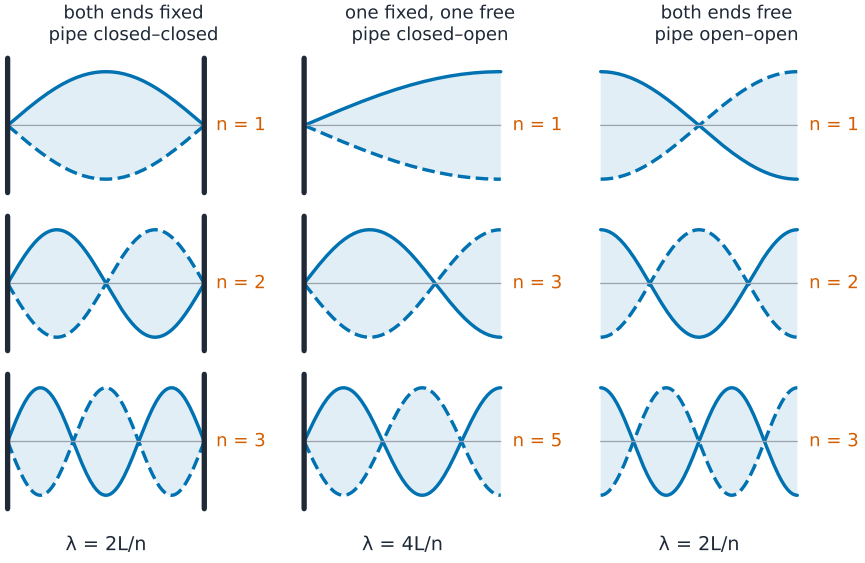
	Travelling wave	Standing wave
Energy	net transfer in the propagation direction	no net transfer along an ideal standing wave
Amplitude	uniform for an ideal undamped plane wave	zero at nodes, greatest at antinodes
Phase	changes steadily along the wave	the same within a loop; π between neighbouring loops
Wave shape	moves along at the wave speed	stays in place

Standing waves on strings

The ends of a string decide which standing waves fit. These are the **boundary conditions** 边界条件:

- A **fixed** end, such as a string clamped at a guitar's bridge, cannot move, so it must be a **node**.
- A **free** end, such as a light ring that can slide on a smooth rod, moves the most, so it is an **antinode**.

Only wavelengths that give a node or antinode at the right places can form. Each allowed pattern is a **harmonic** 谐波. The lowest frequency is the **first harmonic** 第一谐波 ($n = 1$).



The ends decide which harmonics fit; the solid and dashed lines show the extremes of the motion

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Boundaries	Allowed n	Wavelength	Frequency
both fixed, or both free	1, 2, 3, …	$\lambda_n = \dfrac{2L}{n}$	$f_n = \dfrac{nv}{2L}$
one fixed, one free	1, 3, 5, … (odd only)	$\lambda_n = \dfrac{4L}{n}$	$f_n = \dfrac{nv}{4L}$

where L is the length of the string and v is the speed of the wave along it. Every harmonic is a whole-number multiple of the first harmonic frequency.

Worked example: a guitar string of length 0.65 m, fixed at both ends, plays a first harmonic of 110 Hz.

- $\lambda_1 = 2L = 1.30$ m, so $v = f\lambda = 110 \times 1.30 = 143$ m s^{-1}.
- The second and third harmonics are 220 Hz and 330 Hz.
- Touching the middle forces a node there, so the first harmonic cannot form and the string sounds 220 Hz.

Standing waves in pipes

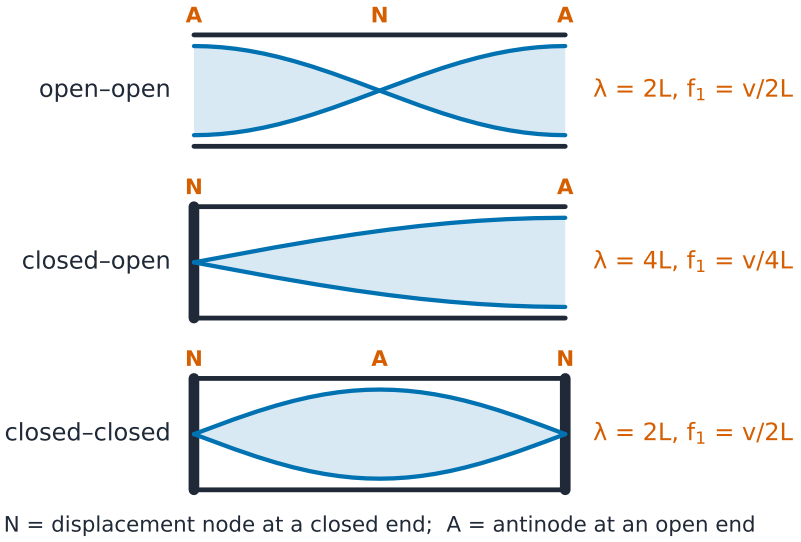
The air in a pipe can hold a standing **sound** wave. It is described by the **displacement** of the air molecules:



Organ pipes: each pipe's length sets the wavelength of its standing wave, and so its note

- At a **closed** end the air cannot move, so there is a displacement **node**.
- At an **open** end the air moves most freely, so there is a displacement **antinode**.

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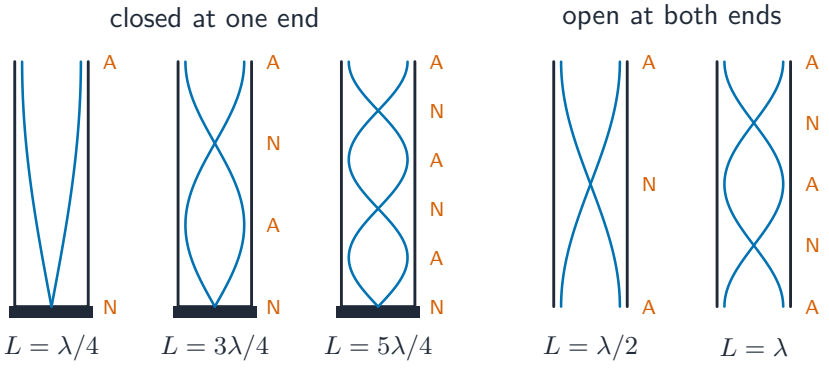


First harmonics in pipes: closed ends are nodes, open ends are antinodes

These are displacement patterns, not transverse paths of the air. Pressure variations have the opposite node/antinode pattern. Neglecting end corrections and losses, a uniform pipe has the same mode-length relationships as the corresponding string boundaries:

- **open–open** and **closed–closed** pipes: all harmonics, $f_n = \dfrac{nv}{2L}$;
- **closed–open** pipe (closed at one end): odd harmonics only, $f_n = \dfrac{nv}{4L}$, $n = 1, 3, 5, \ldots$

Worked examples, for pipes 0.34 m long with $v = 340$ m s^{-1}:



Modes of an air column: a closed end is a node, an open end an antinode

- Open at both ends: $f_1 = 340/(2 \times 0.34) = 500$ Hz, then 1000 Hz, 1500 Hz, …

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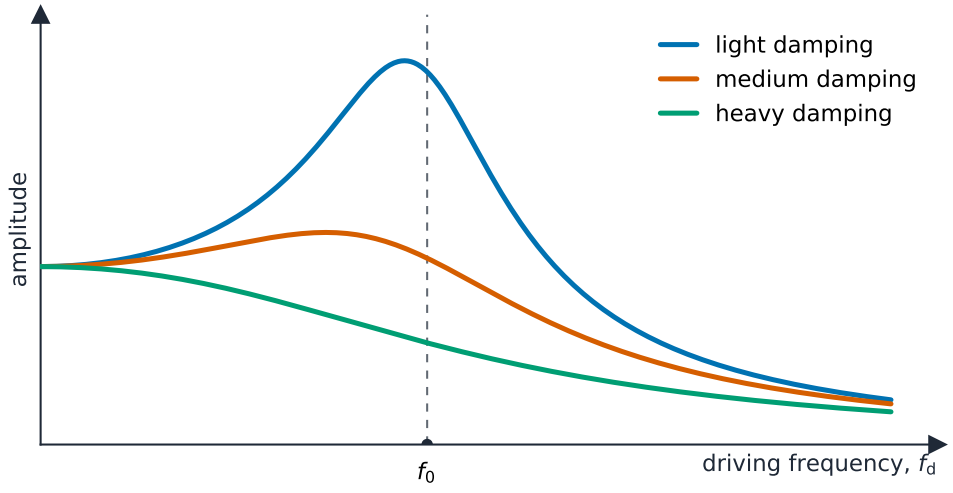
- Closed at one end: $f_1 = 340/(4 \times 0.34) = 250$ Hz, then 750 Hz, 1250 Hz, … Closing one end lowers the first harmonic by an octave and removes the even harmonics.

Worked example with invented measurements, neglecting end correction: measuring the speed of sound. A tuning fork of 512 Hz is held over a tube whose lower end is closed by water. The first loud resonance is heard when the air column is 0.166 m long. This is a closed–open pipe, so $L = \lambda/4$: $\lambda = 0.664$ m and $v = 512 \times 0.664 = 340$ m s^{-1}. The next resonance is at $3\lambda/4 = 0.498$ m.

Resonance

Natural frequency and driving frequency

A **natural frequency** 固有频率 is a frequency of free oscillation of a mode after displacement and release. A simple oscillator has one relevant mode; a glass or bridge can have many.



resonance: max amplitude near f_0 · lighter damping ⇒ sharper peak

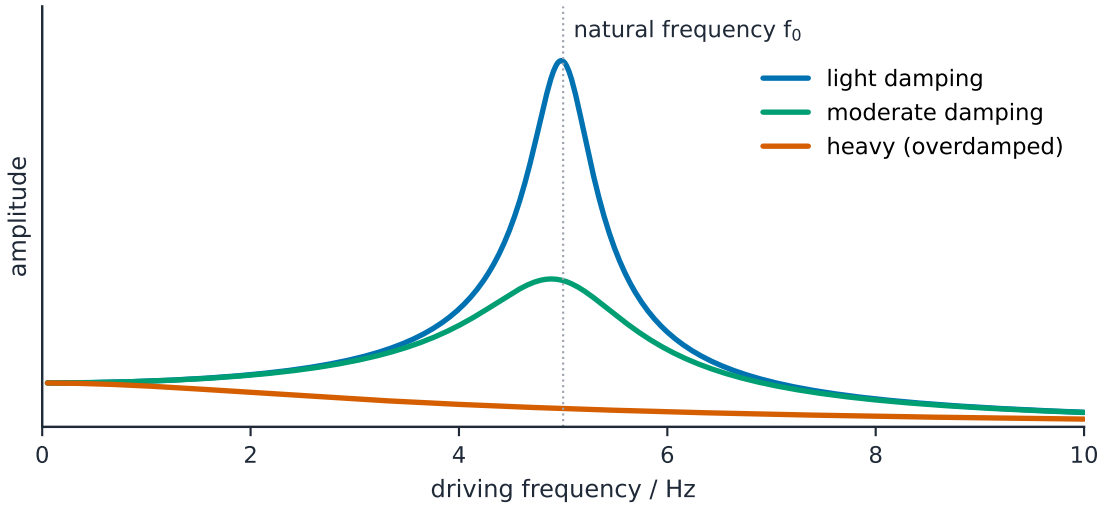
The amplitude of a forced oscillation peaks at resonance, when the driving frequency equals the natural frequency

When a periodic force keeps pushing an oscillator, the oscillator is **forced** to oscillate at the frequency of the push, the **driving frequency** 驱动频率. The amplitude depends on how close the driving frequency is to the natural frequency.

Resonance 共振 is an enhanced response to periodic driving near a natural frequency. For light damping the displacement-amplitude maximum is very close to the undamped natural frequency. With damping its precise position can be lower; in the linear viscous-damping model, maximum average absorbed power is at the undamped natural frequency. Pushing a swing once per swing, at the right moment, makes it go higher and higher; pushing at any other rhythm sometimes helps and sometimes hinders.

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The resonance curve



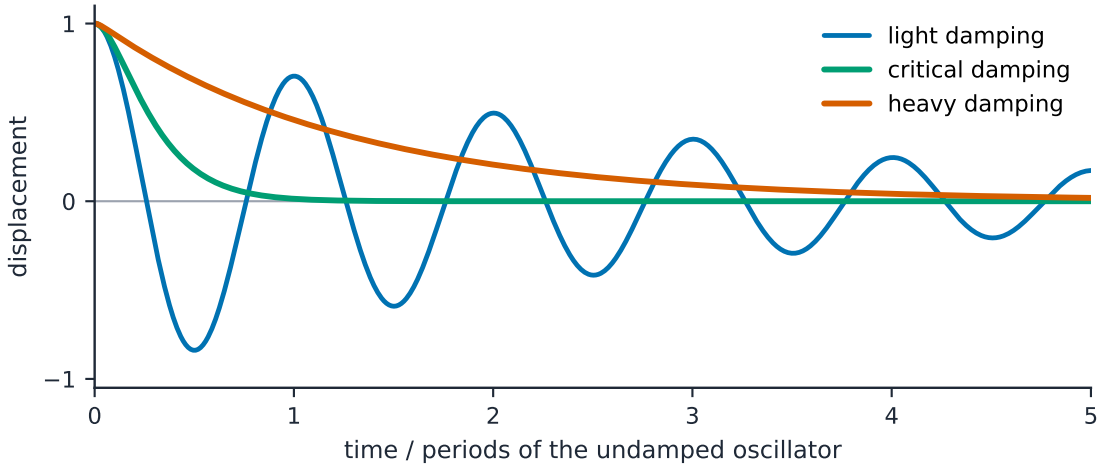
Light and moderate damping have peaks below the natural frequency; the overdamped case has no resonance peak.

- At very low driving frequency the oscillator follows the force almost statically. Its amplitude is finite, about force amplitude divided by stiffness in the spring model.
- With light damping, the displacement peak lies close to the natural frequency; increasing damping shifts it lower.
- At high driving frequencies the oscillator cannot keep up, and the amplitude falls towards zero.

Damping

Damping 阻尼 is the loss of energy from an oscillator, usually by friction or air resistance, which makes the amplitude of free oscillations decrease.

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Three degrees of damping after the same release

An oscillator can have **light damping** 欠阻尼, **critical damping** 临界阻尼 or **heavy damping** 过阻尼:

Degree of damping	Behaviour after release	Example
light	oscillates many times; the amplitude decreases gradually	a pendulum in air, a plucked guitar string
critical	fastest non-oscillatory return for the same ideal oscillator released from rest	a model suspension or door closer
heavy	returns to equilibrium **slowly**, without oscillating	a pendulum in thick oil

■ Damping and resonance

Increasing the damping of a driven oscillator:

- **reduces** the maximum amplitude at resonance;
- makes the resonance peak **broader**, so the oscillator responds over a wider range of frequencies;
- moves the displacement-amplitude peak to a **lower** frequency; sufficiently strong damping removes a distinct peak altogether.

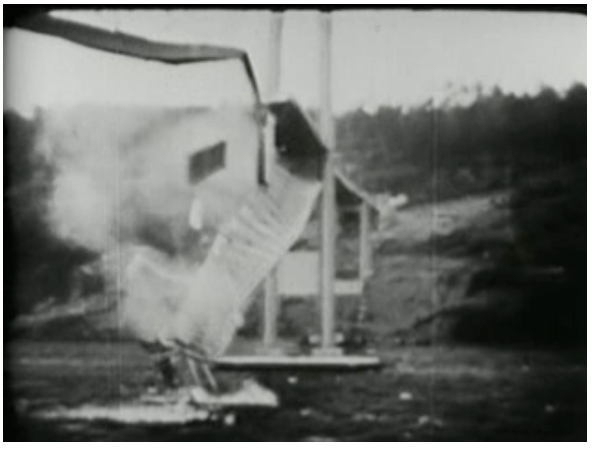
In the lab: release a pendulum through a small angle, keep its path clear and time several complete oscillations. Count each cycle when the bob returns to the same position moving in the same direction, then divide the total time by the number of cycles. Repeat the timing and record how the amplitude decreases, rather than assuming undamped motion. For ideal local SHM, speed is greatest at equilibrium and acceleration magnitude is greatest at the extremes.

Uses and dangers of resonance

Resonance is useful when we want large oscillations at one particular frequency:

- **Musical instruments**: the body of a guitar or the air in a flute resonates to make the sound loud.
- **Tuning a radio**: the circuit is adjusted until its natural frequency matches one station, so only that station is received strongly.
- **Clocks**: many quartz watches use a 32 768 Hz crystal mode as a stable timing reference; quartz resonators do not all share this frequency.
- **MRI scanners** use radio-frequency excitation near the nuclear-spin resonance frequency in a magnetic field. Microwave-oven heating is not a special resonance of water molecules: its alternating electric field drives molecular dipoles and energy is dissipated. See the IAPWS explanation.

Resonance is dangerous when a structure is driven near its natural frequency:



The Tacoma Narrows Bridge collapsed in 1940 through self-excited torsional flutter: the moving deck changed the aerodynamic forces, feeding energy into its twisting motion. This differs from a prescribed periodic force driving simple resonance. See WSDOT's engineering account.

- **Bridges**: London's Millennium Bridge developed lateral sway under crowds in 2000. Pedestrian forces interacted with the moving deck and fed its motion; this was more complex than a fixed external driver at one frequency. Engineers added damping. See the Arup engineers' account.
- **Buildings in earthquakes**: ground motion near a building mode can amplify its response, but damage also depends on strength, duration, damping and the shaking spectrum. A tuned mass damper can reduce selected structural vibrations.
- **Machines and vehicles**: an engine running at a certain speed can make a part of a car vibrate loudly, or a washing machine shake across the floor.

To avoid harmful resonance, engineers change the natural frequency (make the structure stiffer or heavier) or add damping.

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Practice

1. Explain how a standing wave forms on a string fixed at both ends.
2. State three differences between a standing wave and a travelling wave.
3. A string 0.80 m long is fixed at both ends. Waves travel along it at 240 m s^{-1}. Find the wavelength and frequency of the first harmonic, and the frequencies of the next two harmonics.
4. Neighbouring nodes on a vibrating string are 0.25 m apart. The frequency is 400 Hz. Find the wave speed.
5. A pipe 0.25 m long is closed at one end. Find the frequencies of the two lowest harmonics.
6. A pipe open at both ends has a first harmonic of 425 Hz. Find its length.
7. Two neighbouring resonant frequencies of a pipe are 300 Hz and 500 Hz. Show that the pipe must be closed at one end, and find its first harmonic.
8. Points X and Y on a standing wave are on opposite sides of one node. Points Y and Z are in the same loop. State the phase difference between X and Y, and between Y and Z.
9. Define natural frequency and resonance.
10. Describe how the resonance curve of a driven oscillator changes as the damping increases.
11. In an ideal suspension model released from rest, explain the advantage of critical damping over light or heavy damping. State one limit of applying this model to a real car.
12. Give one useful and one harmful example of resonance.
13. A 512 Hz tuning fork gives its first resonance over a tube closed at the bottom when the air column is 0.166 m long. Find the speed of sound, and the length for the next resonance.

Answers.

1. Waves travel along the string and reflect at the fixed ends. The incident and reflected waves have the same frequency, wavelength and amplitude and travel in opposite directions; they superpose to give fixed nodes (including at the ends) and antinodes.
2. An ideal standing wave has zero net energy transfer along the medium, amplitude varying between nodes and antinodes, and in-phase motion within each loop. An ideal undamped plane travelling wave has uniform amplitude and changing phase along its direction of travel.
3. $\lambda_1 = 2L = 1.6$ m; $f_1 = 240/1.6 = 150$ Hz; then 300 Hz and 450 Hz.
4. $\lambda = 2 \times 0.25 = 0.50$ m; $v = 400 \times 0.50 = 200$ m s^{-1}.
5. $f_1 = 340/(4 \times 0.25) = 340$ Hz; next harmonic $n = 3$: 1020 Hz.
6. $L = \frac{v}{2f_1} = \frac{340}{2\times425} = 0.40$ m.

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7. For an open (or closed–closed) pipe, neighbouring harmonics differ by f_1, so $f_1 = 200$ Hz, but 300 Hz is not a whole multiple of 200 Hz. For a closed–open pipe, neighbouring harmonics differ by $2f_1$: $f_1 = 100$ Hz, and 300 Hz and 500 Hz are the 3rd and 5th harmonics.
8. X and Y: π rad (antiphase). Y and Z: zero (in phase).
9. The natural frequency is the frequency at which a system oscillates freely after being displaced. Resonance is an enhanced driven response near a natural frequency; damping limits the response and can shift the displacement-amplitude peak.
10. At fixed driving-force amplitude the displacement peak lowers, broadens and shifts lower; sufficiently strong damping removes a distinct peak.
11. Critical damping gives the fastest non-oscillatory return for this released ideal model. Light damping permits bouncing; heavy damping returns more slowly. A real suspension has several coupled modes and must also maintain tyre contact, so its optimum is not universally exact critical damping.
12. Useful: a radio tuned to one station, a musical instrument body, a quartz watch. Harmful: a footbridge swaying under walkers, buildings in an earthquake, a vibrating car part.
13. $\lambda = 4 \times 0.166 = 0.664$ m; $v = 512 \times 0.664 = 340$ m s^{-1}. Next resonance at $3\lambda/4 = 0.498$ m.

Exam tips

- Neighbouring nodes are $\lambda/2$ apart; a node and the next antinode are $\lambda/4$ apart.
- Draw the boundary first: fixed end or closed pipe end is a node, free end or open pipe end is an antinode. Then fit the pattern.
- A closed–open pipe has odd harmonics only; call the lowest mode the **first harmonic**.
- In a standing wave, points in the same loop are in phase; points in neighbouring loops are in antiphase.
- Resonance: enhanced driven response **near** a natural frequency. Distinguish a displacement-amplitude peak from maximum absorbed power.
- At fixed force amplitude, damping lowers, broadens and shifts the displacement peak lower; strong damping can remove the peak.

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Doppler effect

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches C.5 Doppler effect in full, for the course first assessed in 2025, but HL students must also study the sections marked AHL: the equations for a moving source and a moving observer of sound and other mechanical waves.

Prerequisites: wavefronts from C.3; $v = f\lambda$, sound and electromagnetic waves from C.2; line spectra are met again in E.1. Use $c = 3.00 \times 10^8$ m s^{-1} and 340 m s^{-1} for the speed of sound in air.

The Doppler effect

Why does an ambulance siren change its note as it passes?

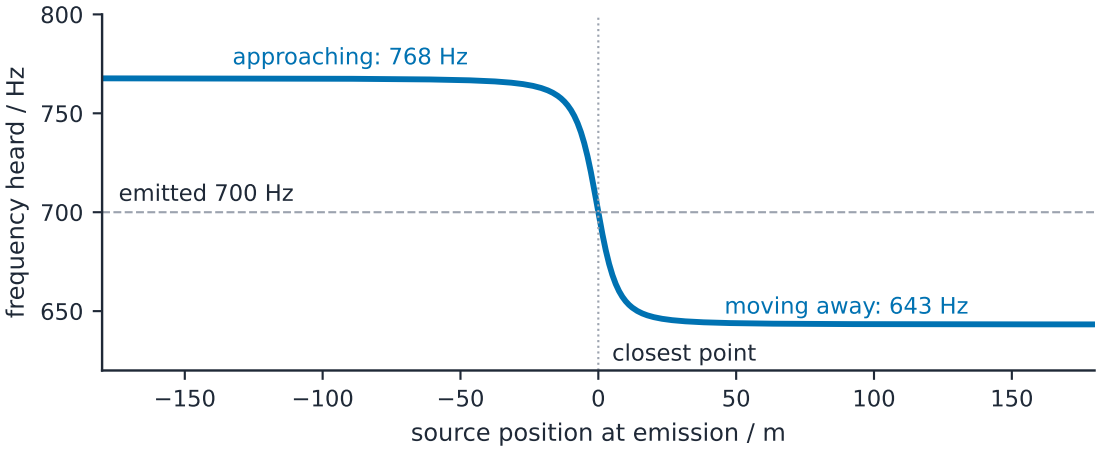
Recall a recording of a passing ambulance; do not stand near moving traffic for an experiment. While it comes towards you the siren sounds higher than it really is; near passage, the note falls rapidly, and it stays low as the ambulance drives away. The driver hears a steady note the whole time. Astronomers use the same effect with light to measure how fast stars and galaxies are moving, and doctors use it with ultrasound to measure how fast blood flows.

What the effect is

The **Doppler effect** 多普勒效应 is a change in observed frequency associated with source–observer motion. For mechanical waves, specify motion relative to the medium as well as the observer. A moving observer can change measured frequency without changing the wavelength in that medium.

- When source and observer move **towards** each other, the observed frequency is **higher**.
- When they move **apart**, the observed frequency is **lower**.
- The first-order shift depends on velocity **along the line** of sight. The light approximation here neglects relativistic transverse shifts; it does not assert that perpendicular motion can never produce a shift.

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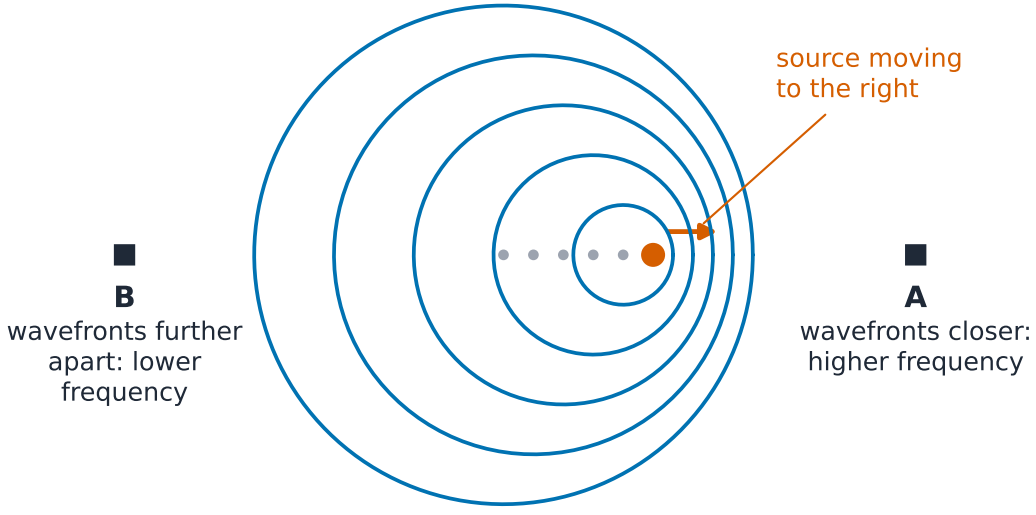


Model received frequency indexed by emission position; zero marks the closest point on the road, not reception time

The effect happens for all waves: sound, water waves and electromagnetic waves.

Wavefront diagrams

A moving source



A moving source squeezes the wavefronts ahead of it and stretches them behind it

Each wavefront spreads out at the wave speed from the point where the source was when it emitted it. Because the source moves between emissions, the circles are not concentric:

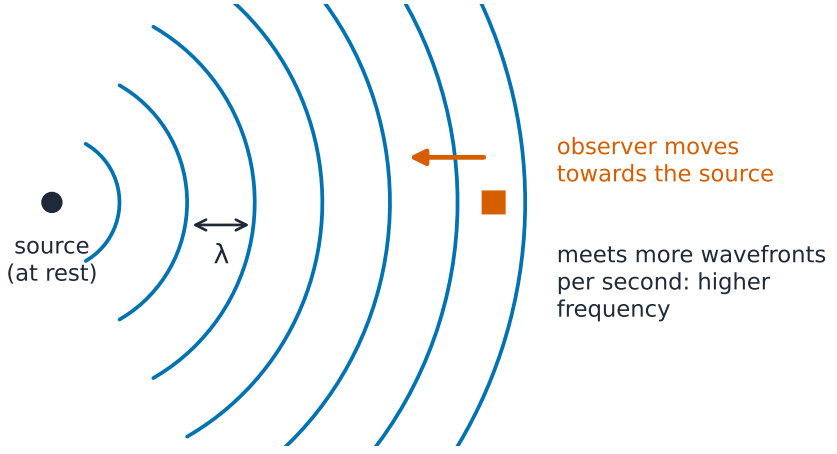
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- **Ahead** of the source (observer A) the wavefronts are closer together. The wavelength is shorter, the waves still travel at the same speed, so more waves arrive each second: the observed frequency is higher.
- **Behind** the source (observer B) the wavefronts are further apart: longer wavelength, lower frequency.

The speed of the waves does not change: it is set by the medium, not by the source.

A moving observer

When the source is at rest and the observer moves, the wavefronts are evenly spaced circles and the wavelength is unchanged. An observer moving **towards** the source meets the wavefronts more often than a stationary observer, so the observed frequency is higher. Moving away, they meet them less often and the frequency is lower.



A moving observer does not change the wavelength, but meets the wavefronts more often

The Doppler effect for light

Light and other electromagnetic waves show the Doppler effect too. When the relative speed v of source and observer is much smaller than the speed of light, the fractional changes in frequency and wavelength are:

$$\frac{|\Delta f|}{f} \approx \frac{|\Delta \lambda|}{\lambda} \approx \frac{|v_r|}{c}$$

Here changes are measured relative to emitted/rest values, and v_r is radial relative velocity. Frequency and wavelength changes have opposite signs because $c = f\lambda$; their fractional magnitudes are equal only approximately at low speed. Signed redshift is $z = (\lambda_{\text{obs}} - \lambda_0)/\lambda_0$, with $v_r \approx cz$ for low-speed Doppler motion, positive for recession.

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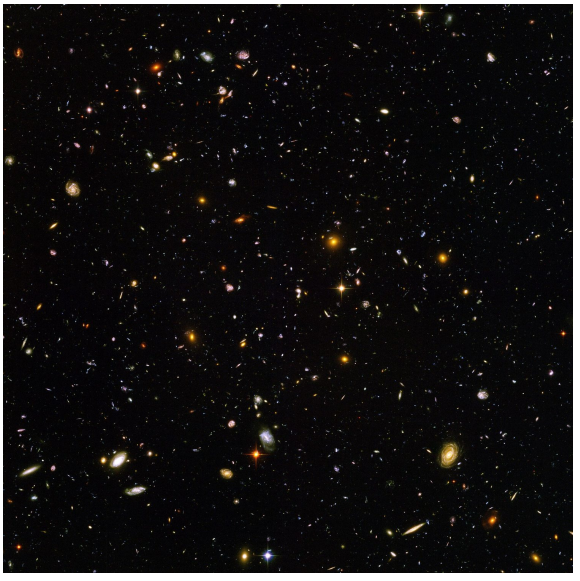
- A source moving **away** has a longer observed wavelength. In visible light this moves spectral lines towards the red end, so it is called **redshift** 红移.
- A source moving **towards** the observer has a shorter observed wavelength: **blueshift** 蓝移.

■ Worked examples

- A line emitted at 500 nm is observed at 501 nm. $\Delta\lambda/\lambda = 1/500 = 0.002$, so $v = 0.002 \times 3.00 \times 10^8 = 6.0 \times 10^5$ m s^{-1}, moving away.
- A satellite moving away at 7.5 km s^{-1} transmits at 100 MHz. $\Delta f = 100 \times 10^6 \times 7.5 \times 10^3 / 3.00 \times 10^8 = 2500$ Hz, so the received frequency is 2.5 kHz lower.
- Police radar sends microwaves at a car and measures the frequency shift of the reflection to find the car's speed.

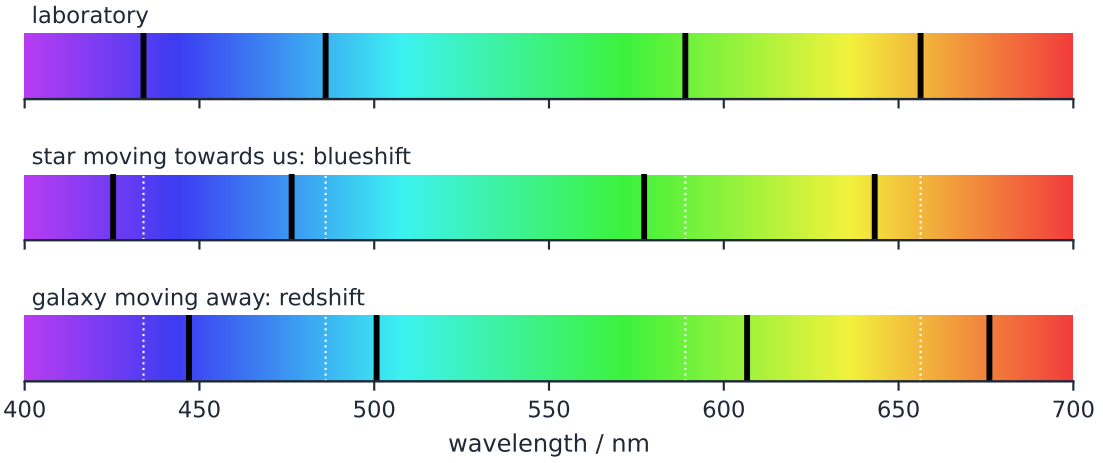
Spectral lines and moving stars and galaxies

Every element absorbs and emits light at its own set of wavelengths, which appear as **spectral lines** 谱线. Measured in a laboratory, hydrogen has a red line at 656.3 nm. Identify several lines by their pattern and compare their fractional shifts. In a low-speed Doppler interpretation, a consistent shift estimates radial motion; it does not measure the transverse velocity or exclude other causes of redshift.



The Hubble Ultra Deep Field: almost every dot is a galaxy, and the light of the faintest is redshifted the most

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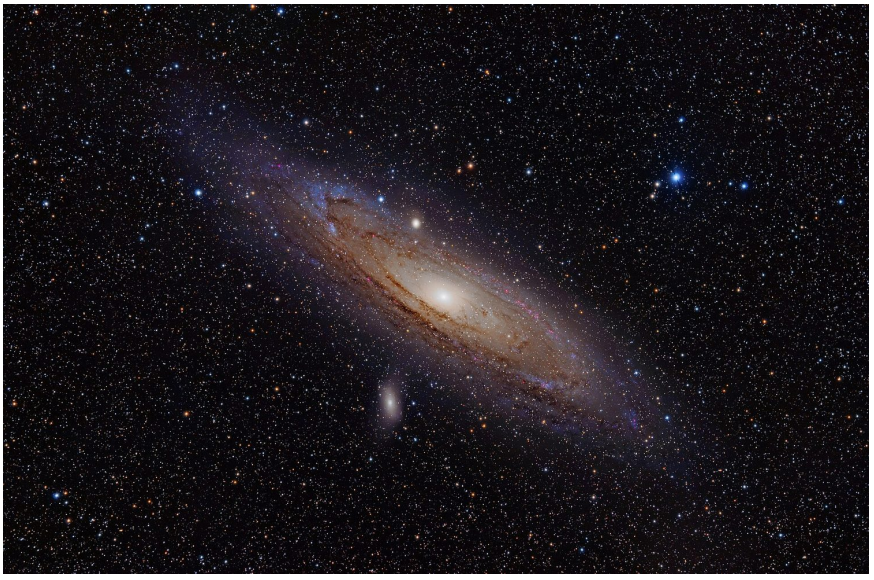


Invented line-pattern comparison with exaggerated fractional shifts; the white dotted marks show the rest positions

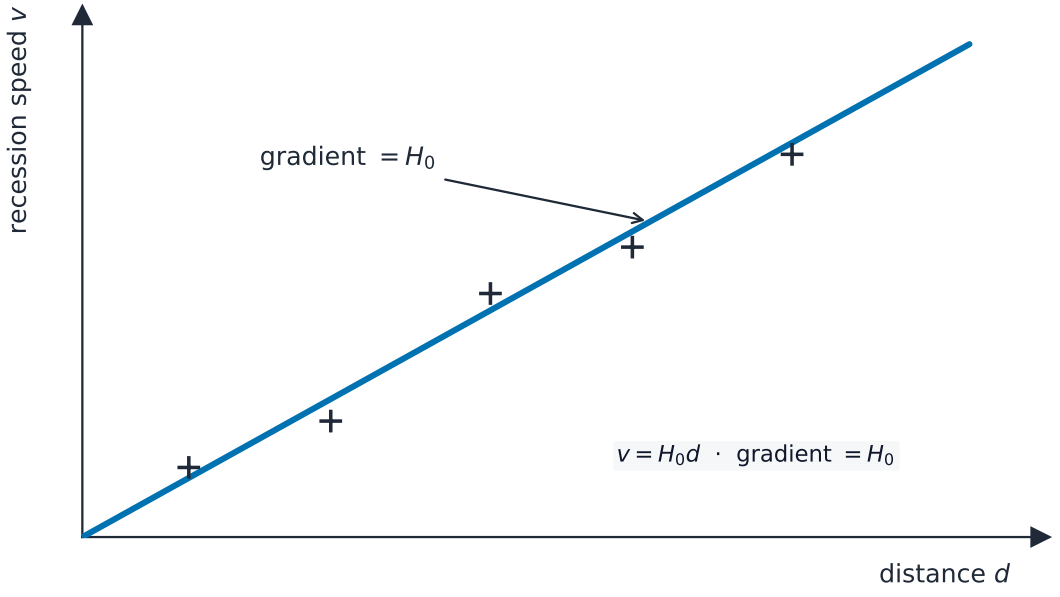
Worked example: the hydrogen line at 656.3 nm is observed at 659.0 nm in a star's spectrum. $\Delta\lambda = 2.7$ nm, so $v = c\,\Delta\lambda/\lambda = 3.00 \times 10^8 \times 2.7/656.3 = 1.2 \times 10^6$ m s^{-1}, moving away from us.

What spectral shifts tell astronomers:

- **Stars in our galaxy** show small redshifts or blueshifts as they move towards or away from the Sun.
- **Rotation**: opposite sides can have opposite radial rotation components relative to the system's mean shift. Inclination matters: a face-on rotating disc gives little radial rotational signal. An unresolved rotating star may show broadened lines rather than separately imaged edges.
- **Binary stars** orbit each other, so their lines shift back and forth regularly. Tiny regular shifts in a star's lines also reveal planets pulling it to and fro.
- **Distant galaxies** show an overall redshift–distance trend associated with cosmic expansion, with local motions adding scatter. Cosmological redshift describes light stretched as space expands, not an exact low-speed source-motion measurement. See NASA's spectroscopy explanation.



The Andromeda galaxy is approaching the Milky Way: its spectrum is blueshifted, against the general trend

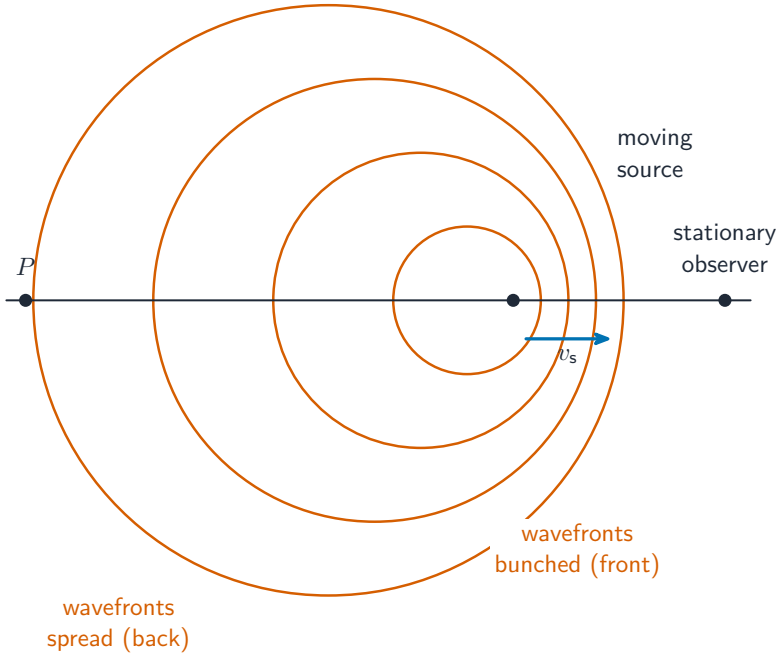


Hubble's law: a galaxy's recession speed is proportional to its distance, $v = H_0 d$

AHL: A moving source

For a subsonic source moving with speed u_s directly towards or away from an observer stationary in a uniform medium, whose wave speed is v:

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A moving source squashes the wavefronts ahead of it, raising the observed frequency

$$f' = f\,\frac{v}{v \pm u_s}$$

where f is the emitted frequency and f' the observed frequency. Use **minus** when the source moves **towards** the observer (smaller denominator, higher frequency) and **plus** when it moves **away**.

Worked example: an ambulance siren emits 700 Hz and the ambulance moves at 30 m s^{-1}.

- Approaching: $f' = 700 \times \dfrac{340}{340 - 30} = 768$ Hz.
- Moving away: $f' = 700 \times \dfrac{340}{340 + 30} = 643$ Hz.

The shift is not symmetrical: the rise when approaching (68 Hz) is larger than the fall when moving away (57 Hz).

Worked example, finding the speed: a train horn of 400 Hz is heard at 420 Hz as the train approaches. $420 = 400 \times \dfrac{340}{340 - u_s}$, so $340 - u_s = 340 \times 400/420 = 323.8$ and $u_s = 16$ m s^{-1}.

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AHL: A moving observer

For an observer moving with speed u_o directly towards or away from a stationary source:

$$f' = f\,\frac{v \pm u_o}{v}$$

Use **plus** when the observer moves **towards** the source and **minus** when moving **away**.

Worked example: a cyclist rides at 20 m s^{-1} past a stationary 500 Hz alarm.

- Towards it: $f' = 500 \times \dfrac{340 + 20}{340} = 529$ Hz.
- Away from it: $f' = 500 \times \dfrac{340 - 20}{340} = 471$ Hz.

For a moving observer the shift is symmetrical, and the wavelength in the medium does not change; only the rate at which the observer meets the wavefronts does.

To choose the sign, ask whether the frequency should go up or down, then pick the sign that does that. The worked equations here treat one moving body at a time in a stationary uniform medium; this is a scope choice, not a physical prohibition on both moving.

Practice

1. State what is meant by the Doppler effect.
2. Draw a wavefront diagram for a source moving to the right. Mark where an observer would hear the highest frequency.
3. Explain, using wavefronts, why an observer moving towards a stationary source hears a higher frequency, although the wavelength does not change.
4. Explain why the note of a passing car's horn drops suddenly as it passes.
5. In an invented low-speed Doppler model, a 486.1 nm laboratory line is observed at 490.0 nm. Estimate radial velocity and state approach or recession.
6. A star moves towards the Earth at 30 km s^{-1}. Find the observed wavelength of a line emitted at 589.0 nm.
7. Explain how Doppler shifts can be used to measure the rotation of a galaxy.
8. State why $\Delta f/f \approx v/c$ can be used for light from stars in our galaxy, and state when it would not be accurate.
9. Invented rest lines at 500.0 and 600.0 nm appear at 500.5 and 600.6 nm. Test whether they have a common fractional shift and estimate radial velocity in the low-speed Doppler model.
10. An invented 500.0 nm rest line has measured shift $+0.20 \pm 0.02$ nm. Treat the rest value and c as exact. Estimate radial velocity and its uncertainty.

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11. A single observed line near 600 nm could be identified with different laboratory lines. Explain why this alone cannot establish a unique Doppler speed.
12. **HL.** An ambulance siren emits 700 Hz. Find the frequency heard by a stationary observer when the ambulance moves at 30 m s^{-1} (a) towards them, (b) away from them.
13. **HL.** An observer moves at 20 m s^{-1} towards, then away from, a stationary 500 Hz source. Find the two observed frequencies.
14. **HL.** A stationary source emits 500 Hz. A moving observer hears 450 Hz. Find the observer's speed and direction.

Answers.

1. A change in observed frequency associated with source–observer motion. For sound, specify the medium frame; a moving observer need not alter the wavelength in the medium.
2. Circles centred on the successive positions of the source, closer together ahead of it; the highest frequency is heard directly ahead.
3. The wavefronts are equally spaced, so the wavelength is unchanged, but the observer's motion means they meet more wavefronts per second, so the frequency they measure is higher.
4. While approaching, the car is moving towards the listener and the frequency heard is higher; as it passes, its velocity along the line to the listener changes quickly from towards to away, so the frequency drops to below the emitted value.
5. $\Delta\lambda = 3.9$ nm; $v = 3.00 \times 10^8 \times 3.9/486.1 = 2.4 \times 10^6$ m s^{-1}, receding (redshift).
6. $\Delta\lambda = 589.0 \times 3.0 \times 10^4/3.00 \times 10^8 = 0.059$ nm; observed 588.94 nm (blueshift).
7. Compare opposite-side radial shifts relative to the galaxy's mean shift. They constrain projected rotation speed; inclination is needed to infer the full rotation speed. A face-on disc has little radial rotational signal.
8. These stars move at tens or hundreds of km s^{-1}, much less than c. For sources moving at a large fraction of c the approximation is not accurate.
9. Both give $z = 0.0010$, consistent with a common shift; $v_r \approx 3.0 \times 10^5$ m s^{-1}, receding. Line identification and calibration must still be checked.
10. $v_r \approx 3.00 \times 10^8 \times 0.20/500.0 = 1.20 \times 10^5$ m s^{-1}; 10% shift uncertainty gives $\pm 1.2 \times 10^4$ m s^{-1}.
11. The rest wavelength sets both shift and fractional denominator. Different identifications give different speeds or directions; use more lines and a calibrated wavelength scale.
12. **HL.** (a) $700 \times 340/310 = 768$ Hz. (b) $700 \times 340/370 = 643$ Hz.
13. **HL.** Towards: $500 \times 360/340 = 529$ Hz. Away: $500 \times 320/340 = 471$ Hz.
14. **HL.** The frequency is lower, so the observer moves away. $450 = 500(340 - u_o)/340$ gives $u_o = 34$ m s^{-1}.

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Exam tips

- The Doppler effect changes the **observed** frequency; the source still emits the same frequency.
- For sound, a moving source changes the wavelength; a moving observer does not.
- In the Doppler interpretation, redshift indicates recession and blueshift approach; identify the pattern and compare fractional shifts. Cosmological redshift needs its expansion model.
- In $z \approx v_r/c$, use signed radial velocity and check the low-speed regime. A zero first-order radial shift does not prove the source is stationary.

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Gravitational fields

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

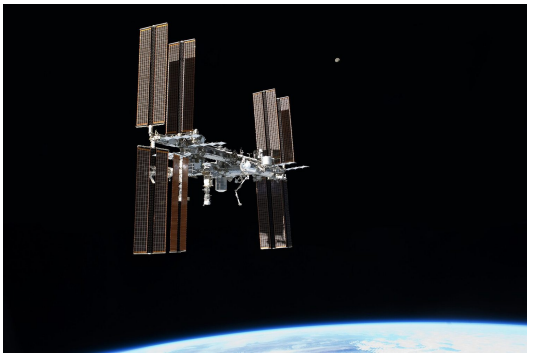
This handout teaches D.1 Gravitational fields in full, for the course first assessed in 2025, but HL students must also study the sections marked AHL: gravitational potential energy and potential, potential gradient, equipotential surfaces, orbital and escape speed, the energy of orbiting satellites and the effect of atmospheric drag.

Prerequisites: circular motion and centripetal force from A.2; work and energy from A.3. Use $G = 6.67\times10^{-11}$ N m^2 kg^{-2}, the Earth's mass 5.97×10^{24} kg and radius 6.37×10^{6} m. Numerical orbit models are circular unless an ellipse is explicitly discussed. Treat satellite mass as negligible beside the central mass and neglect losses unless stated.

Gravitation and orbits

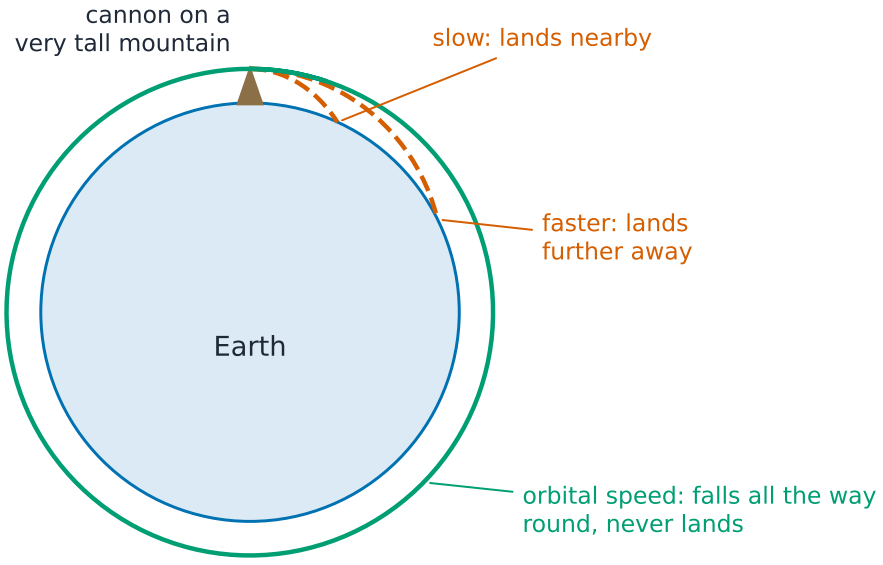
■ Why doesn't the International Space Station fall down?

Use an illustrative space-station orbit about 400 km above a spherical Earth. At that height the Earth's gravity is still 89% as strong as at the surface, so gravity has not "switched off". The station *is* falling, all the time. But it also moves sideways at about 7.7 km every second. As it falls, the curved surface of the Earth drops away beneath it just as fast, so it never gets any closer to the ground. Isaac Newton imagined the same thing with a cannon on a very tall mountain.



The International Space Station orbits Earth, held in its path by gravity.

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Newton's ideal cannon: suitable sideways speed gives an orbit if atmosphere, obstacles and other perturbations are neglected

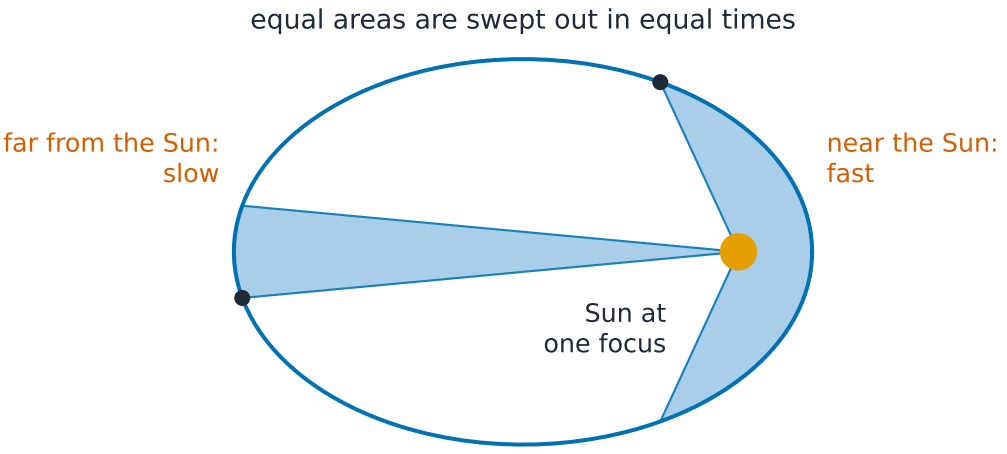
The same force that makes an apple fall keeps the Moon in orbit and the planets moving round the Sun. This unit describes that force, the field it comes from, and the orbits it produces.

Kepler's laws of orbital motion

Johannes Kepler studied careful measurements of the planets' positions and found three rules, now called **Kepler's laws** 开普勒定律:

1. **First law**: each planet moves in an **ellipse** 椭圆 with the Sun at one **focus** 焦点.
2. **Second law**: the line from the Sun to a planet sweeps out equal areas in equal times. So a planet moves fastest when it is closest to the Sun and slowest when it is furthest away.
3. **Third law**: the square of the orbital period T is proportional to the cube of the semi-major axis a of the ellipse: $T^2 \propto a^3$ (for a circle, $a = r$).

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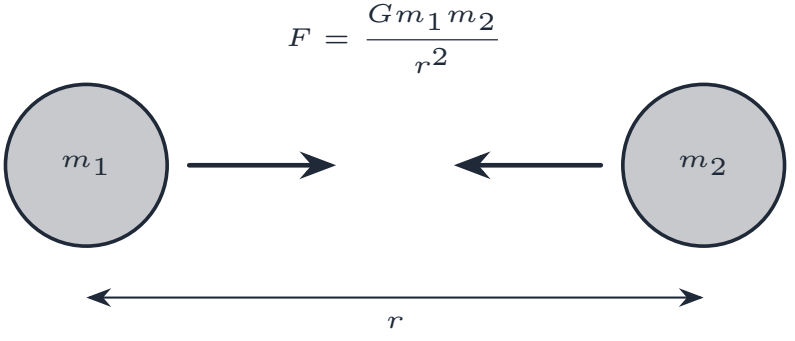
Kepler's second law: equal areas in equal times, so faster near the Sun

Circular approximations are useful for many planetary calculations; ellipticity varies between planets and Kepler's third law uses the semi-major axis, not an arbitrary instantaneous distance. Kepler's laws describe the motion; Newton's law of gravitation, below, explains it.

Worked example (third law): Mars takes 1.88 years to orbit the Sun. The Earth takes 1.00 year at 1.00 AU (the Earth–Sun distance). $r^3 \propto T^2$, so $r_{\text{Mars}} = 1.88^{2/3} = 1.52$ AU.

Newton's universal law of gravitation

Every mass attracts every other mass. For two **point masses** 质点 m_1 and m_2 a distance r apart, **Newton's universal law of gravitation** 万有引力定律 gives the size of the force:



Two masses attract along the line joining them

$$F = \frac{Gm_1m_2}{r^2}$$

- The force on each mass is equal in size and opposite in direction (Newton's third law), and it is always attractive.
- It is an **inverse-square law**: double the distance and the force falls to one-quarter.
- G is the gravitational constant. It is so small that gravity between everyday objects is tiny; ordinary gravitational forces between small objects are usually hard to notice without sensitive apparatus.

Worked examples using the stated spherical-body model:

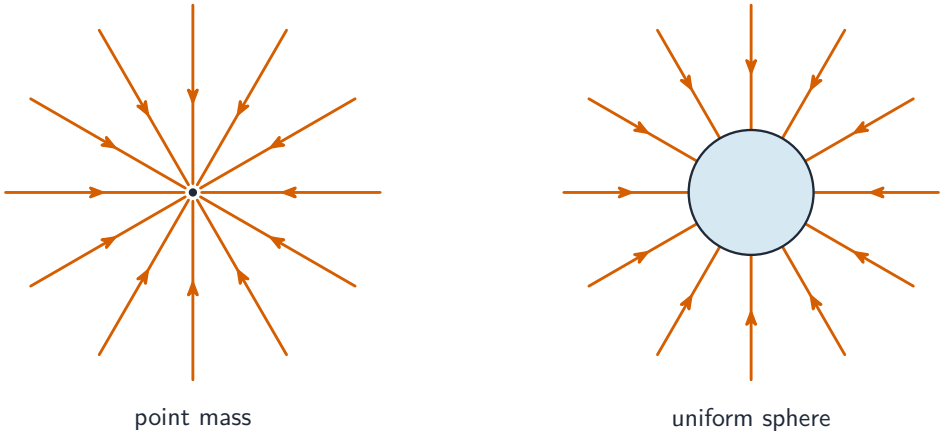
- Two lead spheres of 5.0 kg each have centres 0.10 m apart: $F = 6.67 \times 10^{-11} \times 5.0 \times 5.0/0.10^2 = 1.7 \times 10^{-7}$ N.
- The Earth and the Moon (7.35×10^{22} kg, 3.84×10^8 m apart): $F = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22}/(3.84 \times 10^8)^2 = 2.0 \times 10^{20}$ N.

When can a large body be treated as a point mass?

The law is written for point masses, but we can use it for real bodies when:

- the body is a **sphere of uniform density** (or made of uniform spherical shells). Outside it, it acts as if all its mass were at its centre;
- the bodies are very far apart compared with their sizes, as for the Sun and the planets.

For point masses or these spherical-body approximations, measure r **between the centres**; arbitrary nearby extended shapes require a mass-distribution calculation. For a satellite at height h above the Earth, $r = R_E + h$.



Outside a uniform sphere the field is radial, exactly like a point mass at the centre

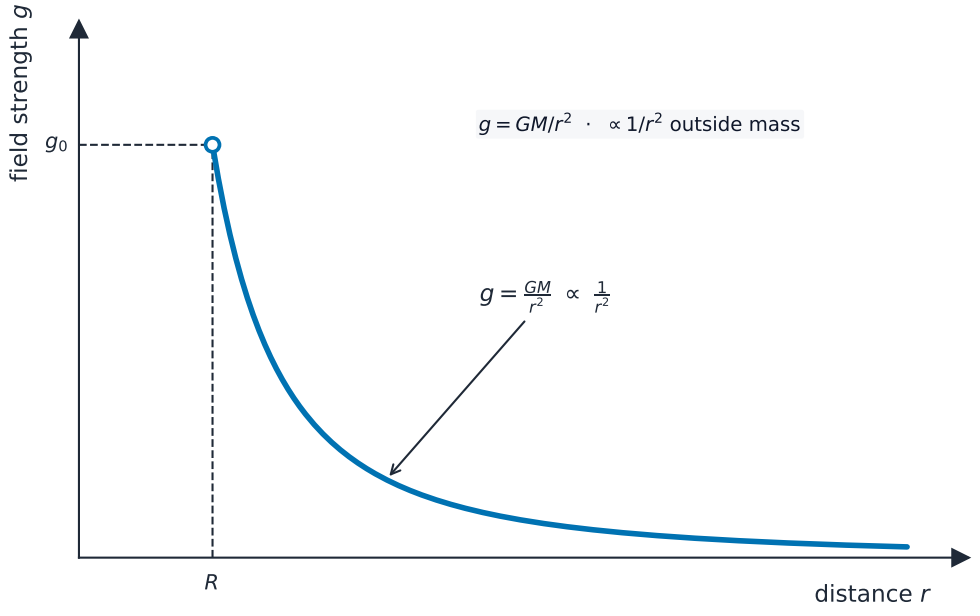
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Gravitational field strength

A mass creates a **gravitational field** 引力场 in the space around it: any other mass placed there feels a force. The **gravitational field strength** 引力场强度 g at a point is the force per unit mass on a small test mass placed there:

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

where M is the mass creating the field. Its unit is N kg^{-1}, which is the same as m s^{-2}. Field strength is a vector: it points towards the mass M.



Field strength falls off as $1/r^2$ with distance from a point mass

Worked examples using the stated spherical-body model:

- At the Earth's surface: $g = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} / (6.37 \times 10^6)^2 = 9.81$ N kg^{-1}.
- At the height of the space station, $r = 6.37 \times 10^6 + 4.0 \times 10^5 = 6.77 \times 10^6$ m: $g = 8.69$ N kg^{-1}, still 89% of the surface value.

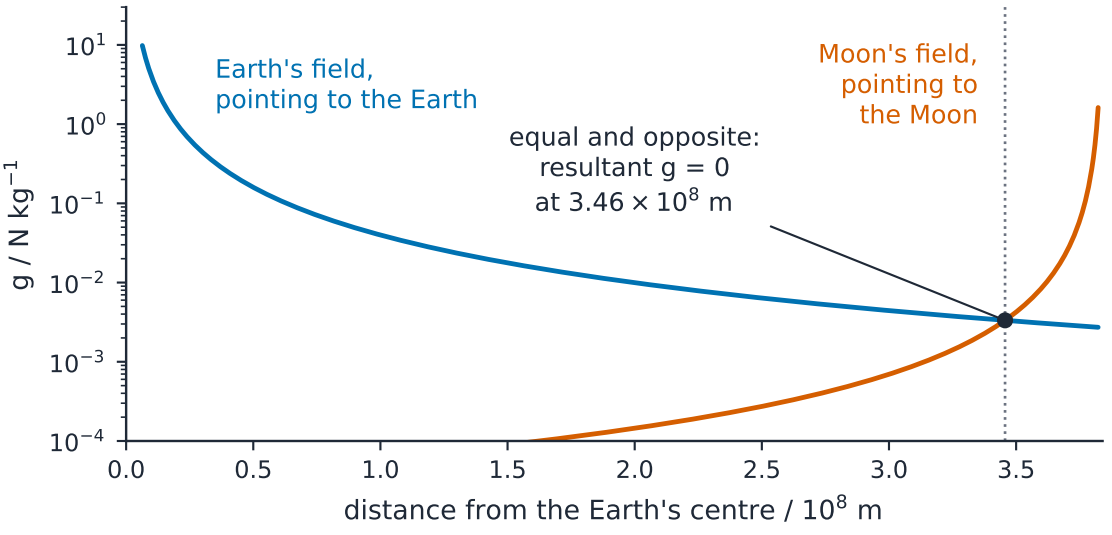
The resultant field of two bodies

Fields are vectors, so where two bodies both pull, add their fields with their directions. Between the Earth and Moon, their fields point in opposite directions towards their respective centres. This is a two-body static field calculation, not the rotating-frame L1 equilibrium calculation. At one point they are equal and opposite and the resultant field is zero:

$$\frac{GM_E}{x^2} = \frac{GM_M}{(d-x)^2} \quad \Rightarrow \quad \frac{x}{d-x} = \sqrt{\frac{M_E}{M_M}} = 9.01$$

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so $x = 0.900\,d = 3.46 \times 10^8$ m from the Earth's centre. The point is much closer to the Moon because the Moon's mass is much smaller.



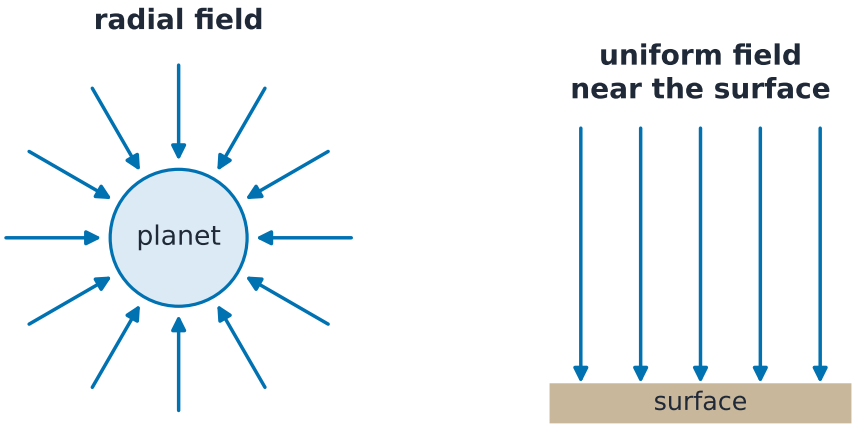
The two fields cancel nine-tenths of the way to the Moon

Gravitational field lines

Field lines 场线 show the direction of the force on a small test mass. Their spacing shows the field strength: closer lines mean a stronger field.

- Around a spherical planet the field is **radial**: the lines point in towards the centre and spread apart further out, as the field gets weaker.
- Close to the surface, over a region small compared with the planet, the lines are almost parallel and equally spaced: a **uniform field** 匀强场, where g is constant.
- Field lines cannot cross where the field has a definite nonzero direction. In this isolated attractive-mass model they point towards masses; their drawn density represents strength, not material strings.

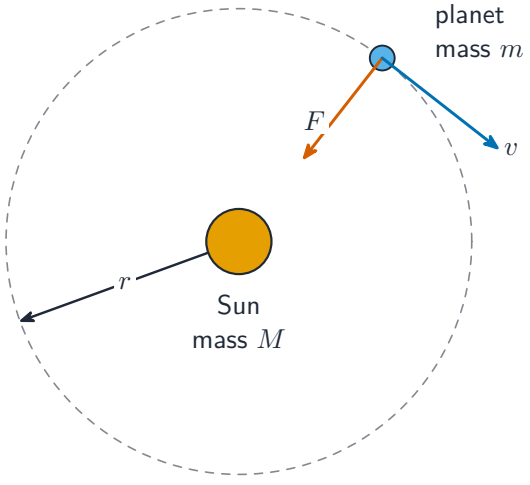
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A radial field far away; an almost uniform field close to the surface

Circular orbits and Kepler's third law

For a satellite in a circular **orbit** 轨道, gravity is the only force acting, and it provides the **centripetal force** 向心力. Using $a = 4\pi^2 r/T^2$ for circular motion:

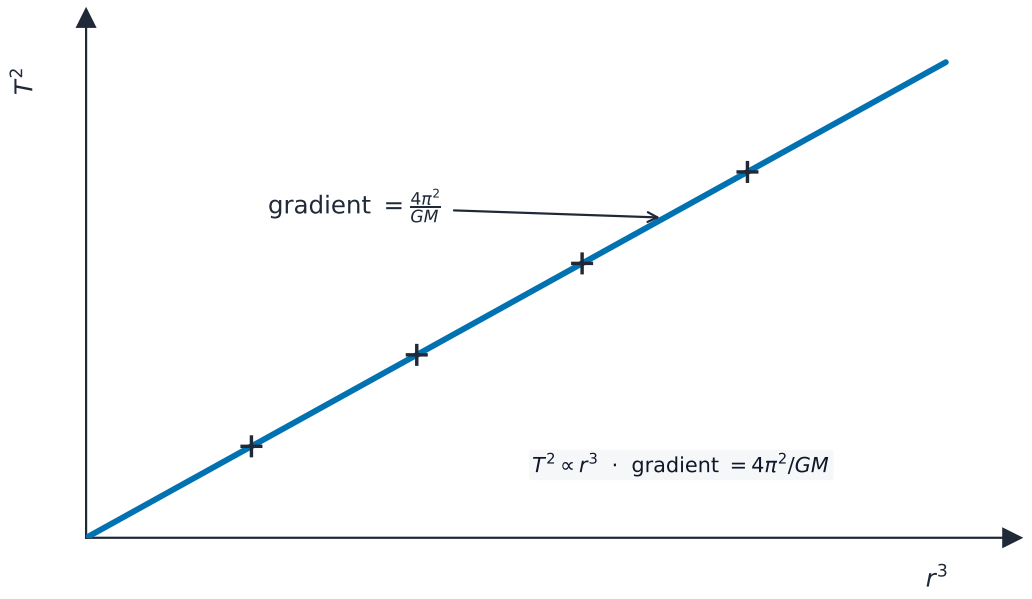


Gravity provides the centripetal force that keeps a planet in a circular orbit

$$\frac{GMm}{r^2} = \frac{4\pi^2 mr}{T^2} \quad \Rightarrow \quad T^2 = \frac{4\pi^2}{GM} r^3$$

This is Kepler's third law, now explained by Newton's law. The constant $4\pi^2/GM$ depends only on the central mass M, so all satellites of the same body share it, and the satellite's own mass cancels.

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Kepler's third law: $T^2 \propto r^3$, a straight line through the origin

Worked examples using the stated spherical-body model:

- **Space station**, $r = 6.77 \times 10^6$ m: $T = 2\pi\sqrt{r^3/GM} = 5.5 \times 10^3$ s, about 92 minutes.
- **Geostationary orbit** 地球静止轨道: a circular equatorial satellite moving eastwards with Earth's rotation has a sidereal period $T = 86164$ s, about 23 h 56 min. Using $r^3 = \frac{GMT^2}{4\pi^2}$ gives $r \approx 4.215 \times 10^7$ m and altitude about 35 800 km for the stated rounded constants. Period alone is insufficient: circularity, plane and direction also matter. See ESA's orbit guide.
- **Mass of the Sun** from the Earth's orbit ($r = 1.50 \times 10^{11}$ m, $T = 3.16 \times 10^7$ s): $M = 4\pi^2 r^3/GT^2 = 2.0 \times 10^{30}$ kg.

Astronauts in orbit feel weightless because they and their spacecraft fall together with the same acceleration, so the floor does not push on them. This is **apparent weightlessness** 表观失重: gravity is still acting.

AHL: Gravitational potential energy

The **gravitational potential energy** 引力势能 E_p of a system is the external work to assemble its parts slowly from infinite separation. For two masses whose centres are r apart:

$$E_p = -\frac{Gm_1m_2}{r}$$

- E_p is zero at infinite separation. Gravity is attractive, so work is done *by* the field as the masses come together, and E_p is negative at any finite distance.

- Moving the masses further apart makes E_p less negative, which is an increase.
- mgh works only for small heights near a surface, where g is almost constant. For large changes in height, use $-Gm_1m_2/r$.

Worked example: a 1000 kg satellite at rest on the Earth's surface has $E_p = -6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1000/6.37 \times 10^6 = -6.25 \times 10^{10}$ J. To take it to infinity and leave it at rest needs at least 6.25×10^{10} J.

AHL: Gravitational potential

The **gravitational potential** 引力势 V_g at a point is the external work per unit mass for bringing a small mass slowly from infinity to that point:

$$V_g = -\frac{GM}{r}$$

Its unit is J kg^{-1}. Potential belongs to a point in the field; potential energy belongs to a mass placed there: $E_p = mV_g$.

The external work for a slow move with no kinetic-energy change is

$$W = m\,\Delta V_g$$

Worked example: at the Earth's surface $V_g = -6.25 \times 10^7$ J kg^{-1}; at the height of the space station $V_g = -5.88 \times 10^7$ J kg^{-1}. Lifting a 1000 kg satellite to that height needs $W = 1000 \times 0.37 \times 10^7 = 3.7 \times 10^9$ J. Using mgh would give 3.9×10^9 J, too much, because g gets smaller with height.

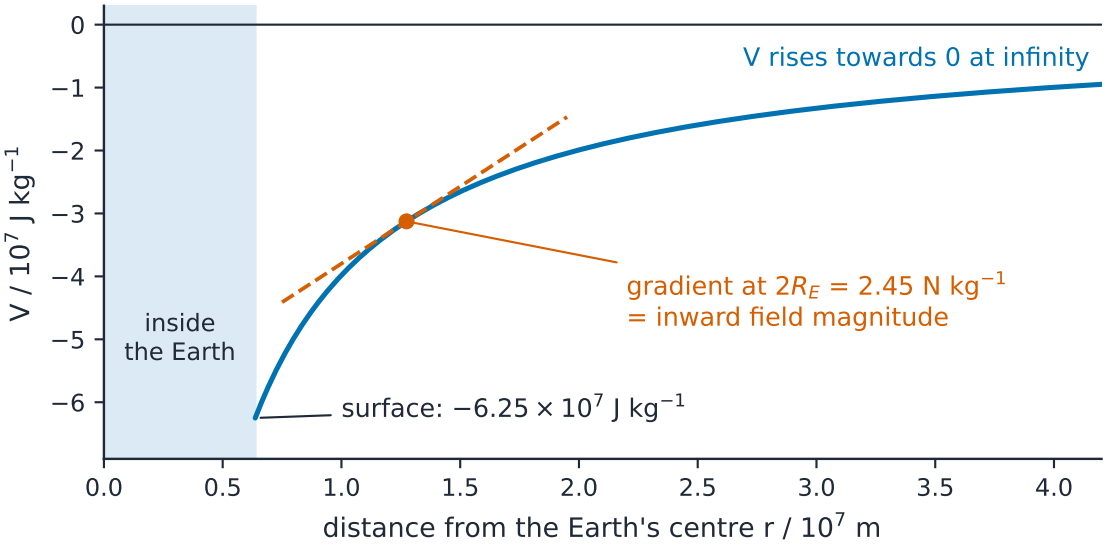
AHL: Field strength as a potential gradient

The field strength is the negative of the **potential gradient** 势梯度:

$$g_r = -\frac{dV_g}{dr} \qquad \overline{g_r} = -\frac{\Delta V_g}{\Delta r}$$

Here outward radial direction is positive. The positive slope of $V_g(r)$ has magnitude GM/r^2; the radial field component g_r is its negative and points inward. A finite difference gives an interval average rather than the exact local derivative. The minus sign means the field points from high potential to low potential, "downhill".

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Positive potential slope gives the field magnitude; its negative gives the inward radial component

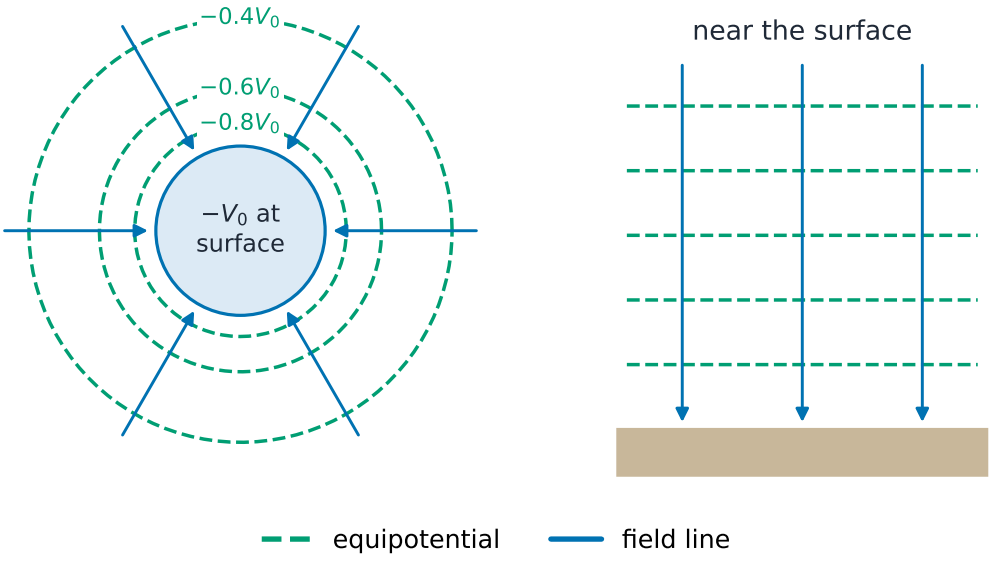
Worked example: 100 km above the surface the potential is -6.155×10^7 J kg^{-1}, compared with -6.251×10^7 J kg^{-1} at the surface. $g = 0.096 \times 10^7 / 1.00 \times 10^5 = 9.6$ N kg^{-1}, the average field over that 100 km.

AHL: Equipotential surfaces

An **equipotential surface** 等势面 joins points with the same potential.

- Around a spherical mass the equipotentials are spheres centred on the mass. Near the surface, over a small region, they are horizontal planes.
- **Field lines cross equipotentials at right angles.**
- Gravity does no work along an equipotential because $\Delta V_g = 0$; an external force could still change kinetic energy.
- Drawn at equal steps of potential, equipotentials are close together where the field is strong and further apart where it is weak. That is how a field is mapped with potential.

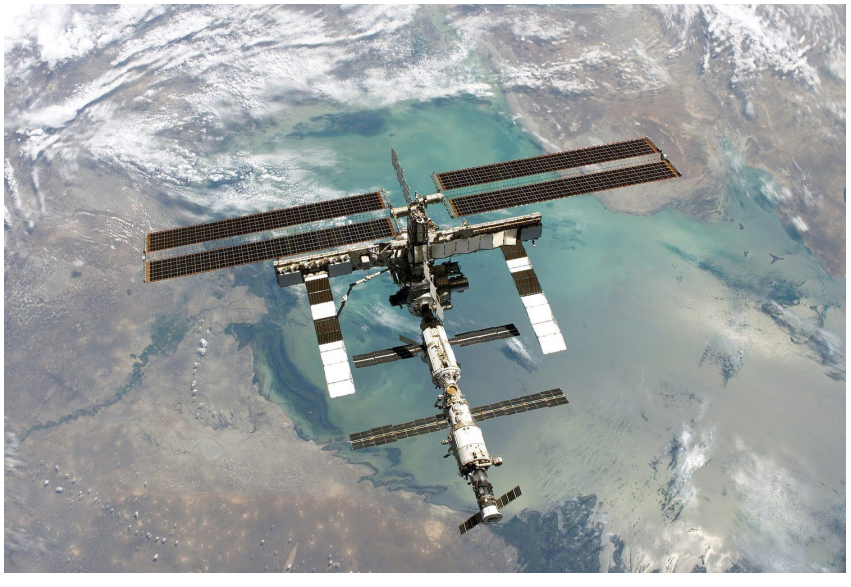
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Field lines meet equipotentials at right angles

AHL: Orbital speed and escape speed

From $GMm/r^2 = mv^2/r$, the **orbital speed** 轨道速度 of a body in a circular orbit of radius r round a large mass M is



The ISS in orbit: orbital speed is the speed that keeps a satellite in free-fall circular motion

$$v_{\text{orbital}} = \sqrt{\frac{GM}{r}}$$

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Closer orbits are faster. The space station, at $r = 6.77 \times 10^6$ m, moves at $v = \sqrt{3.98 \times 10^{14}/6.77 \times 10^6} = 7.67 \times 10^3$ m s^{-1}.

The **escape speed** 逃逸速度 is the smallest speed that lets a body move infinitely far away, with no further push. Its kinetic energy must at least cancel its negative potential energy: $\frac{1}{2}mv^2 - GMm/r = 0$, so

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

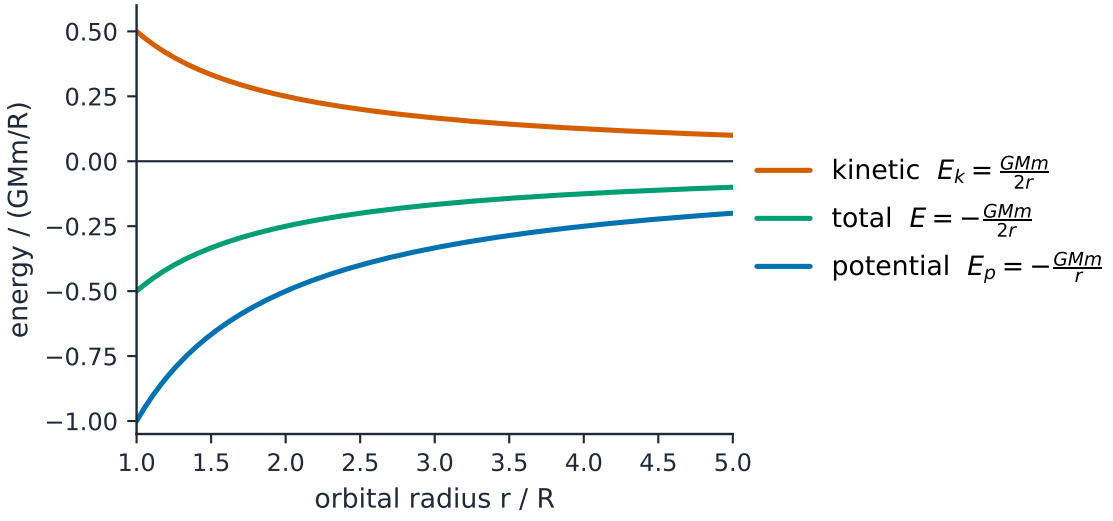
- From the Earth's surface: $v_{\text{esc}} = \sqrt{2 \times 3.98 \times 10^{14}/6.37 \times 10^6} = 1.12 \times 10^4$ m s^{-1} (11.2 km s^{-1}), ignoring air resistance.
- The ideal escape-energy threshold is independent of test mass. A viable path must avoid collision with the central body; ignore atmosphere, rotation and other gravitating bodies here.
- At any orbital radius, $v_{\text{esc}} = \sqrt{2}\, v_{\text{orbital}}$.

AHL: Energy of an orbiting satellite

For a satellite of mass m in a circular orbit of radius r:

$$E_k = \frac{GMm}{2r} \qquad E_p = -\frac{GMm}{r} \qquad E_{\text{total}} = -\frac{GMm}{2r}$$

So $E_k = -E_{\text{total}}$ and $E_p = 2E_{\text{total}}$. The total energy is negative: the satellite is bound to the planet.



A higher orbit has more total energy but less kinetic energy

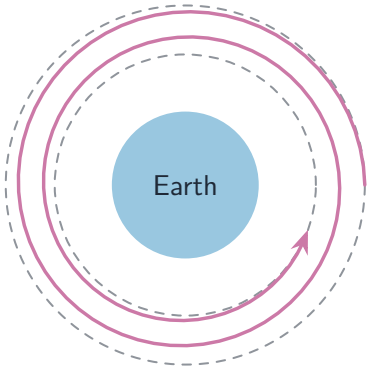
Worked examples, for a 1000 kg satellite ($GMm = 3.98 \times 10^{17}$ J m):

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- **Changing orbit** from $r = 7.0\times10^6$ m to 1.4×10^7 m: E_{total} goes from -2.84×10^{10} J to -1.42×10^{10} J. The satellite's mechanical-energy increase is 1.42×10^{10} J; this is not the total fuel energy of a real transfer. E_p rises by 2.84×10^{10} J, but E_k *falls* by 1.42×10^{10} J: the higher orbit is slower.
- **Going into orbit** from rest on the surface of a non-rotating Earth to $r = 6.77\times10^6$ m: E_{total} goes from -6.25×10^{10} J to -2.94×10^{10} J, so 3.31×10^{10} J is needed. Most of it becomes kinetic energy, not potential energy.
- **Escaping from orbit** at $r = 7.0\times10^6$ m: the total energy must be raised from -2.84×10^{10} J to zero, so at least 2.84×10^{10} J is needed.

AHL: Atmospheric drag on an orbiting satellite

Satellites in low orbits pass through very thin air, which exerts a small **viscous drag** 黏滞阻力 force. In a model of weak drag and slowly evolving, nearly circular orbits, drag removes mechanical energy, so $-GMm/2r$ becomes more negative:



drag removes energy: $E = -\dfrac{GMm}{2r}$ falls
r shrinks $\Rightarrow$ E_p falls by 2Δ
E_k rises by Δ
the other Δ heats the air

Drag makes a satellite speed up

- the orbital radius r decreases, so the satellite slowly spirals lower;
- across this sequence of near-circular states, $E_k = -E_{\text{total}}$ increases and lower-orbit speed is higher. Drag itself opposes motion and initially brakes the satellite; gravity supplies the subsequent kinetic-energy gain during descent;
- the potential energy decreases by twice as much as the kinetic energy increases; the difference is transferred to the air as thermal energy.

This slow-decay model explains the need for orbital reboosts; it is not an exact instantaneous circular-orbit formula for strong atmospheric re-entry. See NASA's orbit-maintenance explanation.

Practice

1. State Kepler's three laws of orbital motion.
2. Calculate the gravitational force between the Earth and the Moon.
3. State two conditions under which a large body can be treated as a point mass in Newton's law of gravitation.
4. Calculate the gravitational field strength 400 km above the Earth's surface.
5. Mars has mass 6.42×10^{23} kg and radius 3.39×10^{6} m. Calculate the gravitational field strength at its surface.
6. Find the distance from the Earth's centre, along the line to the Moon, where the resultant gravitational field is zero.
7. Sketch the gravitational field lines around a planet and close to its surface. Describe the difference.
8. Mars takes 1.88 years to orbit the Sun. Find its orbital radius in AU.
9. Calculate the radius and altitude of a model geostationary orbit, using T = 86164 s, and state the required orbital plane and direction.
10. The Earth orbits the Sun at 1.50×10^{11} m with a period of 3.16×10^{7} s. Calculate the mass of the Sun.
11. An invented small probe has mass 20 kg where Earth's model field strength is 8.0 N/kg. Find its weight. If probe mass doubles at the same point, state what happens to its weight and the field strength.
12. **HL.** A 1000 kg satellite starts at rest on a non-rotating Earth and enters a circular orbit 400 km above the surface. Find initial potential energy, final orbital speed and period, and minimum increase in mechanical energy. Ignore air and propulsion losses.
13. **HL.** A 1000 kg satellite changes circular orbital radius from 7.0×10^{6} m to 1.4×10^{7} m. Find changes in kinetic, potential and total energy. Then explain how slow drag-driven decay reverses those trends, distinguishing immediate braking from later motion.
14. **HL.** For the Moon ($M = 7.35 \times 10^{22}$ kg, $R = 1.74 \times 10^{6}$ m), find surface potential and escape speed. Explain why gravity does no work along an equipotential and give one condition needed for the escape calculation.

Answers.

1. Elliptical orbits have the Sun at one focus; the Sun–planet line sweeps equal areas in equal times; $T^2 \propto a^3$ for semi-major axis a, which equals radius for a circle.
2. $F = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22} / (3.84 \times 10^{8})^2 = 2.0 \times 10^{20}$ N.
3. The body is a sphere of uniform density (the field outside it is then the same as a point mass at its centre), or the distance between the bodies is very large compared with their sizes.
4. $r = 6.77 \times 10^{6}$ m; $g = 3.98 \times 10^{14} / (6.77 \times 10^{6})^2 = 8.7$ N kg^{-1}.

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5. $g = 6.67 \times 10^{-11} \times 6.42 \times 10^{23}/(3.39 \times 10^{6})^2 = 3.7$ N kg^{-1}.
6. $x/(d-x) = \sqrt{5.97 \times 10^{24}/7.35 \times 10^{22}} = 9.01$, so $x = 0.900 \times 3.84 \times 10^{8} = 3.46 \times 10^{8}$ m.
7. Around the planet the lines are radial, pointing to the centre and spreading apart further out (weaker field). Near the surface they are parallel, equally spaced and pointing down (uniform field).
8. $r = 1.88^{2/3} = 1.52$ AU.
9. $r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} \approx 4.215 \times 10^{7}$ m; $h = r - R_E \approx 3.578 \times 10^{7}$ m, about 35 800 km. The orbit is circular, equatorial and prograde.
10. $M = 4\pi^2(1.50 \times 10^{11})^3/(6.67 \times 10^{-11} \times (3.16 \times 10^{7})^2) = 2.0 \times 10^{30}$ kg.
11. $F = mg = 20 \times 8.0 = 160$ N towards Earth. Doubling test mass doubles weight to 320 N; Earth's field remains 8.0 N/kg when the probe's own gravitational effect is negligible.
12. **HL.** $U_i = -GMm/R_E \approx -6.25 \times 10^{10}$ J. $v = \sqrt{GM/r} \approx 7.67 \times 10^{3}$ m/s and $T = 2\pi r/v \approx 5.55 \times 10^{3}$ s (92.4 min), with $r = 6.77 \times 10^{6}$ m. $E_f = -GMm/(2r) \approx -2.94 \times 10^{10}$ J, so $\Delta E \approx +3.31 \times 10^{10}$ J. This is a mechanical-energy minimum, not fuel expenditure.
13. **HL.** $\Delta K \approx -1.42 \times 10^{10}$ J, $\Delta U \approx +2.84 \times 10^{10}$ J and $\Delta E \approx +1.42 \times 10^{10}$ J. During weak-drag near-circular decay, radius and total energy decrease, potential decreases and kinetic energy increases across successive states. Drag itself initially brakes motion; gravity accelerates the subsequent descent.
14. **HL.** $V = -GM/R \approx -2.82 \times 10^{6}$ J/kg; $v_{\text{esc}} = \sqrt{-2V} \approx 2.37 \times 10^{3}$ m/s. Equipotential motion has $\Delta U = m\Delta V = 0$, so gravity does zero work. The ideal escape path must avoid collision and neglect other forces/central rotation.

Exam tips

- Measure r from the **centre** of the planet: add the radius to the height.
- Inverse square: double r and both F and g fall to one-quarter.
- In a circular orbit gravity provides the centripetal force. Never add an extra outward force.
- Orbiting astronauts are weightless only in appearance: gravity still acts on them.

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Electric and magnetic fields

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches D.2 Electric and magnetic fields in full, for the course first assessed in 2025, but HL students must also study the sections marked AHL: electric potential energy, electric potential, potential gradient, work done in electric fields and equipotential surfaces.

Prerequisites: forces and Newton's laws from A.2; work and energy from A.3; current and potential difference from B.5; gravitational fields from D.1, which follow the same patterns. Use $e = 1.60 \times 10^{-19}$ C, $k = 8.99 \times 10^{9}$ N m^2 C^{-2} and $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}.

Electric charge

Why do you sometimes get a shock from a car door?

On a dry day you slide out of a car seat, touch the metal door and feel a sharp snap. Sliding across the seat rubbed electrons from one surface onto the other, so your body gained an electric charge. When your finger came close to the metal, the charge jumped across the gap as a tiny spark. Lightning is the same thing on a huge scale. To understand it we need electric charge, the forces between charges, and the fields that carry those forces.

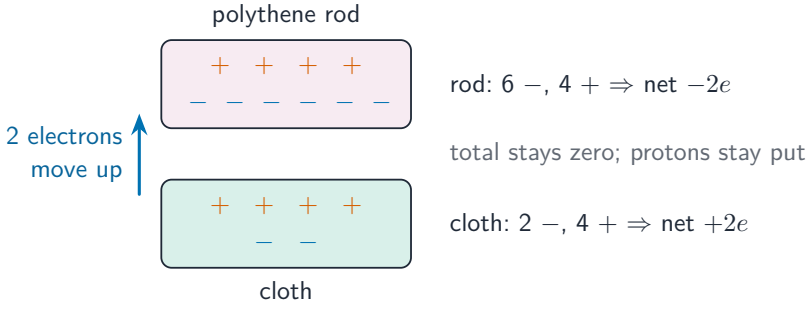


Lightning is a giant spark driven by a huge electric field.

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Two types of charge

Electric charge 电荷 comes in two types, **positive** and **negative**. Protons carry a positive charge $+e$, electrons a negative charge $-e$, where $e = 1.60 \times 10^{-19}$ C is the **elementary charge** 元电荷.



Rubbing moves electrons

- **Like charges repel; unlike charges attract.**
- A neutral object has equal amounts of positive and negative charge. It becomes charged when electrons are added (negative) or removed (positive). Protons stay in the nuclei.
- **Charge is conserved** 电荷守恒: the total net charge of an isolated system stays the same. In these ordinary solid-body charging examples electrons transfer; nuclei remain in the solids.

Charging by friction, contact and induction

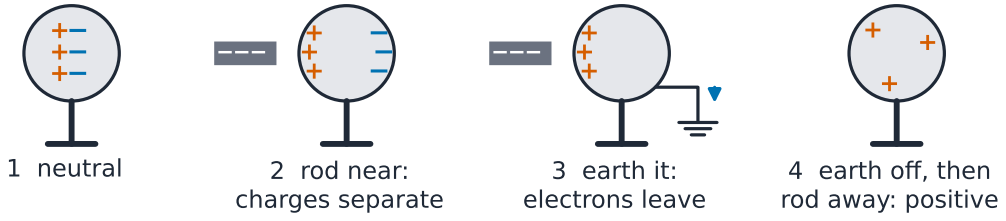
There are three ways to charge a body:



A Van de Graaff generator stores a large static charge on its metal dome, making a strong electric field around it

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- **Friction**: rubbing two different insulators together transfers electrons from one to the other. A balloon rubbed on hair takes electrons and becomes negative; the hair is left with an equal positive charge.
- **Contact**: touching a charged conductor to an uncharged one lets charge flow until they share it. Two identical, isolated metal spheres, with no nearby charged object, carrying $+6.0$ nC and -2.0 nC, touched and separated, each end up with $(6.0 - 2.0)/2 = +2.0$ nC.
- **Electrostatic induction** 静电感应: a charged object brought near a conductor, without touching it, pushes the conductor's free electrons to one side. If the conductor is then **earthed** 接地 (connected to the ground), electrons flow to or from the Earth, and the conductor is left with a charge opposite to the charged object.



Charging by induction: the sphere ends up with the opposite charge to the rod

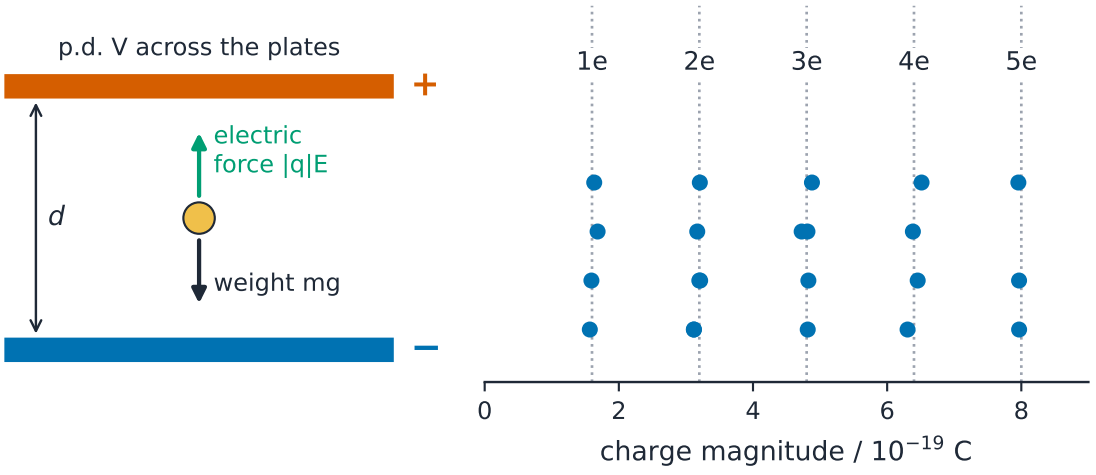
The order of the last step matters: remove the earth connection **before** taking the rod away. Otherwise electrons flow back from the earth and the sphere is left neutral.

Millikan's oil-drop experiment

Is charge continuous, or does it come in lumps? Millikan's oil-drop work tested discrete charge; its final report appeared in 1913. In the simplified suspension model, a small charged oil drop is observed between horizontal plates. By adjusting the potential difference V across the plates, separated by d, he could hold a drop still. For a stationary drop, neglecting air buoyancy, the upward electric-force magnitude balances weight:

$$|q|E = mg \quad \Rightarrow \quad |q| = \frac{mgd}{V}$$

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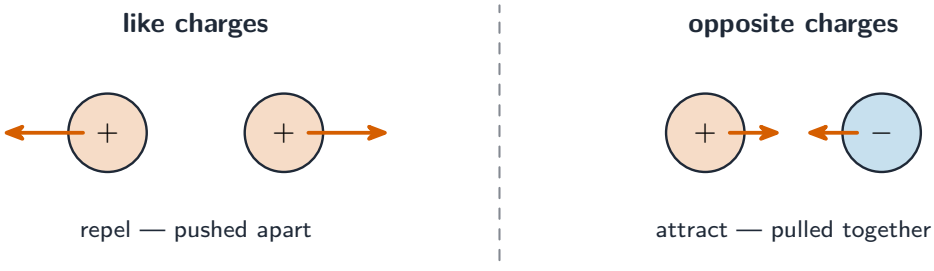
Invented charge-magnitude estimates cluster near multiples of e; plate polarity determines the suspended drop's sign

Measurements, with their uncertainties, supported charges in integer multiples of e, rather than arbitrary continuous values. This is evidence that **charge is quantized** 电荷量子化 for the isolated bodies studied here. The plotted points are invented illustrative estimates, not Millikan's original data.

Worked example with invented values: a drop of mass 4.9×10^{-15} kg is stationary between plates 1.0 cm apart with 1000 V across them, upper plate positive. Neglect buoyancy. The field is downward, so an upward electric force needs negative charge. $|q| = 4.9 \times 10^{-15} \times 9.81 \times 0.010/1000 \approx 4.8 \times 10^{-19}$ C; thus $q \approx -3e$, about three extra electrons.

Coulomb's law

The force between two point charges q_1 and q_2 a distance r apart is given by **Coulomb's law** 库仑定律:



Like charges repel; opposite charges attract – the force is along the line joining them

$$F = \frac{k|q_1 q_2|}{r^2} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

- Like gravity, it is an inverse-square law. Unlike gravity, it can attract or repel.
- An isolated spherically symmetric charge distribution acts as a point charge outside it. Nearby charges can redistribute charge on a conducting sphere, invalidating that simple external pattern.
- ε_0 is the **permittivity** 电容率 of free space. An ideal homogeneous linear dielectric has $\varepsilon = \varepsilon_r \varepsilon_0$, reducing the point-charge force by ε_r. The approximate table values are for electrostatic conditions and depend on material and temperature; conducting liquids and dissolved ions require more than this simple model.

Medium	Relative permittivity
vacuum	1 (exactly)
air	1.0006
paper	about 3
glass	about 5 to 10
water	about 80

Worked examples:

- $+3.0\ \mu$C and $-2.0\ \mu$C, 0.20 m apart in air: $F = 8.99 \times 10^9 \times 3.0 \times 10^{-6} \times 2.0 \times 10^{-6}/0.20^2 = 1.35$ N, attractive. In the ideal dielectric model with $\varepsilon_r = 80$, the force is about 80 times smaller, 0.017 N. Salt solubility additionally depends on lattice, hydration and entropy changes; this force comparison alone does not predict it.
- The electron and proton in a hydrogen atom, 5.3×10^{-11} m apart: $F = 8.99 \times 10^9 \times (1.60 \times 10^{-19})^2/(5.3 \times 10^{-11})^2 = 8.2 \times 10^{-8}$ N.

Electric field strength

A charge creates an **electric field** 电场 around it. The **electric field strength** 电场强度 E at a point is the force per unit charge on a small positive test charge placed there:

$$E = \frac{F}{q}$$

Its unit is N C^{-1} (the same as V m^{-1}). The field is a vector, in the direction of the force on a **positive** charge. A negative charge feels a force in the opposite direction to the field.

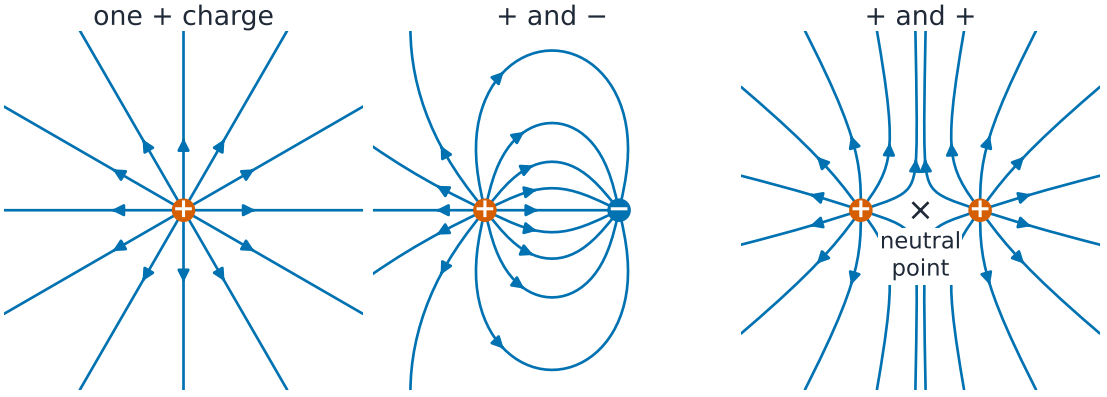
For a point charge Q, combining with Coulomb's law gives $E = kQ/r^2$. For example, 0.10 m from a +2.0 nC charge, $E = 8.99 \times 10^9 \times 2.0 \times 10^{-9}/0.10^2 = 1.8 \times 10^3$ N C^{-1}, pointing away from the charge.

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Electric field lines

Electric field lines 电场线 show the direction of the force on a positive test charge.

- They point away from positive charges and towards negative charges; lines may also extend to or from infinity. They do not cross where the field has a definite nonzero direction.
- The **density of the lines shows the field strength**: where they are closer together, the field is stronger.
- Around a single point charge the field is radial: outwards from a positive charge, inwards to a negative one.
- Between like charges the lines push apart, and there is a **neutral point** where the field is zero.

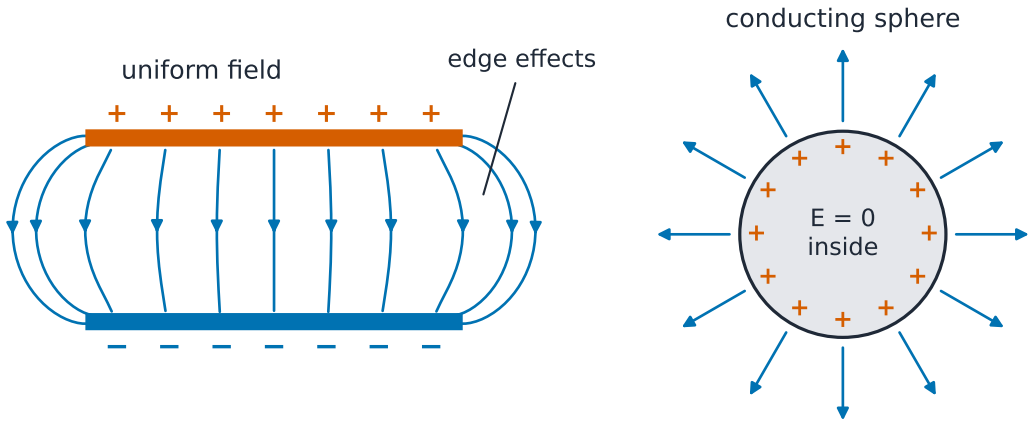


Lines point away from positive charges and towards negative charges; some extend to infinity

Two more patterns to know:

- **Parallel plates**: between two oppositely charged plates the lines are straight, parallel and equally spaced. The field there is uniform. Near the ends of the plates the lines bulge outwards: these are the **edge effects** 边缘效应.
- **An isolated charged conducting sphere in electrostatic equilibrium**: excess charge is on its surface and the exterior field is radial. The field inside conducting material is **zero**. A closed empty cavity also has zero field; a charge inside that cavity changes this conclusion about the cavity, not the conducting material.

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A uniform field between plates; no field inside a charged conductor

The uniform field between parallel plates

For ideal broad parallel plates, away from edges, let V be the positive magnitude of the potential difference across gap d. The uniform field magnitude is:

$$E = \frac{V}{d}$$

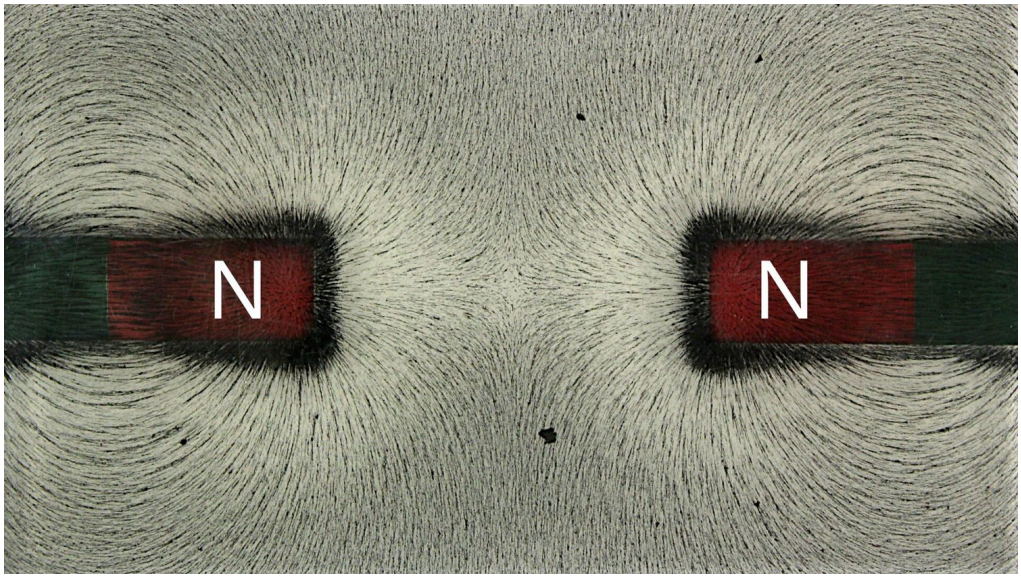
Before calculating, sketch the plate polarities, field direction and charge sign; convert the gap to metres. In school, use a simulation or approved low-voltage model rather than exposed high-voltage equipment.

Worked example: 2.0 kV across plates 4.0 cm apart gives $E = 2000/0.040 = 5.0 \times 10^4$ V m^{-1}. An electron between the plates feels $F = eE = 1.60 \times 10^{-19} \times 5.0 \times 10^4 = 8.0 \times 10^{-15}$ N, towards the positive plate, wherever it is between them.

Magnetic field lines

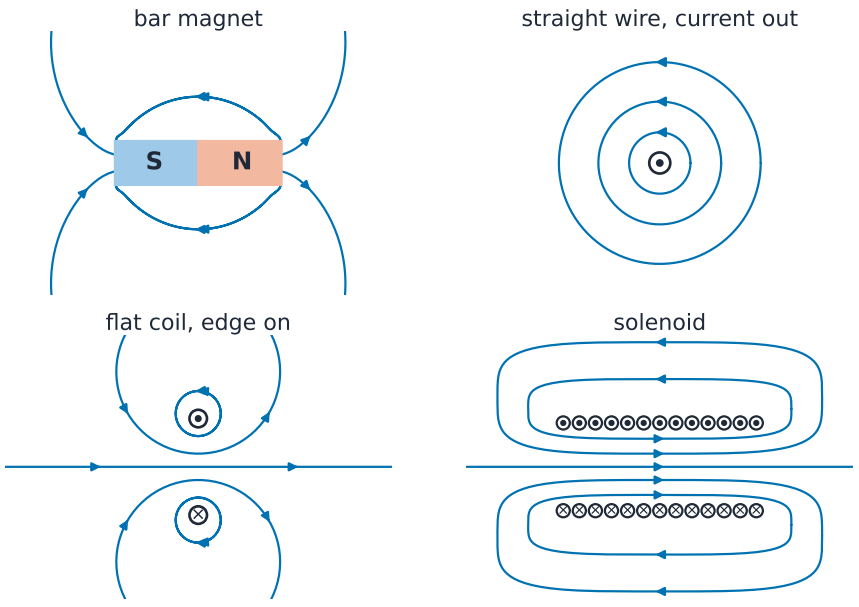
Magnets and electric currents create **magnetic fields** 磁场. **Magnetic field lines** 磁感线 show the direction a compass needle's north pole would point.

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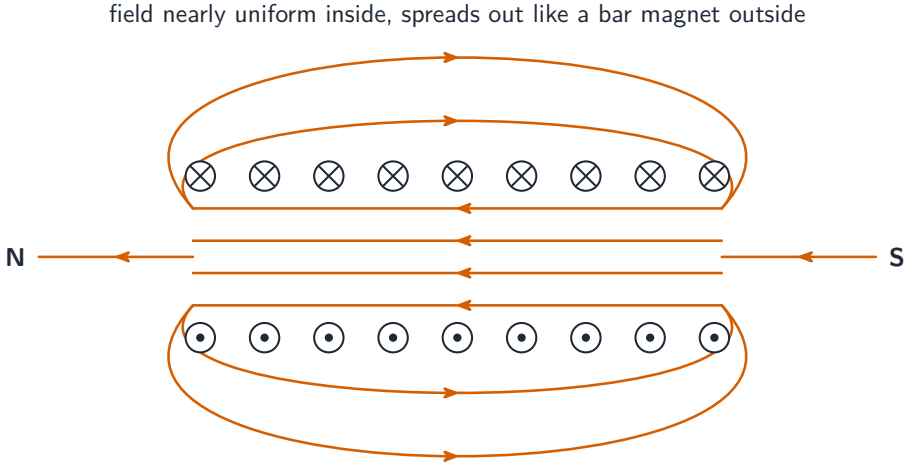
Iron filings trace the field lines around bar magnets.

- Outside a magnet the lines go from the **north pole to the south pole**. They are closest together, so the field is strongest, at the poles.
- Around a long **straight wire** the lines are circles centred on the wire, closer together near the wire. Find their direction with the **right-hand grip rule** 右手螺旋定则: point your right thumb along the current; your fingers curl in the direction of the field.
- Through a flat **circular coil** the lines pass straight through the middle and loop round each side.
- Inside a long **solenoid** 螺线管 (a coil of many turns, here with an air core) the field is strong and uniform: straight, parallel, equally spaced lines. Outside, the pattern is like a bar magnet's. Use the grip rule with the fingers along the current in the turns: the thumb then points to the north end.



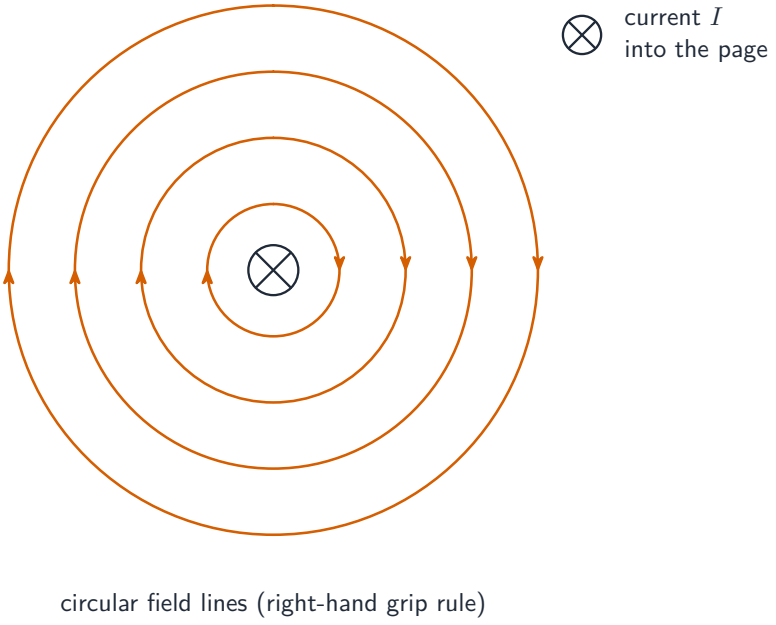
The four magnetic field patterns to know

In diagrams, a dot ⊙ means a current coming out of the page and a cross ⊗ means a current going into it. For a current out of the page, the field circles anticlockwise.



Field of a solenoid

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Field around a long straight wire

AHL: Electric potential energy

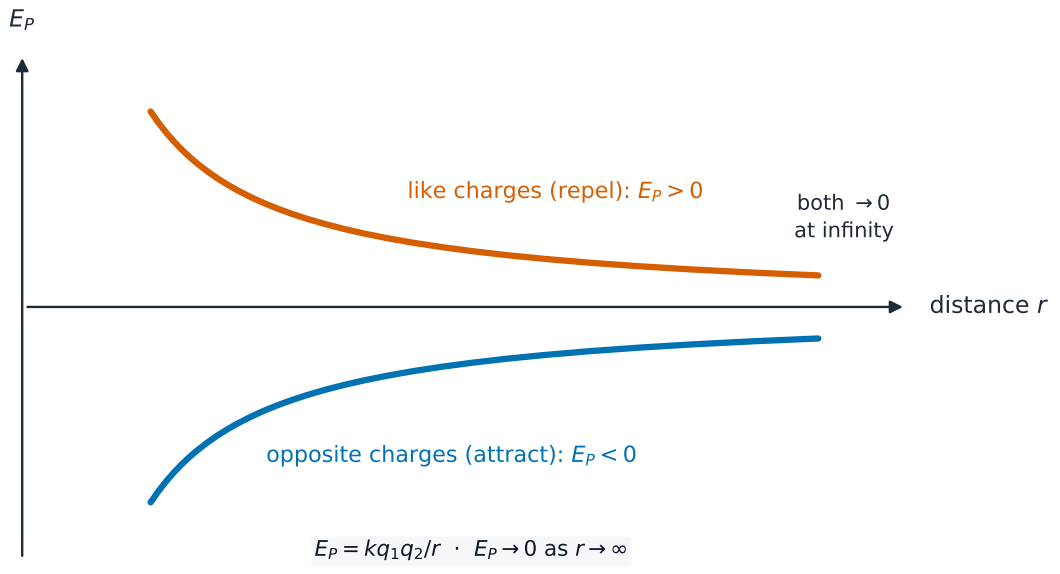
The **electric potential energy** 电势能 E_p of a system of charges is the external work to assemble the system slowly from infinite separation. For two point charges r apart:

$$E_p = \frac{kq_1q_2}{r}$$

- Put in the signs of the charges. Like charges give a **positive** E_p: work must be done to push them together. Unlike charges give a **negative** E_p: they attract, so work is done by the field as they come together.
- E_p is zero when the charges are infinitely far apart.

Worked example: $+3.0\ \mu$C and $-2.0\ \mu$C, 0.20 m apart: $E_p = 8.99 \times 10^9 \times (3.0 \times 10^{-6}) \times (-2.0 \times 10^{-6})/0.20 = -0.27$ J. Separating them completely needs at least 0.27 J. For the hydrogen atom, $E_p = -4.3 \times 10^{-18}$ J.

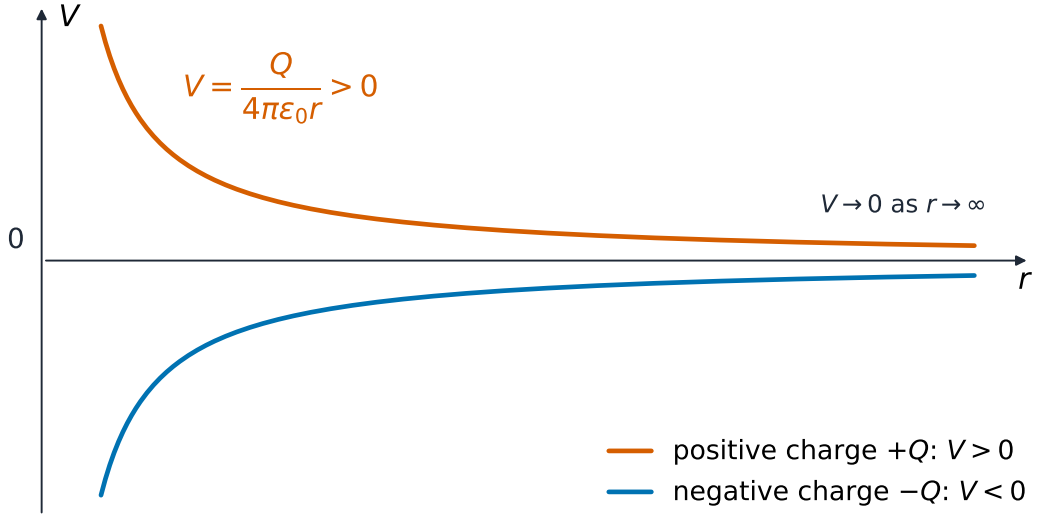
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Electric PE of two charges: positive for like charges, a negative well for opposite charges

AHL: Electric potential

The **electric potential** 电势 V_e at a point is the external work per unit charge to bring a small positive test charge slowly from infinity to that point. For a point charge Q:



The potential near a point charge varies as 1/r – positive for a positive charge, negative for a negative one

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$$V_e = \frac{kQ}{r}$$

- Its unit is the volt, V (J C^{-1}). Potential is zero at infinity.
- It is positive near a positive charge and negative near a negative one.
- **Potential is a scalar** 标量. The potential due to several charges is the simple sum of their potentials, with signs. Field strengths, being vectors, must be added with their directions.

Worked example: midway between +4.0 nC and −2.0 nC that are 0.20 m apart, each charge is 0.10 m away.

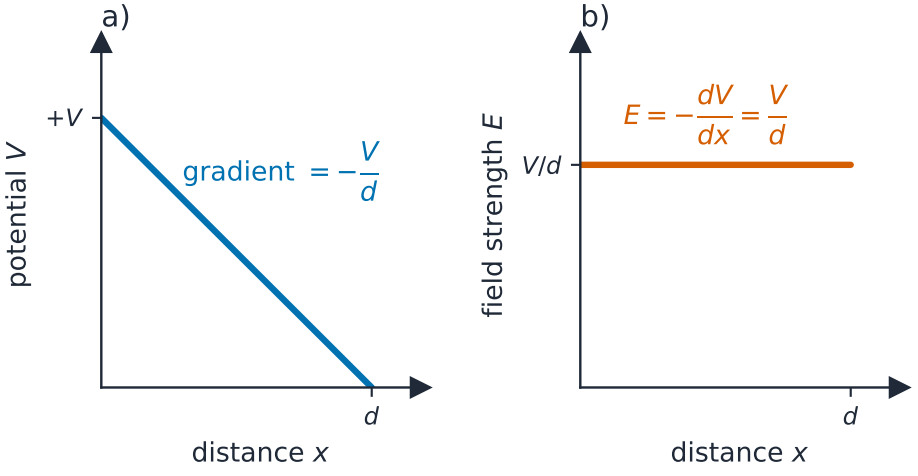
- Potential: $V = 8.99 \times 10^9 \times (4.0 - 2.0) \times 10^{-9} / 0.10 = 180$ V.
- Field: both fields point towards the negative charge, so they add: $E = 3.6 \times 10^3 + 1.8 \times 10^3 = 5.4 \times 10^3$ N C^{-1}.

AHL: Potential gradient and work done

The field strength is the negative of the **potential gradient** 电势梯度:

$$E_r = -\frac{dV_e}{dr} \qquad \overline{E_r} = -\frac{\Delta V_e}{\Delta r}$$

For outward radial coordinate r, the local component is the negative derivative; a finite difference gives its interval average. The field points towards lower potential. Between parallel plates the potential changes steadily, and $E = V/d$ is a special case.



In a uniform field the potential falls steadily with distance, so the field strength V/d is constant

The external **work done** for a slow move at unchanged kinetic energy is

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$$W = q\,\Delta V_e$$

Worked examples:

- 0.20 m from a +2.0 nC charge, $V = 90$ V; at 0.10 m, $V = 180$ V. Moving a +1.0 nC charge inwards needs $W = 1.0 \times 10^{-9} \times 90 = 9.0 \times 10^{-8}$ J.
- In an ideal uniform field, two equipotentials 50 V apart have a perpendicular separation of 2.0 cm. The field magnitude is $E = 50/0.020 = 2.5 \times 10^{3}$ V m^{-1}.

The electronvolt

For particles, work is often given in **electronvolts** 电子伏特 (eV). One electronvolt is the energy gained by an electron moving through a potential difference of 1 V: 1 eV $= 1.60 \times 10^{-19}$ J.

Worked example: an electron accelerated from rest through 2.0 kV gains 2.0 keV $= 2000 \times 1.60 \times 10^{-19} = 3.2 \times 10^{-16}$ J. Then $\frac{1}{2}mv^2 = 3.2 \times 10^{-16}$ gives $v = 2.7 \times 10^{7}$ m s^{-1}.

AHL: Equipotential surfaces

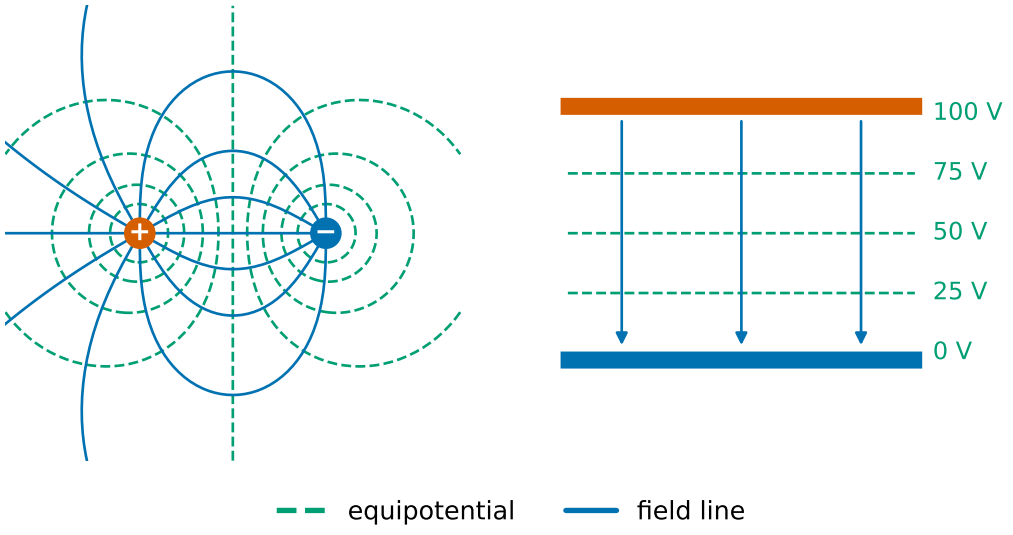
An **equipotential surface** 等势面 joins points at the same potential.

- **No work is done** by the electric field along an equipotential because $\Delta V_e = 0$; another force may still change kinetic energy.
- **Field lines cross equipotentials at right angles.** If the field had a part along the surface, moving along it would need work.
- Drawn at equal steps of potential, equipotentials are closer together where the field is stronger.

Patterns to recognize:

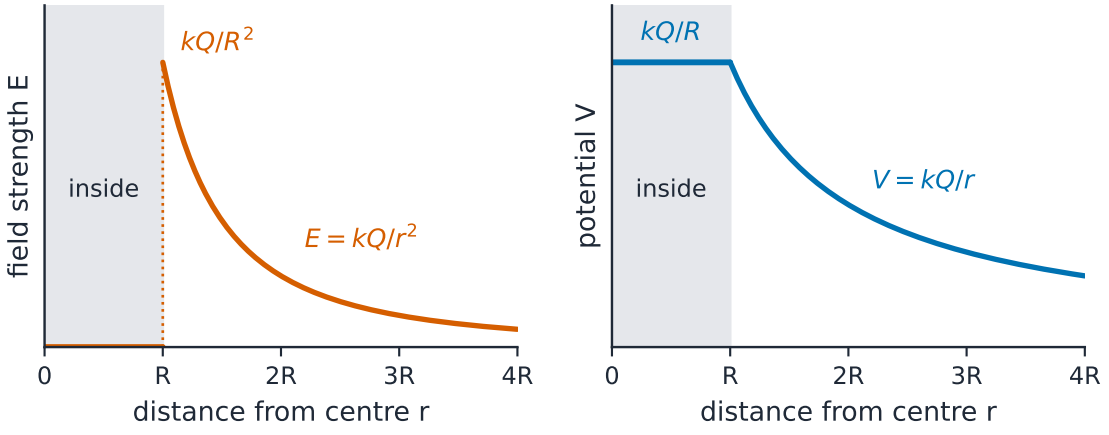
- **Point charge**: concentric spheres, further apart at larger distances.
- **Several point charges**: add potentials as scalars with signs. Equipotential shapes depend on the whole arrangement and need not be closed; for an equal opposite pair the zero-potential surface includes the perpendicular bisector plane.
- **Parallel plates**: flat planes parallel to the plates and equally spaced, because the field is uniform.
- **Isolated charged conducting sphere**: the conducting material is equipotential; so is an empty cavity. With spherical symmetry the exterior equipotentials are concentric spheres.

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Equipotentials always meet field lines at right angles

For an isolated conducting sphere in electrostatic equilibrium, with an empty cavity if hollow, the internal field is zero and potential is constant: it equals the surface value kQ/R everywhere inside.



Inside a conductor: zero field, constant potential

Worked example: an isolated conducting sphere of radius 0.10 m carries +5.0 nC. Just outside its surface $V = 8.99 \times 10^9 \times 5.0 \times 10^{-9} / 0.10 = 450$ V and $E = 4.5 \times 10^3$ V m^{-1}. At its centre, $V = 450$ V and $E = 0$.

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Practice

1. A balloon rubbed on hair becomes negatively charged. Explain what happens to the charge, and why the hair is left positive.
2. Two identical metal spheres carry $+6.0$ nC and -2.0 nC. They are touched together and separated. Find the charge on each.
3. Describe how a negatively charged rod can be used to give an insulated metal sphere a positive charge.
4. Find the force between $+3.0\ \mu$C and $-2.0\ \mu$C 0.20 m apart in air, and in water (relative permittivity 80).
5. An invented oil drop of mass 4.9×10^{-15} kg is stationary between plates 1.0 cm apart with 1000 V across them, upper plate positive. Neglect buoyancy and use g $= 9.81$ m/s^2. Find signed charge and the number of extra electrons.
6. Find the electric field strength 0.10 m from a $+2.0$ nC point charge.
7. 2.0 kV is applied across plates 4.0 cm apart. Find the field strength and the force on an electron between them.
8. Sketch the field lines between a positive and a negative point charge, and between two oppositely charged parallel plates, including edge effects.
9. A vertical wire carries a current upwards. State the direction of the magnetic field lines round it, seen from above.
10. Sketch the magnetic field of a solenoid and state how it compares with that of a bar magnet.
11. An invented particle with charge -3.2×10^{-19} C is in a uniform field of magnitude 5.0×10^4 N/C directed right. Find its force magnitude and direction, and its charge in units of e.
12. **HL.** Point charges $+4.0$ nC and -2.0 nC are 0.20 m apart in vacuum. Find their interaction potential energy, external work for slow separation to infinity, and midpoint potential and field direction/magnitude.
13. **HL.** An electron is released from rest just beside the negative plate of two broad plates 4.0 cm apart with 2.0 kV across them. It crosses the full gap to the positive plate; neglect other forces and edge effects. Find field magnitude, which plate it accelerates towards, kinetic-energy gain in eV and J, and non-relativistic final speed. Use electron mass 9.11×10^{-31} kg.
14. **HL.** An isolated conducting sphere of radius 0.10 m carries $+5.0$ nC. Find V and E at its centre, just outside the surface and at r $= 0.20$ m. Explain why electrostatic field lines meet a regular equipotential at right angles.

Answers.

1. Rubbing transfers electrons from the hair to the balloon. Charge is conserved, so the hair loses exactly as much negative charge as the balloon gains and is left positive.
2. Total charge $+4.0$ nC is shared equally: $+2.0$ nC each.

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3. Bring the rod near (not touching): electrons are pushed to the far side. Earth the far side: electrons flow to earth. Remove the earth connection, then the rod: the sphere is left positive.
4. Air: $F = 8.99 \times 10^9 \times 6.0 \times 10^{-12}/0.040 = 1.35$ N, attractive. Water: $1.35/80 = 0.017$ N.
5. $|q| = mgd/V \approx 4.8 \times 10^{-19}$ C. The field is downward but electric force upward, so $q \approx -4.8 \times 10^{-19}$ C, about three extra electrons.
6. $E = 8.99 \times 10^9 \times 2.0 \times 10^{-9}/0.10^2 = 1.8 \times 10^3$ N C^{-1}, away from the charge.
7. $E = 2000/0.040 = 5.0 \times 10^4$ V m^{-1}; $F = 1.60 \times 10^{-19} \times 5.0 \times 10^4 = 8.0 \times 10^{-15}$ N, towards the positive plate.
8. Lines start on the positive charge and curve round to end on the negative charge. Between plates: straight, parallel, equally spaced lines from + to −, bulging outwards at the ends.
9. Anticlockwise circles centred on the wire (right-hand grip rule).
10. Inside: straight, parallel, equally spaced lines (uniform field). Outside: the same pattern as a bar magnet, with a north pole at one end.
11. $|F| = |q|E = 1.6 \times 10^{-14}$ N, left because the charge is negative; $q/e = -2$.
12. **HL.** $U = kq_1q_2/r = -3.60 \times 10^{-7}$ J, so slow external separation work is $+3.60 \times 10^{-7}$ J. Midpoint $V = k(4.0 - 2.0) \times 10^{-9}/.10 \approx 180$ V. Field contributions point from positive towards negative and add: $E = k(6.0 \times 10^{-9})/.10^2 \approx 5.4 \times 10^3$ N/C.
13. **HL.** $E = 2000/.040 = 5.0 \times 10^4$ N/C. The electron accelerates towards the positive plate; $\Delta K = 2000$ eV $= 3.2 \times 10^{-16}$ J. $v = \sqrt{2\Delta K/m} \approx 2.65 \times 10^7$ m/s, about .089c; this is the non-relativistic estimate.
14. **HL.** Centre: V ≈ 450 V and E = 0. Just outside R: V ≈ 450 V and E ≈ 4500 N/C outward. At .20 m: V ≈ 225 V and E ≈ 1120 N/C outward. Potential is continuous at the surface while field has a jump. A tangential field would do nonzero electric work along equal potential, so at nonzero field the direction is normal.

Exam tips

- Field direction is the force direction on a **positive** charge; the force on an electron is opposite.
- Field lines never cross; their density shows the field strength.
- In electrostatic equilibrium the field in conducting material is zero; specify an empty cavity when extending that statement to a hollow interior.
- Use the right-hand grip rule for straight wires and solenoids, and draw ⊙ and ⊗ for currents out of and into the page.

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Motion in electromagnetic fields

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches D.3 Motion in electromagnetic fields in full, for the course first assessed in 2025. SL and HL students study the same material in D.3.

Prerequisites: projectile motion and circular motion from A.1 and A.2; kinetic energy from A.3; electric and magnetic fields from D.2. Use $e = 1.60 \times 10^{-19}$ C, electron mass 9.11×10^{-31} kg, proton mass 1.67×10^{-27} kg and $\mu_0 \approx 4\pi \times 10^{-7}$ T m A^{-1}.

Charged particles in fields

What makes the shapes of the northern lights?

Near the polar regions the night sky sometimes fills with moving curtains of light: the aurora. Solar-wind energy enters Earth's magnetosphere and energizes particles, mainly electrons. Earth's magnetic field guides their motion; collisions with the upper atmosphere excite atoms and molecules, which release light as they return to lower energy states. This is not simply a direct stream of solar electrons reaching the ground, nor does magnetic steering alone supply their kinetic energy. NOAA's aurora explanation distinguishes electron acceleration from magnetic guidance. In a laboratory, magnetic fields steer accelerated particles in a cyclotron and bend ion paths in a mass spectrometer.



An aurora: Earth's magnetic field guides energized electrons towards the polar regions, where collisions make the upper air glow

This unit is about how charged particles move in electric fields and in magnetic fields,

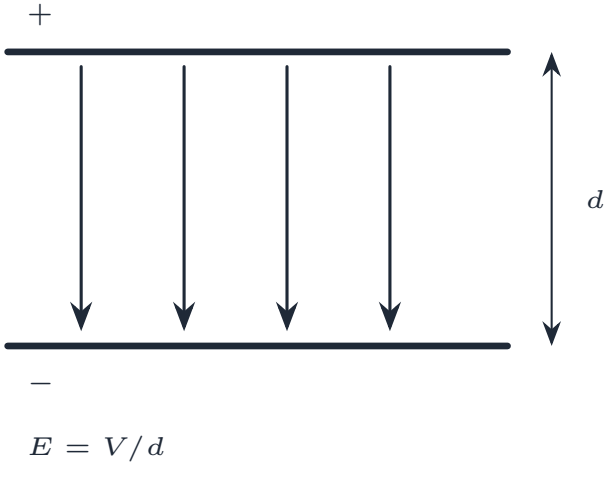
and about the magnetic forces on wires that carry currents.

Motion in a uniform electric field

In a uniform electric field E a charge q feels a constant vector force $\mathbf{F} = q\mathbf{E}$, so it has a constant acceleration $\mathbf{a} = q\mathbf{E}/m$ when other forces are negligible. This is just like a mass in the Earth's uniform gravitational field.



Where the particles come from: a linear accelerator uses electric fields to drive charged particles fast enough to probe the nucleus



Between parallel plates the field is uniform, $E = V/d$

Moving along the field. A charge released from rest between parallel plates ac-

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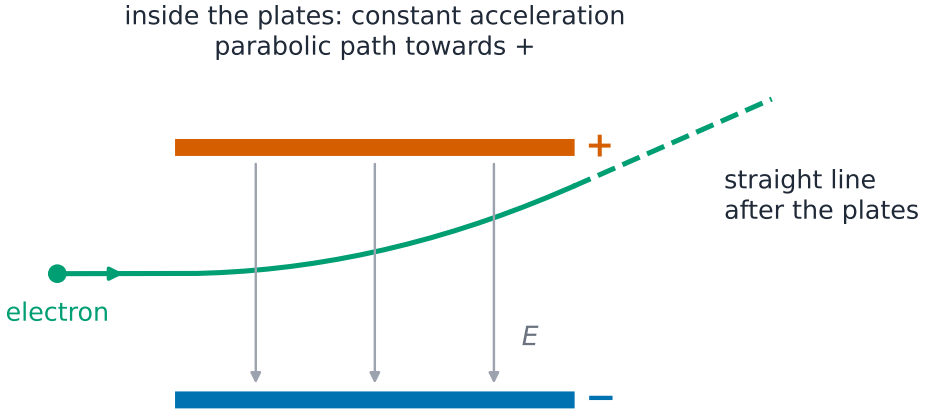
celerates in a straight line: along the field for positive charge, against it for negative charge. If V is the positive magnitude of the accelerating potential difference, the kinetic energy gained is

$$\tfrac{1}{2}mv^2 = |q|V$$

For a signed potential change instead, $\Delta K = -q\Delta V$ when the electric force alone supplies the work. These speed calculations use the non-relativistic model.

Worked example: a proton accelerated from rest through 500 V gains $1.60 \times 10^{-19} \times 500 = 8.0 \times 10^{-17}$ J (500 eV), so $v = \sqrt{2 \times 8.0 \times 10^{-17} / 1.67 \times 10^{-27}} = 3.1 \times 10^{5}$ m s^{-1}.

Entering at right angles to the field. A charge moving between the plates, parallel to them, keeps a constant velocity along the plates but accelerates across them. Its path is a **parabola** 抛物线, exactly like a ball thrown horizontally. After it leaves the ideal field region it moves along a straight tangent, if other forces are negligible. This model neglects fringe fields. A beam must also clear the plate: otherwise it strikes the plate before reaching the calculated exit.



In a uniform electric field the path is a parabola, like a projectile

Worked example: an electron enters the field ($E = 2.0 \times 10^{4}$ V m^{-1}) between plates 5.0 cm long at 2.0×10^{7} m s^{-1}.

- Time between the plates: $t = 0.050/(2.0 \times 10^{7}) = 2.5 \times 10^{-9}$ s.
- Acceleration across: $a = eE/m = 1.60 \times 10^{-19} \times 2.0 \times 10^{4} / 9.11 \times 10^{-31} = 3.5 \times 10^{15}$ m s^{-2}.
- Sideways deflection: $y = \tfrac{1}{2}at^2 = 0.011$ m (1.1 cm), and sideways velocity $at = 8.8 \times 10^{6}$ m s^{-1} at exit.

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The magnetic force on a moving charge

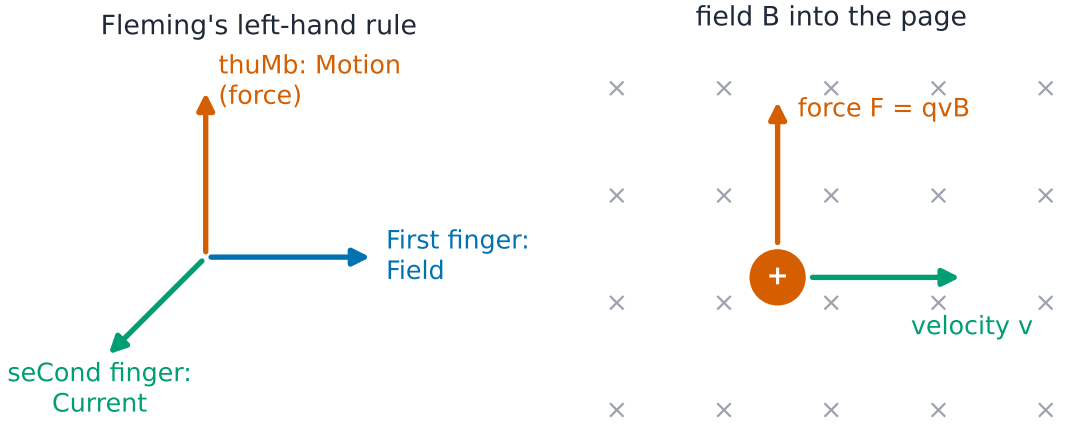
A charge q moving with speed v through a magnetic field B feels a force

$$F = |q|vB\sin\theta$$

where θ is the angle between the velocity and the field. The unit of B is the **tesla** 特斯拉 (T).

- The force is greatest when the charge moves at right angles to the field ($F = |q|vB$) and zero when it moves along the field.
- A charge at rest feels no magnetic force at all.
- The force is always at **right angles to both the velocity and the field**.

To find the direction, use **Fleming's left-hand rule** 弗莱明左手定则: hold the first finger, second finger and thumb of your left hand at right angles. Point the **F**irst finger along the **F**ield and the se**C**ond finger along the **C**urrent, that is the direction a **positive** charge moves. The thu**M**b gives the force (the **M**otion it causes). For a negative charge, such as an electron, the force is in the opposite direction.



The force is at right angles to both the velocity and the field

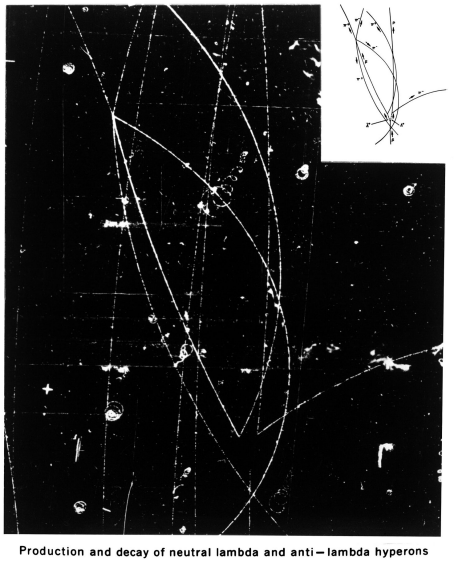
Worked example: a proton moving at 3.0×10^6 m s^{-1} at right angles to a 0.20 T field feels $F = 1.60 \times 10^{-19} \times 3.0 \times 10^6 \times 0.20 = 9.6 \times 10^{-14}$ N. At 30° to the field the force is half as big, 4.8×10^{-14} N.

Exam tips

- State charge sign, field directions and classical-model assumptions before calculating.
- Use radius and perpendicular speed; convert units before squaring.

Circular motion in a magnetic field

Because the magnetic force is always at right angles to the velocity, the magnetic force alone changes the particle's direction but not its speed. **It does no work, so the kinetic energy stays constant.** A particle moving at right angles to a uniform field therefore moves in a circle, with the magnetic force providing the centripetal force:



A bubble chamber reveals the curved tracks of charged particles.

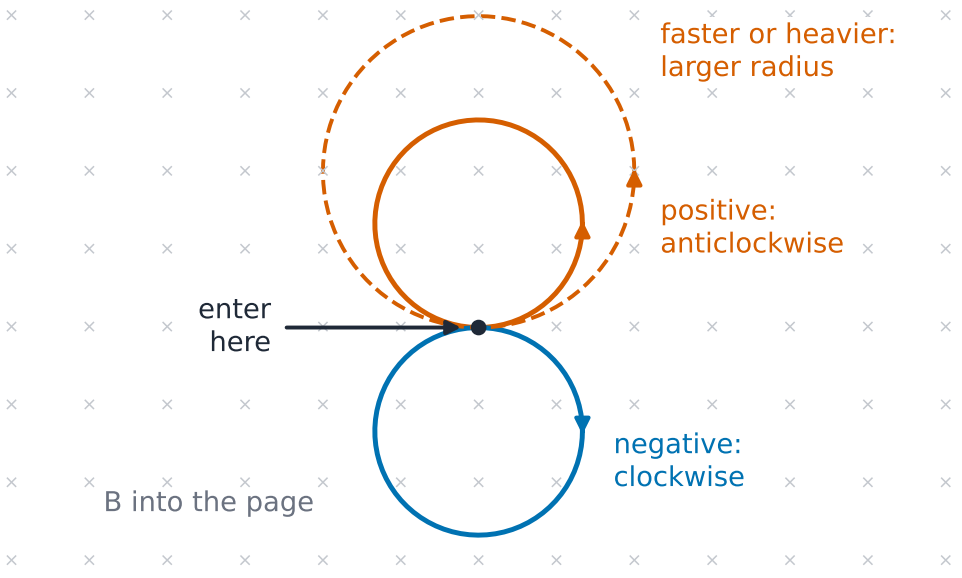
$$|q|vB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{|q|B}$$

The time for one orbit is

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}$$

in the non-relativistic uniform-field model. It does not depend on the speed: a faster particle moves on a bigger circle in the same time.

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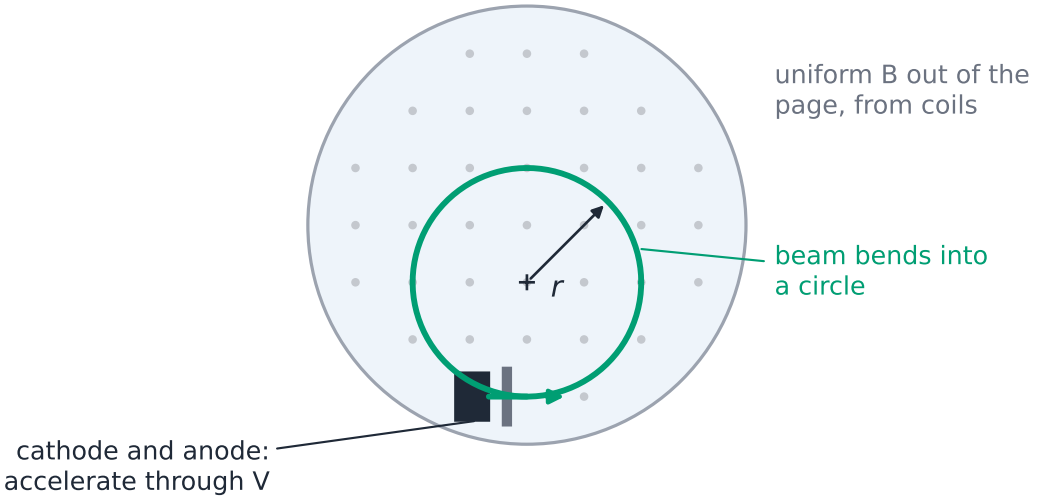
Opposite charges curve opposite ways; more momentum means a larger circle

- Positive and negative particles curve in opposite directions.
- At fixed other quantities, larger momentum gives a larger radius and larger charge magnitude or field gives a smaller one. Inferring mass from a spectrometer needs known speed and charge state; different masses alone do not guarantee different radii.
- For oblique entry, $v_{\parallel} = v\cos\theta$ stays constant while $v_{\perp} = v\sin\theta$ produces circular motion with $r = mv_{\perp}/(|q|B)$. The result is a helix of pitch $|v_{\parallel}|T$. This describes a local uniform-field model; the real auroral field is nonuniform.

Worked example: the proton above (3.0×10^{6} m s^{-1}, 0.20 T) moves on a circle of radius $r = 1.67 \times 10^{-27} \times 3.0 \times 10^{6}/(1.60 \times 10^{-19} \times 0.20) = 0.16$ m, with period $T = 2\pi \times 1.67 \times 10^{-27}/(1.60 \times 10^{-19} \times 0.20) = 3.3 \times 10^{-7}$ s.

Measuring the charge-to-mass ratio

The path of a charged particle in a magnetic field lets us measure the magnitude of its **charge-to-mass ratio** 荷质比 $|q|/m$. Electron e/m below is positive; signed electron q/m is negative. In a fine-beam tube, electrons are accelerated through a potential difference V and then enter a uniform magnetic field made by a pair of coils. The tube holds a little gas, so the beam glows and the radius r of its circle can be measured with a ruler.



Measure V, B and r to find the charge-to-mass ratio of the electron

From the energy gained, $\frac{1}{2}mv^2 = eV$, so $v = \sqrt{2eV/m}$. Putting this into $r = mv/eB$ and rearranging gives

$$\frac{e}{m} = \frac{2V}{B^2 r^2}$$

Worked example: with $V = 200$ V and $B = 1.0$ mT the beam radius is 4.8 cm. Then $e/m = 2 \times 200/((1.0 \times 10^{-3})^2 \times 0.048^2) = 1.7 \times 10^{11}$ C kg^{-1}, close to the accepted magnitude 1.76×10^{11} C kg^{-1}. This assumes negligible initial kinetic energy and non-relativistic, perpendicular motion. Use radius, not diameter; check field calibration and viewing parallax. At fixed B, plotting r^2 against V gives gradient $2/[(e/m)B^2]$, allowing several readings to test the model.

Crossed electric and magnetic fields

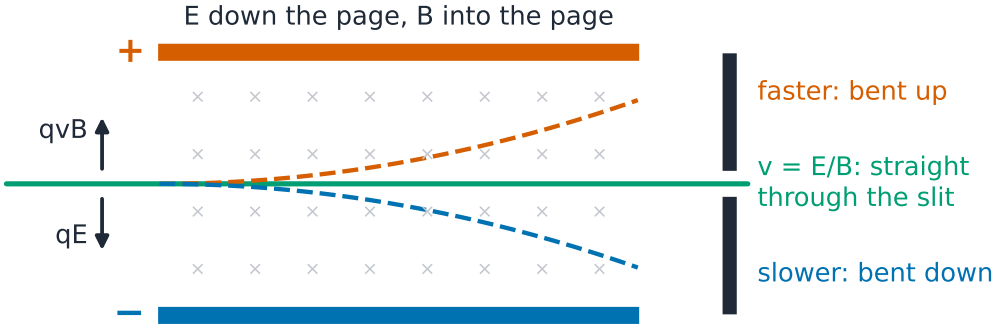
Suppose a uniform electric field and a uniform magnetic field are at right angles to each other and to a beam of charged particles. Electric and magnetic forces can oppose each other. Their magnitudes are $|q|E$ and $|q|vB$ for the perpendicular beam. They balance when

$$|q|E = |q|vB \quad \Rightarrow \quad v = \frac{E}{B}$$

Charged particles entering in the stated perpendicular direction pass undeflected at this speed, independent of their nonzero charge magnitude and mass. Neutral particles also pass straight because neither field acts on them. Slower charged particles initially deflect towards their electric force; faster ones initially deflect the other way. Both forces reverse for negative charge, so the balanced speed is unchanged, while the off-

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speed deflections reverse. This arrangement is called a **velocity selector** 速度选择器: a narrow exit slit passes a small range around the selected speed.

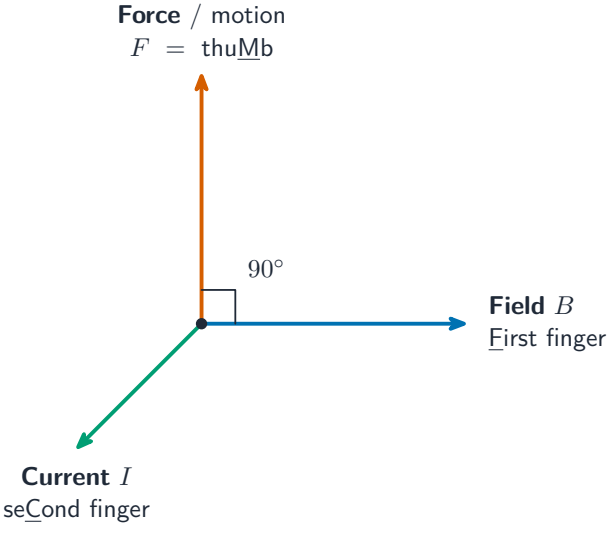


Only particles with $v = E/B$ go straight through

Worked example: with $E = 3.0 \times 10^4$ V m^{-1} and $B = 0.010$ T, only particles with $v = 3.0 \times 10^4 / 0.010 = 3.0 \times 10^6$ m s^{-1} pass straight through.

The force on a current-carrying conductor

A current is a flow of moving charges, so a wire carrying a current I through a magnetic field also feels a force. For a straight length L of wire actually within a uniform field, at angle θ to the field:



use the **left** hand; hold the three at right angles

Fleming's left-hand rule: thumb = force, first finger = field, second finger = current

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$$F = BIL\sin\theta$$

The direction comes from Fleming's left-hand rule, with the second finger along the conventional current. This is the **motor effect** 电动机效应, used in many electric motors and moving-coil loudspeakers. Energy for mechanical work comes from the electrical supply; this does not contradict zero magnetic work on a free charge.

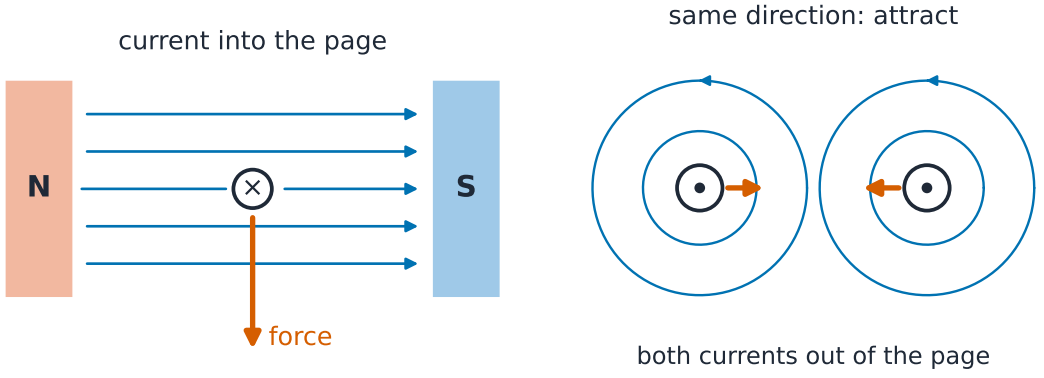
Worked example: 5.0 cm of wire carrying 3.0 A at right angles to a 0.40 T field feels $F = 0.40 \times 3.0 \times 0.050 = 0.060$ N. At 30° to the field the force is 0.030 N; along the field it is zero.

The force between parallel wires

Each current-carrying wire makes a magnetic field, and the other wire sits in that field. For two long parallel wires in vacuum (approximately air), a distance r apart, the field from one wire at the other is $B_1 = \mu_0 I_1/(2\pi r)$. Using $F/L = I_2 B_1$ gives the force-per-length magnitude below. Ignore end effects and nearby return wires; use current magnitudes and centre-to-centre separation.

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

- **Currents in the same direction attract; currents in opposite directions repel.** (This is the opposite of the rule for charges.)
- The forces on the two wires are equal and opposite, as Newton's third law requires.
- Find the direction by using the right-hand grip rule to find the field of one wire at the other, then Fleming's left-hand rule for the force.



The motor effect, and why parallel currents attract

Worked example: two long wires 5.0 cm apart each carry 10 A in the same direction. $F/L = 4\pi \times 10^{-7} \times 10 \times 10/(2\pi \times 0.050) = 4.0 \times 10^{-4}$ N m^{-1}, attractive.

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Practice

1. A proton is accelerated from rest through 500 V. Find its kinetic energy and its speed.
2. An electron enters the uniform field between plates 5.0 cm long at 2.0×10^7 m s^{-1}, moving parallel to the plates. The field is 2.0×10^4 V m^{-1}. Find the time between the plates, its sideways deflection and its sideways velocity when it leaves.
3. Find the force on a proton moving at 3.0×10^6 m s^{-1} in a 0.20 T field when it moves (a) at right angles to the field, (b) at 30° to it, (c) along it.
4. Explain why a magnetic field cannot change the kinetic energy of a charged particle.
5. Find the radius and the period of the circular path of a proton moving at 3.0×10^6 m s^{-1} at right angles to a 0.20 T field.
6. The proton's speed is doubled. State what happens to the radius and to the period of its path.
7. In a fine-beam tube, electrons accelerated through 200 V move in a circle of radius 4.8 cm in a 1.0 mT field. Find the charge-to-mass ratio of the electron.
8. In a velocity selector $E = 3.0 \times 10^4$ V m^{-1} and $B = 0.010$ T. Find the speed of the particles that pass straight through, and state what happens to slower positive ions.
9. Find the force on 5.0 cm of wire carrying 3.0 A in a 0.40 T field when the wire is (a) at right angles to the field, (b) at 30° to it.
10. Two long parallel wires 5.0 cm apart each carry 10 A in the same direction. Find the force per metre between them and state whether it is attractive or repulsive.
11. A proton and an alpha particle (twice the charge magnitude, four times the mass) enter the same uniform field perpendicularly. Compare their radii (a) at the same speed and (b) at the same kinetic energy, in the classical model.
12. An electron moves due north through a magnetic field pointing vertically downwards. State the direction of the force on it.
13. An invented proton has perpendicular speed 3.0×10^6 m/s and parallel speed 1.0×10^6 m/s in a uniform .20 T field. Find helix radius and pitch. Explain which speed component determines each.
14. In an invented beam-tube measurement, percentage uncertainties in V, B and r are 1%, 2% and 3%. Estimate a conservative percentage uncertainty in e/m. Explain why measuring a diameter as though it were a radius changes the result by a factor of four.

Answers.

1. $E_k = 500$ eV $= 8.0 \times 10^{-17}$ J; $v = \sqrt{2 \times 8.0 \times 10^{-17} / 1.67 \times 10^{-27}} = 3.1 \times 10^5$ m s^{-1}.
2. $t = 2.5 \times 10^{-9}$ s; $a = 3.5 \times 10^{15}$ m s^{-2}, so $y = \frac{1}{2}at^2 = 1.1$ cm and $v_y = at = 8.8 \times 10^6$ m s^{-1}.
3. (a) 9.6×10^{-14} N; (b) 4.8×10^{-14} N; (c) zero.

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4. The magnetic force is always at right angles to the velocity, so it does no work on the particle; the speed, and so the kinetic energy, cannot change.
5. $r = mv/(|q|B) = 0.16$ m; $T = 2\pi m/(|q|B) = 3.3 \times 10^{-7}$ s.
6. The radius doubles; the period stays the same.
7. $e/m = 2V/B^2r^2 = 400/((1.0 \times 10^{-3})^2 \times 0.048^2) = 1.7 \times 10^{11}$ C kg^{-1}.
8. $v = E/B = 3.0 \times 10^6$ m s^{-1}. Slower ions feel a smaller magnetic force, so the electric force wins and they are bent in the direction of the electric field, missing the slit.
9. (a) $F = 0.40 \times 3.0 \times 0.050 = 0.060$ N; (b) 0.030 N.
10. $F/L = 4\pi \times 10^{-7} \times 100/(2\pi \times 0.050) = 4.0 \times 10^{-4}$ N m^{-1}, attractive.
11. (a) $r = mv/(|q|B)$: at equal speed the alpha radius is $4/2 = 2$ times the proton's. (b) At equal K, $p = \sqrt{2mK}$ doubles, as does $|q|$, so the radii are equal.
12. For a positive charge the left-hand rule gives a force to the west; the electron is negative, so the force is to the **east**.
13. $r = mv_\perp/(eB) = .1566$ m. $T = 2\pi m/(eB) = 3.28 \times 10^{-7}$ s; pitch $= |v_\parallel|T = .328$ m. Perpendicular motion sets radius; parallel motion sets advance.
14. Conservative uncertainty $= 1 + 2(2) + 2(3) = 11\%$. Using diameter doubles r, quadruples the denominator, and makes inferred e/m one quarter of its correct value.

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Induction

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

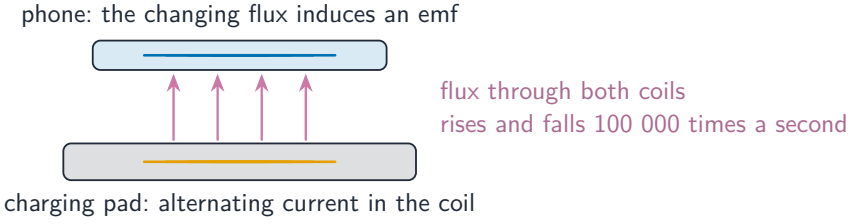
This handout teaches D.4 Induction in full, for the course first assessed in 2025. The whole unit is additional higher level (AHL); there is no SL counterpart.

Prerequisites: magnetic field lines and the force on a current-carrying conductor from D.2 and D.3; emf and circuits from B.5; energy conservation from A.3; sine curves and angular velocity $\omega = 2\pi f$ from A.2 and C.1.

AHL: Magnetic flux

How does a phone charge without a cable?

Put a phone on a wireless charging pad and it starts charging, although no wire joins them. A coil in the pad carries an alternating current, so the magnetic field through a second coil inside the phone keeps changing. A changing magnetic field through a coil makes an emf in it, and that emf drives the charging current. The same idea runs power-station generators, transformers, electric guitar pick-ups, induction cookers and the brakes of some trains. This unit explains how it works.



Changing flux links two coils

Flux through an area

For a uniform field through a flat area, the signed **magnetic flux** 磁通量 Φ is the normal component of magnetic flux density multiplied by area:

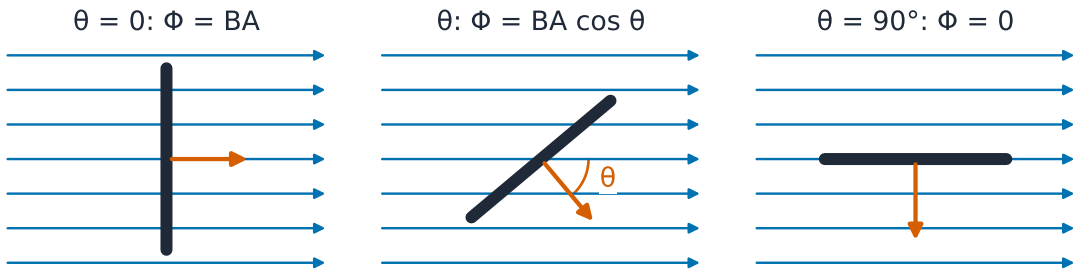
$$\Phi = BA\cos\theta$$

where B is the magnetic flux density and θ is the angle between the field and the **normal** 法线 to the area (the line at right angles to the surface).

- The unit of flux is the **weber** 韦伯 (Wb): 1 Wb = 1 T m^2.
- The flux is greatest ($\Phi = BA$) when the field passes straight through the area ($\theta = 0$).

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- The flux is zero when the field runs along the surface ($\theta = 90°$).



edge-on view: black = coil, red = normal to the coil, blue = field B

Only the part of the field along the normal passes through the coil

Choose and retain one surface normal; reversing the field reverses the flux sign. For a coil of N turns each linking the same flux Φ, the product $N\Phi$ is called the **flux linkage** 磁链 and is measured in Wb (sometimes written Wb-turns).

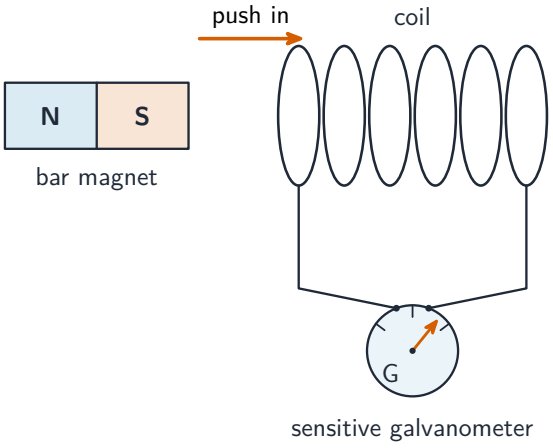
Worked examples

- A coil of area 2.0×10^{-3} m^2 lies at right angles to a field of 0.20 T. $\Phi = 0.20 \times 2.0 \times 10^{-3} = 4.0 \times 10^{-4}$ Wb.
- The coil is turned so its normal is at 60° to the field. $\Phi = 4.0 \times 10^{-4} \times \cos 60° = 2.0 \times 10^{-4}$ Wb.
- If it has 150 turns, the flux linkage at 60° is $150 \times 2.0 \times 10^{-4} = 0.030$ Wb.

AHL: Faraday's law of induction

When the flux through a circuit changes, an emf is **induced** 感应 in it. If the circuit is complete, the emf drives an **induced current** 感应电流. **Faraday's law** 法拉第定律 gives instantaneous emf as the negative derivative of flux linkage. A finite interval gives its **average**:

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Moving a magnet into a coil induces an e.m.f. that drives a current

$$\bar{\varepsilon} = -\frac{\Delta(N\Phi)}{\Delta t}$$

The minus sign is Lenz's law (see below). For magnitudes use $|\bar{\varepsilon}| = |\Delta(N\Phi)|/\Delta t$. For constant N this is $N|\Delta\Phi|/\Delta t$. An average equals the instantaneous emf throughout the interval only for a constant rate of linkage change. Do not multiply by N again if the supplied graph already shows linkage.

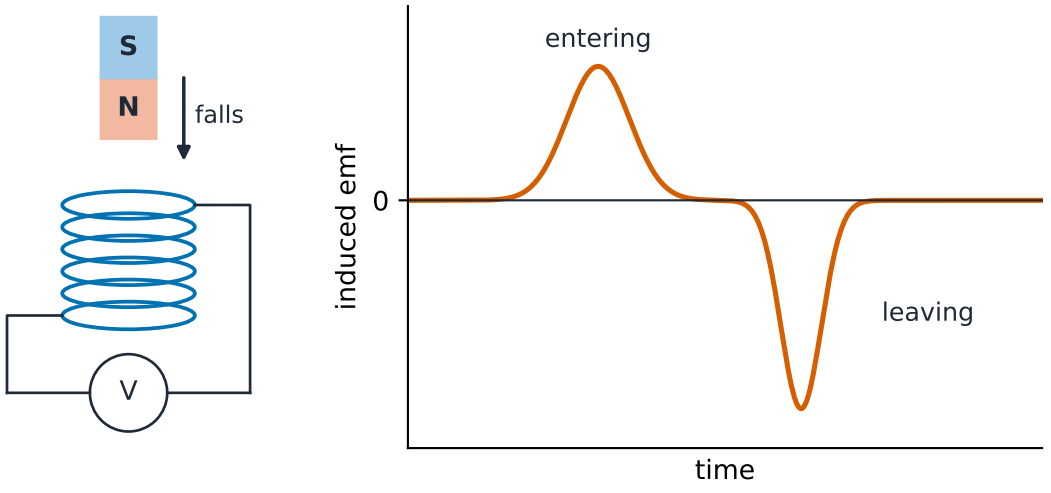
The flux $\Phi = BA\cos\theta$ can change in three ways, and each gives an emf:

- B **changes**: the field through a still coil gets stronger or weaker (a magnet moves near it, or the current in a nearby coil changes).
- A **changes**: the area of the circuit inside the field grows or shrinks (a rod slides along rails, or a coil moves into or out of a field).
- θ **changes**: the coil rotates in the field (a generator).

A steady field through a still coil gives **no** emf, however strong it is. Only change matters.

For a school investigation, connect a coil to a sensitive centre-zero meter. Move the same magnet into and out of the coil, keeping its pole orientation fixed, and record the sign of each deflection. Hold the magnet stationary and record no induced signal. Use approved low-voltage supplies for any powered motor demonstration.

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Illustrative weakly loaded fall: opposite pulses; the faster exit can give a taller pulse

What the graph shows:

- The emf is in **opposite directions** as the magnet enters and leaves: the flux first increases, then decreases.
- In this weakly loaded example the second pulse is **taller and narrower** because the magnet leaves faster. Significant electromagnetic braking or controlled constant-speed motion can change this comparison.
- If initial and final linkage are equal and negligible, the signed total area is zero; the two separate pulse areas then have equal magnitudes. The smooth pulses shown are invented, not measured data.

For a magnet oscillating near a coil, emf depends on speed multiplied by the spatial linkage gradient. Reversing the motion reverses the induced polarity at the same position, but maximum speed alone does not locate every voltage peak.

■ Worked example

A coil of 200 turns has a flux of 4.0×10^{-4} Wb through it. The field is switched off and the flux falls to zero in 0.050 s.

$$|\bar{\varepsilon}| = N\frac{|\Delta\Phi|}{\Delta t} = \frac{200 \times 4.0 \times 10^{-4}}{0.050} = 1.6 \text{ V}$$

Doing the same change in half the time doubles the emf; doubling the number of turns also doubles it.

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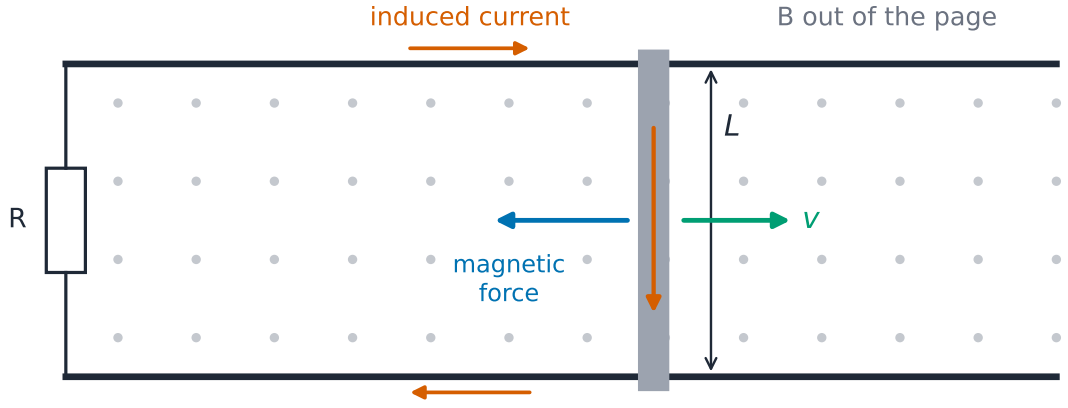
AHL: Emf in a moving straight conductor

For a straight rod, velocity and uniform field mutually perpendicular, with rod length L along the magnetic-force direction, the end-to-end emf magnitude is:

$$\varepsilon = BvL$$

Two ways to see why:

- **Force on charges.** The free electrons in the rod move with it, so it experiences a magnetic force of magnitude evB along the rod. Electrons pile up at one end until the electric field stops more moving. An emf exists between the ends even without a closed circuit; its energy transfer in a loaded circuit requires a source maintaining the motion.
- **Flux cut.** In a time Δt the rod sweeps out an area $Lv\,\Delta t$, so the flux through the circuit changes by $\Delta\Phi = BLv\,\Delta t$, and $\varepsilon = \Delta\Phi/\Delta t = BvL$.



The circuit area grows as the rod moves, so the flux changes and a current flows

When the rails are joined by a resistor R, the current is $I = BvL/R$. In this passive resistive circuit, the current in the rod experiences a force BIL **against** the imposed motion. To keep the rod moving at a steady speed, something must push it with an equal force and do work. Ignoring friction, inductive transients and other energy stores, that work becomes thermal energy in the total circuit resistance: the power is $Fv = BILv = \varepsilon I$.

■ Worked examples

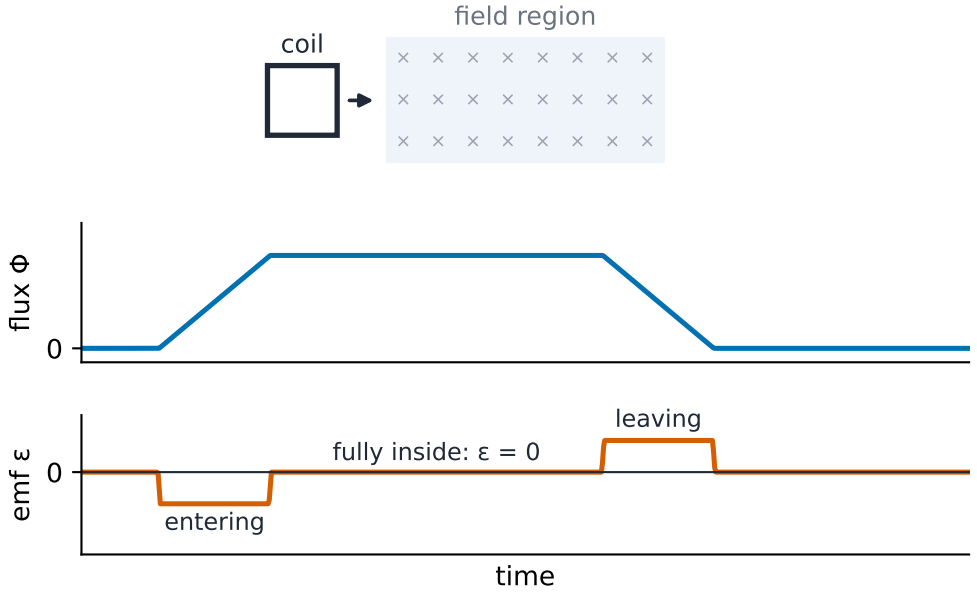
- A rod 0.50 m long moves at 4.0 m s^{-1} across a 0.30 T field. $\varepsilon = 0.30 \times 4.0 \times 0.50 = 0.60$ V. With a 2.0 Ω resistor, $I = 0.30$ A and the opposing force is $0.30 \times 0.30 \times 0.50 = 0.045$ N.
- An aircraft with a wingspan of 60 m flies at 250 m s^{-1} where the vertical part of the Earth's field is 5.0×10^{-5} T. Between the wing tips, $\varepsilon = 5.0 \times 10^{-5} \times 250 \times 60 =$

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0.75 V. In this ideal open-path model, charge separation gives an end-to-end emf but no sustained circuit current. A closed loop moving rigidly through a uniform field can have zero net loop emf; do not treat the wing segment alone as a complete generator circuit.

AHL: Rectangular coils moving into and out of a field

A rectangular coil moving at a steady speed into a region of uniform field has an emf only while the flux through it is changing.



Emf is the negative gradient of flux linkage: constant during ideal entry or exit, zero while fully enclosed

- **Entering**: only the leading side is inside the field, so it acts like the moving rod: $\varepsilon = NBvL$ for a coil of N turns with that side of length L. The flux rises steadily.
- **Fully inside**: both sides cut the field. Their emfs cancel round the loop, and the flux is constant, so $\varepsilon = 0$.
- **Leaving**: only the trailing side is in the field. The flux falls, so the emf has the same size in the opposite direction.

The emf–time graph is the gradient of the flux–time graph, multiplied by $-N$.

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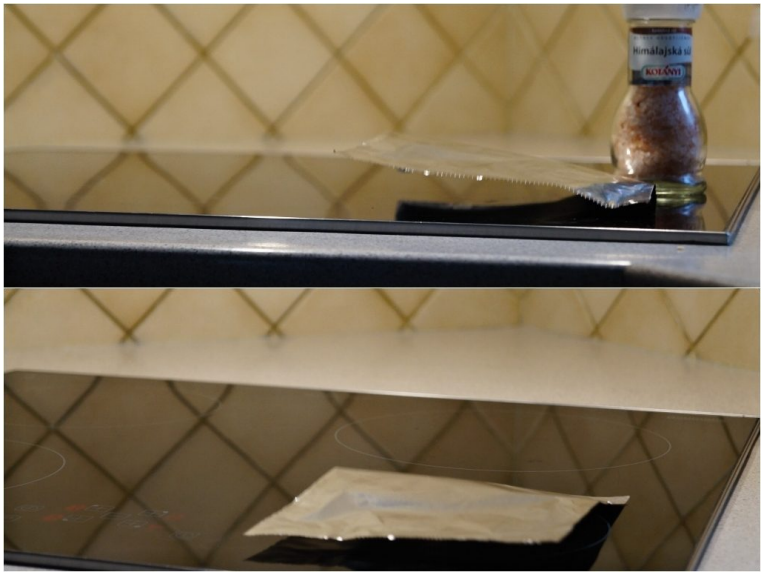
■ **Worked example**

A square coil of 50 turns, side 0.10 m, moves at 2.0 m s^{-1} through a 0.40 T field region 0.30 m wide.

- Entering: $\varepsilon = NBvL = 50 \times 0.40 \times 2.0 \times 0.10 = 4.0$ V, lasting $0.10/2.0 = 0.050$ s.
- Check with Faraday's law: $\Delta(N\Phi) = 50 \times 0.40 \times 0.010 = 0.20$ Wb, and $0.20/0.050 = 4.0$ V.
- Fully inside for $(0.30 - 0.10)/2.0 = 0.10$ s with $\varepsilon = 0$; then 4.0 V the other way for 0.050 s.

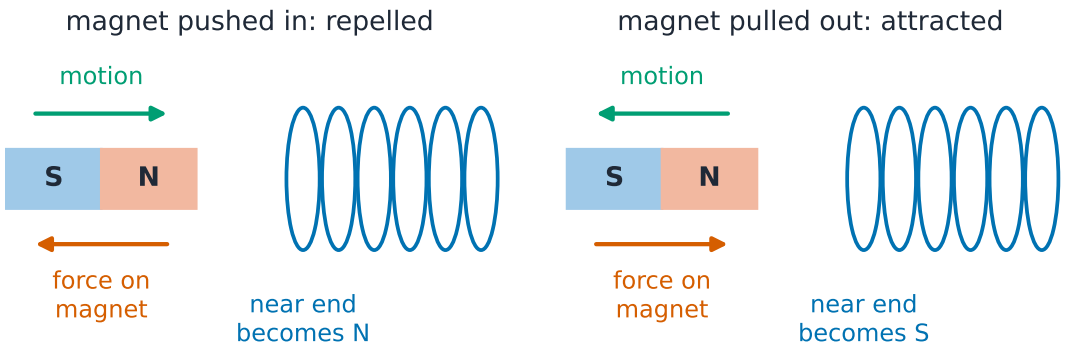
AHL: Lenz's law

Lenz's law 楞次定律: the induced current flows in the direction that **opposes the change** that produced it.



Aluminium foil floating above an induction hob: the changing field induces eddy currents in the foil, and their field repels the coil's (Lenz's law)

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A passive closed coil resists the specified approach or withdrawal

- Push a north pole **towards** a coil: the induced current makes the near end of the coil a **north** pole, which repels the magnet.
- Pull the north pole **away**: the near end becomes a **south** pole, which attracts the magnet back.

To find the direction of the current, decide which pole the near end must be, then use the right-hand grip rule on the coil.

■ Why it must be so: energy conservation

Suppose the induced current helped the change instead. Then a magnet nudged towards a coil would be pulled in faster, which would make a larger current, which would pull harder: kinetic energy and electrical energy would appear from nothing. Lenz's law is energy conservation. In the passive examples here, induced forces resist the imposed motion. Mechanical input supplies transferred electrical energy and any changes in kinetic or magnetic energy. Only in a steady resistive model with negligible other losses or stores does input work equal resistor heating. Lenz polarity applies to emf even when an open circuit prevents sustained current; an induced field can support an existing field that is decreasing.

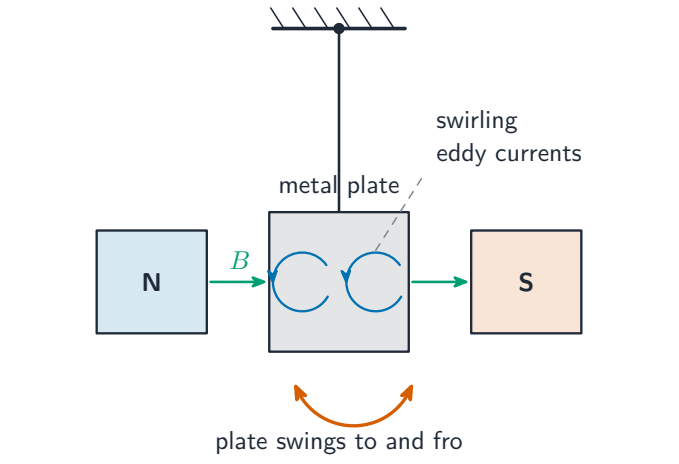
For a finite interval this polarity is represented by $\bar{\varepsilon} = -\Delta(N\Phi)/\Delta t$.

■ Examples of Lenz's law

- **Eddy-current braking.** A metal disc or plate moving through a magnetic field has currents (eddy currents 涡流) induced in it. For the changing-flux geometry, the induced effect opposes motion, so the plate slows down and its kinetic energy becomes thermal energy. Some trains and roller coasters brake this way, with no parts touching.
- **A magnet dropped down a copper pipe** falls slowly: currents induced in the pipe walls oppose its motion.
- **The rod on rails** feels a force against its motion (previous section).

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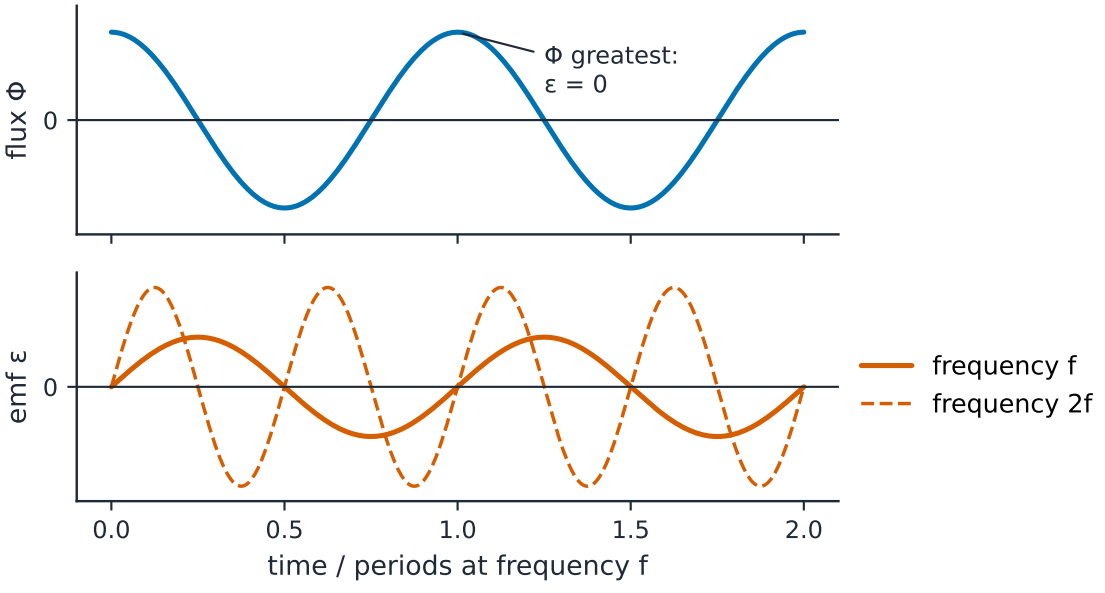
- **A generator** is harder to turn when it is supplying a current than when its circuit is open.



Eddy-current damping

AHL: A coil rotating in a magnetic field

A **generator** 发电机 has a coil turning at a steady rate in a magnetic field. As it turns, the angle θ changes, so the flux $\Phi = BA\cos\omega t$ keeps changing and an alternating emf is induced.



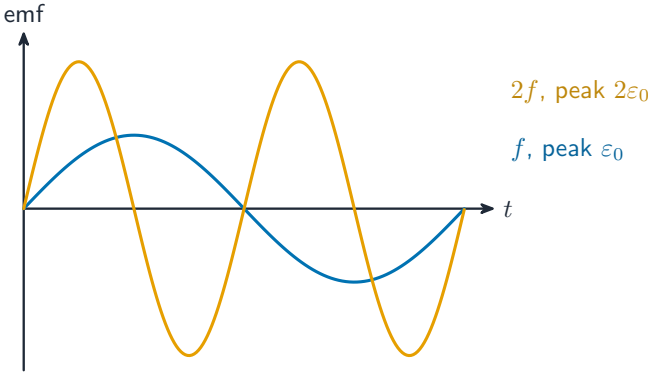
Emf is greatest where the flux changes fastest, not where it is largest

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- When the coil plane is perpendicular to the field, flux is at a positive or negative extremum and its instantaneous rate of change is zero, so emf is **zero**.
- When the plane of the coil is parallel to the field, the flux is **zero** but changing fastest, so the emf is **greatest**.
- The emf graph is the flux graph's gradient (times $-N$): a cosine flux gives a sine emf.

The effect of the rotation frequency

If the coil turns twice as fast:



$\varepsilon_0 = NBA\omega$: more N, B or A raises the peak but not the frequency

Twice as fast: double frequency, double peak

- the **frequency** of the alternating emf doubles (twice as many cycles each second);
- the **peak emf** doubles, because the same change of flux happens in half the time.

For a rectangular coil the peak emf is $\varepsilon_0 = NBA\omega$, where $\omega = 2\pi f$. Differentiating $N\Phi = NBA\cos\omega t$ gives this result and shows that the peak is proportional to the frequency. A bigger field, a larger coil area or more turns also increase the peak emf, but they do not change its frequency.

Worked example

A coil of 100 turns and area 0.010 m^2 turns at 50 revolutions per second in a field of 0.20 T. $\omega = 2\pi \times 50 = 314$ rad s^{-1}, so $\varepsilon_0 = 100 \times 0.20 \times 0.010 \times 314 = 63$ V. At 25 revolutions per second the peak would be 31 V and the frequency 25 Hz.

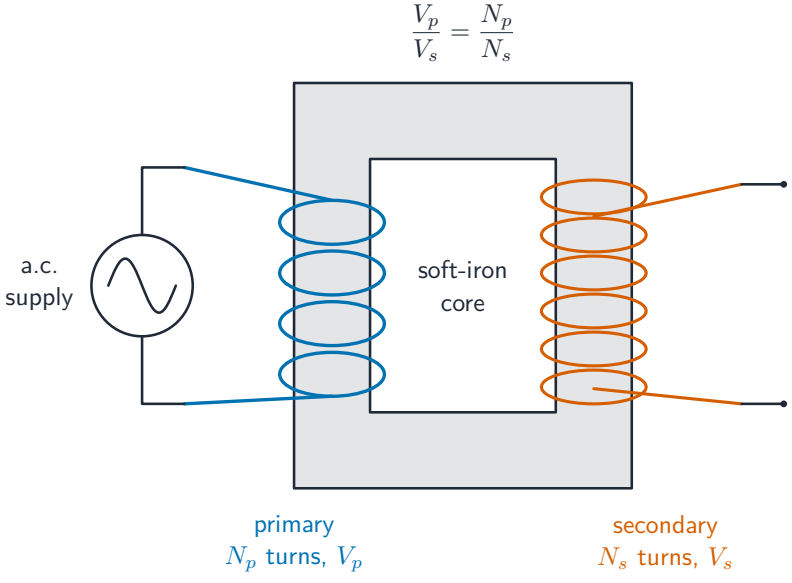
AHL: Changing fields, transformers and self-induction

An emf is induced by a changing field even when nothing moves. A **transformer** 变压器 uses this: an alternating current in the primary coil makes an alternating flux in an iron core, and that changing flux passes through the secondary coil and induces an alternating emf there.

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A substation transformer: changing magnetic flux in coils induces the voltages that power the grid



The changing flux in the core links the two coils

- In the ideal fully linked model, each turn has the same changing flux, so induced emf per turn is equal: $V_s/V_p = N_s/N_p$ for an ideal transformer. Use matching quantities, such as rms voltages of the same waveform, and neglect winding resistance and leakage for terminal-voltage equality.
- A steady **current** makes steady flux, so it gives no continuing secondary emf. Alternating or suitably pulsed current provides changing flux. A maintained DC voltage on an ideal zero-resistance primary would demand continuously changing

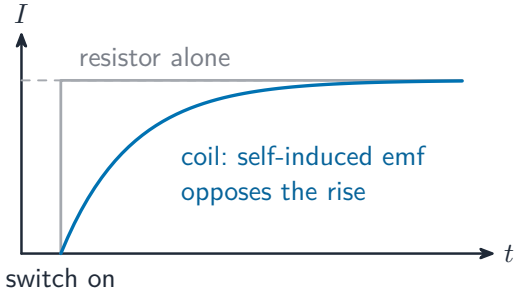
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flux, not an ordinary steady operating state; a real DC-connected winding has transients, resistance and possible core saturation. It is not a suitable substitute for an AC supply.

- Wireless charging uses coupled coils separated by a gap, with driving/control and receiving circuits; alignment, coupling and losses matter.

■ Self-induction (qualitative)

A coil's own changing current changes the flux through **itself**, so it induces an emf in itself. This is **self-induction** 自感. By Lenz's law, this emf opposes the change in current:



A coil delays the rise of current

- when a current is switched on in a coil, the self-induced emf opposes the rise, so the current grows gradually instead of instantly;
- when the current is switched off, the self-induced emf tries to keep it flowing. With many turns and an iron core this emf can be large enough to make a spark across the switch.

Self-induction is required only qualitatively.

Practice

1. **HL.** Define magnetic flux, give its unit and state the uniform-field assumption in $\Phi = BA\cos\theta$. An invented .0050 m^2 coil is in .40 T. Find flux when its normal is at 0°, 30° and 90° to the field.
2. **HL.** State Faraday's law and Lenz's law, distinguishing an average emf from an instantaneous emf.
3. **HL.** In an invented 500-turn coil, flux reverses from $+3.0 \times 10^{-4}$ to -3.0×10^{-4} Wb per turn in .020 s. Find average emf magnitude and explain why the flux change is doubled.
4. **HL.** A magnet falls through a weakly loaded coil, giving opposite voltage pulses, the second taller and narrower. Explain their signs, conditional height comparison and equal area magnitudes if initial and final linkage are equal.

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5. **HL.** An invented .25 m rod moves at 6.0 m/s in .50 T, with rod, velocity and field mutually perpendicular. Rails close it through total resistance 1.5 Ω. Find emf, current, maintaining force and resistor power; explain the energy source.
6. **HL.** An invented square coil has 20 turns, side .050 m and speed .50 m/s through a sharp-edged .80 T region wider than the coil. Find entry emf and its duration, state the fully enclosed emf, and sketch flux and signed emf through entry and exit using a stated normal.
7. **HL.** A north pole is pulled away from a passive closed coil. State its near-face polarity and viewed current direction; explain using Lenz's law and energy conservation. Must an induced field always oppose the original field?
8. **HL.** Explain why eddy currents slow a magnet in a copper pipe, but a comparable plastic pipe lacks this strong braking. State one way to reduce eddy-current loops in a metal plate.
9. **HL.** A coil rotates uniformly in a fixed uniform field. Give its plane orientation when emf is zero and when emf magnitude is greatest. At fixed coil and field, what happens to emf frequency and peak if rotation frequency halves?
10. **HL.** An invented 80-turn coil of area .020 m^2 rotates at 20 revolutions per second in .15 T. Find peak emf.
11. **HL.** An invented ideal transformer has 250 primary turns, 100 secondary turns and 20 V rms input. Find output rms voltage. Explain why a steady primary current gives no continuing secondary emf, while a switching transient can.
12. **HL.** Explain the gradual current rise when a circuit containing resistance and a large coil is switched on, and the possibility of a spark when switched off. Identify the energy store supplying a discharge.
13. **HL.** Invented flux-linkage readings are 0 Wb at 0 s, .12 Wb at .040 s, .12 Wb at .100 s and 0 Wb at .120 s. Assuming linear changes between readings, find signed emf on each interval, and the signed total voltage-time area. The stated positive normal fixes the flux signs.
14. **HL.** The rod in question 5 is disconnected from the resistor while its speed is maintained. What happens to its open-circuit emf and sustained current? Explain why an isolated aircraft wing segment can have an end-to-end emf without constituting a complete generator circuit.

Answers.

1. **HL.** Signed normal field component times area, for a uniform field over a flat area; 1 Wb = 1 T m^2. Fluxes are .0020 Wb, $.0020\cos 30° = .00173$ Wb and zero.
2. **HL.** Instantaneous $\varepsilon = -d(N\Phi)/dt$; average $\bar{\varepsilon} = -\Delta(N\Phi)/\Delta t$. Lenz polarity opposes the change producing induction. A finite-difference average need not equal the local gradient.
3. **HL.** $\Delta\Phi = (-3.0-3.0)\times 10^{-4} = -6.0\times 10^{-4}$ Wb; $|\bar{\varepsilon}| = 500(6.0\times 10^{-4})/.020 = 15$ V. Reversal crosses zero rather than merely reaching it.

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4. **HL.** Linked flux first increases, then decreases, reversing the rate's sign. With weak braking the magnet can leave faster, making a taller, shorter pulse. Equal endpoint linkage gives $\int \varepsilon \, dt = 0$, so the two isolated pulse areas cancel; equal heights are unnecessary.
5. **HL.** $\varepsilon = BvL = .75$ V; $I = .75/1.5 = .50$ A; $F = BIL = .0625$ N, about .063 N. $P = \varepsilon I = I^2R = Fv = .375$ W, about .38 W. The external maintaining force supplies resistor heating in the stated steady ideal model.
6. **HL.** $|\varepsilon| = NBvL = .40$ V during entry for $.050/.50 = .10$ s. Fully enclosed emf is zero. Flux rises linearly, stays constant, then falls linearly; for positive inward normal and inward field the entry emf is negative and exit positive. Exit duration and magnitude equal entry at constant speed.
7. **HL.** Near face south; conventional current clockwise viewed from the withdrawing north pole. Attraction resists withdrawal; external work supplies the transferred energy. The induced field supports the original field as it decreases, so it need not oppose that field itself.
8. **HL.** Changing flux drives conducting loops in copper; their effect opposes the fall and energy is dissipated as heat. Plastic cannot support comparable conduction-current loops. Slots interrupting the paths reduce eddy currents in a metal plate.
9. **HL.** Perpendicular coil plane gives positive or negative flux extrema and zero emf. Parallel plane gives zero flux and greatest emf magnitude. Halving frequency halves both emf frequency and peak; linkage amplitude is unchanged.
10. **HL.** $\omega = 2\pi(20) = 125.66$ rad/s; $\varepsilon_0 = NBA\omega = 80(.15)(.020)(125.66) = 30.16$ V, about 30 V.
11. **HL.** $V_s = 20(100/250) = 8.0$ V rms. Steady current gives constant flux; its time derivative is zero. Switching changes flux and can induce a transient. Do not confuse fixed current with a maintained voltage on an ideal zero-resistance winding.
12. **HL.** Rising current changes self-flux, inducing an emf opposing the rise. At opening, the induced polarity supports the previous current and can create a large voltage across the gap. Stored magnetic energy supplies the subsequent discharge; the coil is not an unlimited source.
13. **HL.** $-.12/.040 = -3.0$ V, zero, then $-(0 - .12)/.020 = +6.0$ V. Areas are $-.12$, 0 and $+.12$ V s, summing to zero. Linkage already includes turns, so do not multiply by N again.
14. **HL.** Ideal open-circuit emf remains .75 V after charge separation; sustained circuit current becomes zero. The wing segment can likewise separate end charges. A complete rigid loop in a uniform field may have cancelling segment emfs, and requires a full circuit analysis rather than one isolated segment.

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Exam tips

- **HL.** Flux uses the angle to the **normal**: $\Phi = BA\cos\theta$. If a question gives the angle between the field and the plane of the coil, use sin instead.
- **HL.** Emf depends on the **rate** of change of flux, not on the size of the flux. A strong steady field induces nothing.

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Structure of the atom

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches E.1 Structure of the atom in full, for the course first assessed in 2025. Sections marked **AHL** are for Higher Level only.

Prerequisites: electric force and Coulomb's law from D.2; electric potential energy (HL) from D.2; kinetic energy and energy conservation from A.3; the electromagnetic spectrum and $c = f\lambda$ from C.2. Use $k = 8.99 \times 10^{9}$ N m^2/C^2, $e = 1.60 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s^{-1} and 1 eV $= 1.60 \times 10^{-19}$ J.

Inside the atom

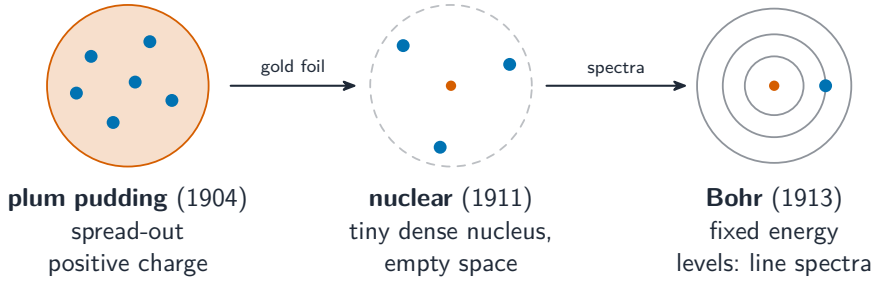
How do you look inside something too small to see?

An atom is of order 10^{-10} m across, too small for ordinary visible light to resolve its internal structure. Geiger and Marsden studied alpha-particle scattering under Rutherford's supervision. Most probes passed through thin foil with little deflection, but a small fraction turned through large angles. This evidence led towards a small, massive **nucleus** 原子核 rather than positive charge and mass spread throughout the whole atom. In the positive-nucleus model a sufficiently close alpha particle is strongly repelled.

The 1909 Geiger–Marsden paper reports about 1 in 8000 reflected under specified conditions using a **platinum** reflector. It also studies gold foils and other materials. Do not transfer that fraction to every gold-foil arrangement: the rate depends on the foil, beam energy and geometry. The apparatus below is a simplified school model, not an exact reconstruction of every historical arrangement.

From the plum-pudding model to the nuclear model

Models of the atom changed as new evidence appeared:



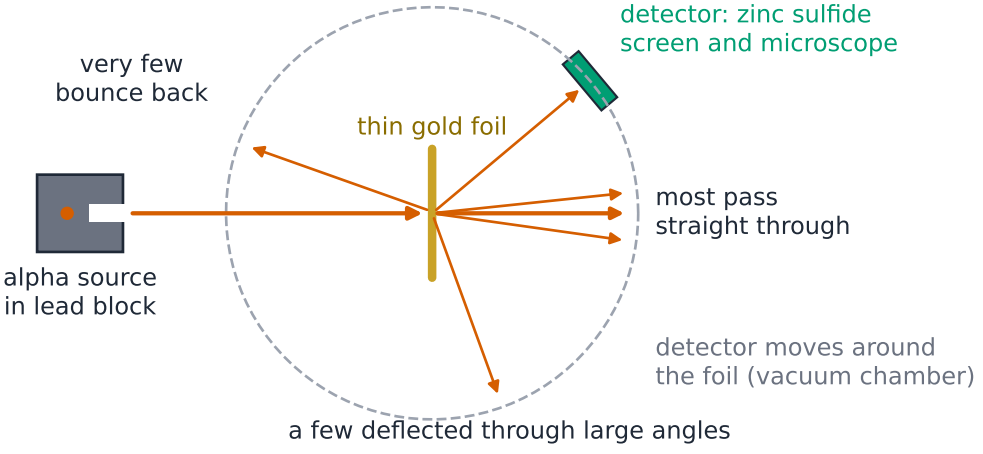
Each model kept what still worked

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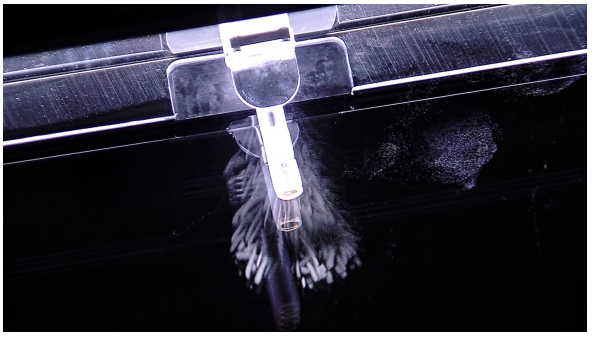
- **Plum-pudding model** 葡萄干布丁模型 (Thomson, 1904): the atom is a sphere of spread-out positive charge with electrons dotted through it. The positive charge and mass are spread over the whole atom, so its electric field is weak everywhere.
- **Nuclear model** 核式模型 (Rutherford, 1911): almost all the mass and all the positive charge are in a tiny nucleus at the centre; the electrons move around it, and most of the atom is empty space.
- **Bohr model** (Bohr, 1913): electrons can only have certain fixed energies, which explains line spectra.

Each model kept what the earlier one got right and changed what the new evidence contradicted.

The Geiger–Marsden–Rutherford experiment



Counting alpha particles at every angle around a thin gold foil



Alpha-particle tracks in a cloud chamber, fanning out from an americium-241 source

The apparatus:

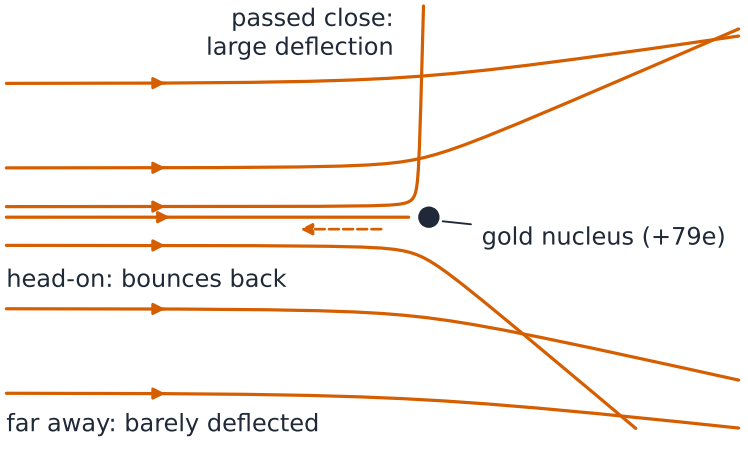
- A narrow beam of **alpha particles** 阿尔法粒子 (helium nuclei, charge $+2e$) from a radioactive source.

- A very thin **gold foil** 金箔, to reduce repeated large deflections and energy loss; most alpha particles do not pass close to any nucleus.
- A small zinc sulfide screen that gives a tiny flash of light when an alpha particle hits it. The flashes were counted by eye through a microscope, at each angle in turn.
- An **evacuated** 抽真空的 chamber, so the alpha particles are not stopped or scattered by air.

Results and what they show

Observation	Conclusion
Almost all alpha particles pass straight through or are deflected by very small angles.	Most of the atom is empty space.
A few are deflected through large angles.	There is a very strong electric field in a small region: the positive charge is concentrated.
A very small fraction return through large angles (more than 90°).	The positive charge and most of the mass are in a tiny, massive nucleus that a fast alpha particle cannot push aside.

The plum-pudding model predicts only tiny deflections, because its spread-out positive charge gives a weak field everywhere. It cannot account for the observed frequency of very large deflections under these conditions.



The closer an alpha particle passes the nucleus, the larger its deflection

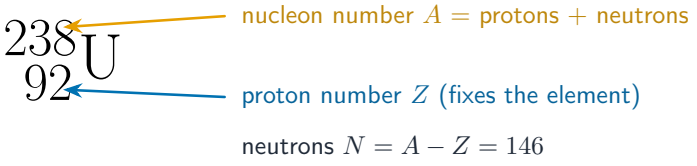
The alpha particles are repelled by the positive nucleus. The force is very large only very close to it, so only the few particles aimed almost straight at a nucleus are deflected strongly. Angular scattering data constrain the small scale of the central charge region. This experiment alone did not establish neutrons or quantised electron

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energy levels. Typical atomic and nuclear linear sizes differ by roughly four to five orders of magnitude, depending on the atom and the radius convention.

Nuclear notation

A **nuclide** 核素 (one particular type of nucleus) is written as



Reading nuclear notation

$$^{A}_{Z}\mathrm{X}$$

- A is the **nucleon number** 核子数 (mass number): the number of protons plus neutrons.
- Z is the **proton number** 质子数 (atomic number): the number of protons. It fixes which element it is.
- X is the chemical symbol. The number of neutrons is $N = A - Z$.

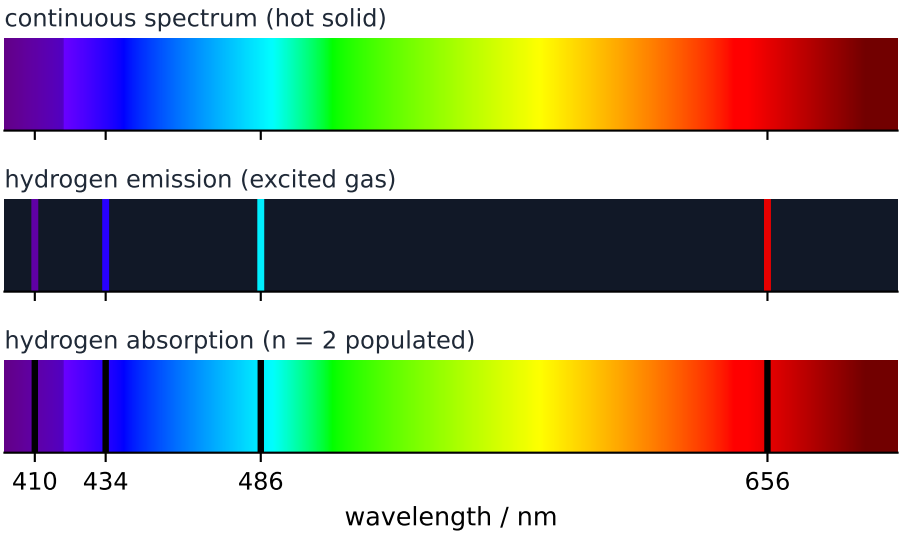
Examples:

- $^{197}_{79}$Au: gold, 79 protons and $197 - 79 = 118$ neutrons. A neutral atom also has 79 electrons.
- $^{4}_{2}$He: an alpha particle, 2 protons and 2 neutrons.
- $^{12}_{6}$C and $^{14}_{6}$C: both carbon, with 6 and 8 neutrons. Nuclei of the same element with different numbers of neutrons are **isotopes** 同位素.

You do not need to remember chemical symbols; they are given in questions.

Emission and absorption spectra

When light from a source passes through a prism or diffraction grating, it spreads into a **spectrum** 光谱. There are three kinds:



The same atomic transitions have matching emission and absorption wavelengths; lower-state populations determine which absorptions appear

- A **continuous spectrum** 连续光谱 contains all wavelengths. It comes from a hot solid or dense gas, such as a lamp filament.
- An **emission spectrum** 发射光谱 is a set of bright lines on a dark background. It comes from a hot, low-pressure gas, for example in a discharge tube.
- An **absorption spectrum** 吸收光谱 is a continuous spectrum crossed by dark lines. It is seen when white light passes through a cooler gas.

For the same atomic transition, emission and absorption correspond to the same level separation and therefore matching wavelengths. Visible line strengths differ with excitation, ionisation, populations and line broadening; an emission line need not appear strongly in every absorption spectrum. In the pictured hydrogen model, the lower $n = 2$ state must be populated for the visible absorptions shown. This is evidence that the electrons in an atom can only have certain **discrete** 分立的 energies, called **energy levels** 能级.

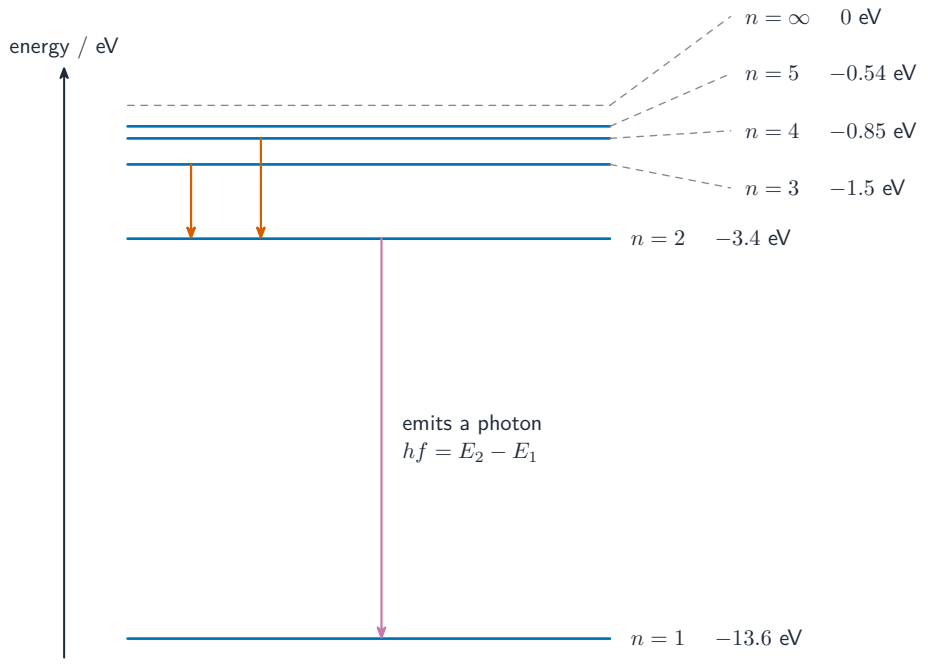
Energy levels and photons

Light is emitted and absorbed in packets of energy called **photons** 光子. The energy of one photon is

$$E = hf = \frac{hc}{\lambda}$$

where h is the Planck constant and f the frequency.

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A downward transition emits one photon carrying exactly the energy difference

- **Emission.** An electron drops from a higher level to a lower one and emits one photon. The photon's energy equals the difference between the levels: $hf = E_{\text{upper}} - E_{\text{lower}}$.
- **Absorption.** An electron absorbs a photon and moves up to a higher level. For idealised bound-to-bound absorption the photon energy must match an allowed level gap and the lower state must be occupied. Above the ionisation threshold a photon can also free the electron, with surplus energy becoming kinetic energy (neglecting recoil); not every absorption is a bound-to-bound line.
- Because the levels are fixed, only certain energy differences are possible, so only certain frequencies appear: a **line spectrum** 线状光谱.
- The **ground state** 基态 is the lowest level. Energies are negative because 0 is defined as the electron just free of the atom. The energy needed to free an electron from the ground state is the **ionisation energy** 电离能: 13.6 eV for hydrogen.

Line darkness depends on optical depth, emission and instrumental resolution. Re-emission in different directions helps reduce the original beam at line wavelengths; absorption lines need not have identical depths or be perfectly black.

■ Worked examples

- An electron in hydrogen falls from -1.51 eV to -3.40 eV. $\Delta E = 1.89$ eV $= 1.89 \times 1.60 \times 10^{-19} = 3.02 \times 10^{-19}$ J. Then $f = E/h = 3.02 \times 10^{-19}/6.63 \times 10^{-34} = 4.56 \times 10^{14}$ Hz and $\lambda = c/f = 6.58 \times 10^{-7}$ m: the red line near 656 nm.

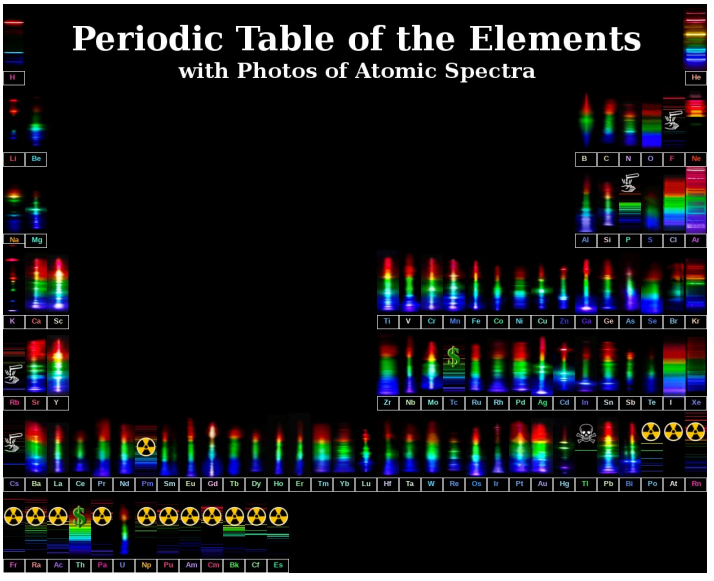
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- The fall from -3.40 to -13.6 eV gives $\Delta E = 10.2$ eV $= 1.63 \times 10^{-18}$ J, so $\lambda = hc/E = 1.22 \times 10^{-7}$ m, in the ultraviolet.
- Across an ensemble with suitable populated levels, four levels can give up to six different downward transitions if all are allowed. One electron in level 4 emits at most three photons in a single descent through these four levels. The possible transitions are 4→3, 4→2, 4→1, 3→2, 3→1 and 2→1: six possible transitions, with distinct line energies only when their gaps differ.

Spectra and chemical composition

Every element has its own set of energy levels, so its line spectrum is unique, like a fingerprint.

For a school comparison, use attributed reference spectra on a common wavelength scale and match several labelled lines. Use teacher-approved light sources and viewing equipment; never look directly at the Sun or another unsafe light source.



Each element's emission lines form its own pattern, so a spectrum identifies the elements in a hot gas or a star

- Comparing the lines in the spectrum of a gas or a star with laboratory spectra shows which elements are present.
- Comparing multiple solar lines with laboratory references identifies elements in the line-forming material. Helium was identified in solar spectra before it was identified in terrestrial samples; the conditions and spectrum type must be considered rather than treating every element as a strong dark line in every solar spectrum.

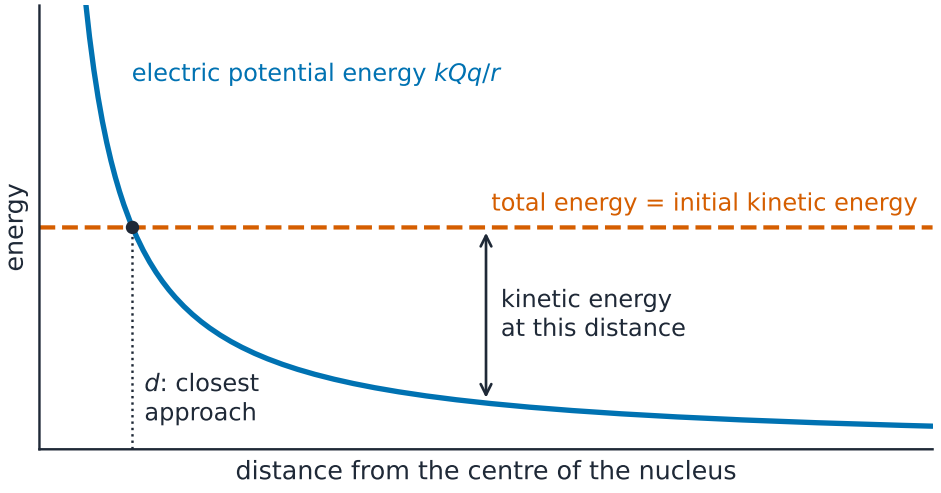
- The same method finds elements in distant stars and gas clouds, and in a laboratory **flame test**. After line identification and calibration, shifts can give radial-motion information under an appropriate Doppler model (C.5); line strength alone is not a direct abundance measurement.

AHL: Distance of closest approach

An alpha particle fired straight at a nucleus slows down as the repulsion grows. It stops at the **distance of closest approach** 最近距离 d, then is pushed straight back. At that point all of its initial kinetic energy has become electric potential energy:

$$E_{\mathrm{k}} = \frac{kQq}{d} \qquad \text{so} \qquad d = \frac{kQq}{E_{\mathrm{k}}}$$

where $Q = Ze$ is the charge of the nucleus and $q = 2e$ that of the alpha particle. This approximation requires a target much heavier than the alpha particle so recoil is small, negligible initial potential energy, and a purely electric interaction outside nuclear range.



At closest approach, all the kinetic energy has become electric potential energy

■ Worked example

A 5.0 MeV alpha particle is fired head-on at a gold nucleus ($Z = 79$).

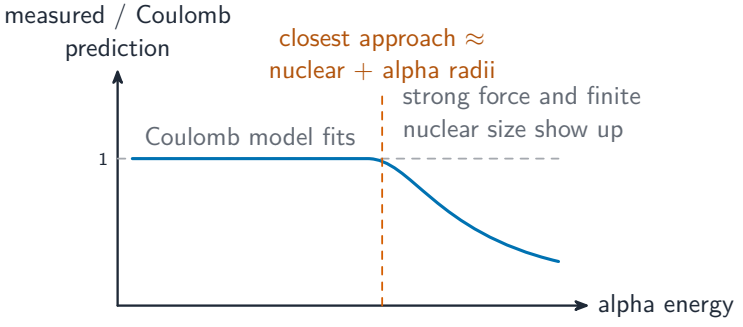
- $E_{\mathrm{k}} = 5.0 \times 10^{6} \times 1.60 \times 10^{-19} = 8.0 \times 10^{-13}$ J.
- $d = \dfrac{8.99 \times 10^{9} \times 79 \times 2 \times (1.60 \times 10^{-19})^{2}}{8.0 \times 10^{-13}} = 4.5 \times 10^{-14}$ m.

The alpha particle never touches the nucleus, so the radius of the gold nucleus must be **less than** 4.5×10^{-14} m. The distance of closest approach gives an upper limit for the nuclear radius. A faster alpha particle, or a nucleus with a smaller charge, gives a smaller d.

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AHL: Deviations from Rutherford scattering

Rutherford's scattering formula assumes that the only force on the alpha particle is the electric repulsion from a point charge. It is a useful model while the probe remains outside the nuclear charge distribution and other interactions are negligible.



Where the point-charge model stops working

At greater probe energies, deviations from the point-charge electric model can reveal finite nuclear charge extent and the short-range **strong nuclear force** 强核力. The change in angular counts need not always be downward; its form depends on energy, target and the scattering process.

Interpreting the onset requires a scattering model. A simple contact-scale estimate compares the centre-to-centre approach distance with the sum of target and alpha radii, rather than treating the alpha particle as a point touching a hard nuclear surface. The closest-approach energy equation applies only while its stated electric, head-on and stationary-target assumptions remain justified.

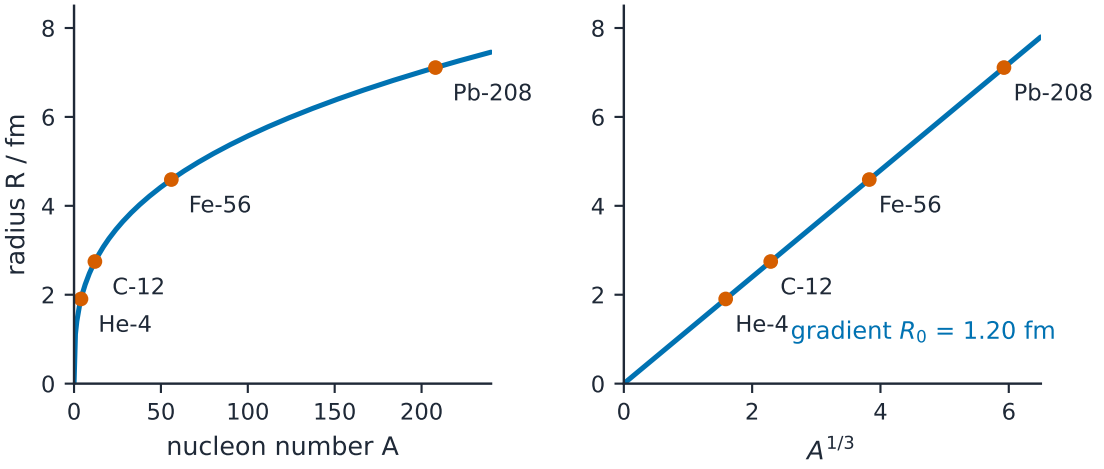
AHL: Nuclear radius and density

Scattering experiments (with alpha particles and, more accurately, with high-energy electrons) support an approximate nuclear-size trend with nucleon number:

$$R = R_0 A^{1/3}$$

where $R_0 = 1.20 \times 10^{-15}$ m (given in the data booklet). A length of 10^{-15} m is called a **femtometre** 飞米 (fm).

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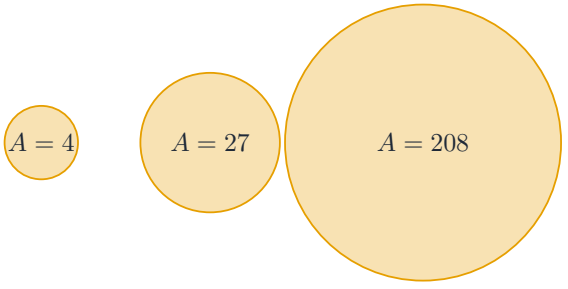


Plotting R against the cube root of A gives a straight line with gradient R_0

- Gold-197: $R = 1.20 \times 10^{-15} \times 197^{1/3} = 1.20 \times 10^{-15} \times 5.82 = 7.0 \times 10^{-15}$ m. This is less than the closest-approach distance of a 5.0 MeV alpha particle, as it must be.
- A nucleus with 8 times as many nucleons has twice the radius, since $8^{1/3} = 2$.

Nuclear density

Each nucleon has a mass of about $m_u = 1.66 \times 10^{-27}$ kg (the atomic mass unit). The volume of a nucleus is $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$. So



$R = R_0 A^{1/3}$, so $V \propto A$: each nucleon takes about the same room
$\rho \approx 2.3 \times 10^{17}$ kg m^{-3} for every nucleus, 10^{14} times water

Every nucleus has about the same density

$$\rho = \frac{Am_u}{\frac{4}{3}\pi R_0^3 A} = \frac{3m_u}{4\pi R_0^3} = \frac{3 \times 1.66 \times 10^{-27}}{4\pi \times (1.20 \times 10^{-15})^3} = 2.3 \times 10^{17} \text{ kg m}^{-3}$$

Implications:

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- A cancels, so **the model predicts approximately common nuclear density**. Light nuclei, binding-energy mass differences, distributions and nonspherical shapes limit this approximation. Nucleons are packed together like balls in a bag, each taking up about the same volume.
- Nuclear matter is about 10^{14} times denser than water. A notional 5 cm^3 sample at this density would have a mass of about 10^{12} kg; this is a dimensional illustration, not a claim that a stable isolated sample can be kept on a desk.
- Since typical atomic and nuclear linear sizes differ by about four to five orders of magnitude, almost all of the volume of ordinary matter is empty space. Neutron stars are made of matter close to nuclear density.

AHL: The Bohr model of hydrogen

In 1913 Niels Bohr proposed a model of the hydrogen atom that explains its line spectrum. The electron moves in a circular orbit around the proton, held by the electric force, but:

- only certain orbits are allowed, and in these orbits the electron does not radiate energy;
- the electron emits or absorbs a photon only when it moves between allowed orbits, with $hf = \Delta E$.

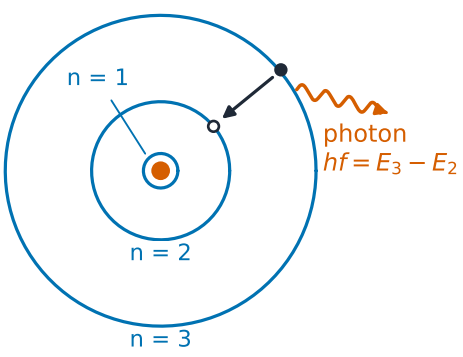
■ Quantised angular momentum

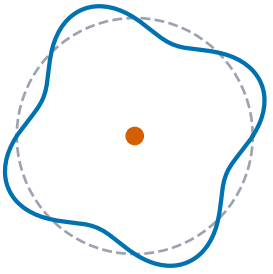
Bohr's key condition is that the **angular momentum** 角动量 mvr of the electron can only be a whole-number multiple of $h/2\pi$:

$$mvr = \frac{nh}{2\pi}$$

where $n = 1, 2, 3, \ldots$ is the **principal quantum number** 主量子数. Combining this condition with the electric force providing the centripetal force gives only certain allowed radii ($r_n = n^2 r_1$, with $r_1 = 5.3 \times 10^{-11}$ m) and therefore only certain energies. **Quantised angular momentum leads to quantised orbits and quantised energy.**

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n = 4: four whole wavelengths fit around the orbit

Allowed orbits: angular momentum is a whole number of $h/2\pi$

A later explanation (E.2): an electron behaves as a wave with wavelength $\lambda = h/mv$. In this historical standing-wave consistency picture, a whole number of wavelengths fits around the circumference: $2\pi r = n\lambda = nh/mv$, which is the same as $mvr = nh/2\pi$.

Worked example: in the ground state, $r = 5.3 \times 10^{-11}$ m, so the speed is $v = \dfrac{h}{2\pi mr} = \dfrac{6.63 \times 10^{-34}}{2\pi \times 9.11 \times 10^{-31} \times 5.3 \times 10^{-11}} = 2.2 \times 10^{6}$ m s^{-1}.

The energy levels

The energy of the electron in orbit n is

$$E = -\frac{13.6}{n^2} \text{ eV}$$

n	**1**	**2**	**3**	**4**	∞
E / eV	−13.6	−3.40	−1.51	−0.850	0

Worked example: the photon emitted in the transition from $n = 3$ to $n = 2$ has energy $13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 1.89$ eV, the red line at 656 nm. The predicted wavelengths agree with the measured hydrogen spectrum, which was the great success of the model.

Limits of the Bohr model

This form gives hydrogen energies. The hydrogen-like extension to other one-electron ions requires a nuclear-charge factor Z^2; the −13.6 eV coefficient is not unchanged for every ion. It cannot explain why some lines are brighter than others, and it describes the electron as a particle in a definite orbit, which quantum physics shows is not correct. It is still used because it gives the right energy levels for hydrogen in a simple way.

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Practice

1. Describe the simplified alpha-scattering experiment, including why the foil is thin and the chamber evacuated. Explain why the historical "1 in 8000" fraction is not a universal gold-foil result.
2. State the main small-angle, large-angle and backward-scattering observations and the limited conclusions they support. Name one atomic property not established by scattering alone.
3. Explain why the spread-out positive charge in the plum-pudding model cannot explain the observed rate of very large deflections.
4. State proton, neutron and electron counts in neutral uranium-235 with $Z = 92$.
5. Compare emission and absorption spectra. Explain why the same transition gives matching wavelengths, but not every emission line must be equally strong in absorption.
6. Find photon energy in eV for wavelength 434 nm. Use the constants in the prerequisites.
7. Invented levels are -5.0, -2.0 and $-.50$ eV. Find the three pair gaps and the highest photon frequency, assuming all three transitions are allowed.
8. Explain how a pattern of laboratory reference lines helps identify sodium in a solar spectrum, and why line strength alone does not directly give abundance.
9. In a counting example, iron-56 has $Z = 26$ and charge 3+. Find proton, neutron and electron counts; distinguish an isotope change from ionisation.
10. An invented atom starts at -5.0 eV with an allowed bound state at -2.0 eV. Explain the different outcomes of absorbing 3.0 eV and 6.0 eV photons, neglecting recoil.
11. Four distinct levels allow all six pair transitions, with different gaps. Compare the number of possible lines across a sample with the maximum photons from one electron descending from level 4 to level 1.
12. **AHL.** A 4.0 MeV alpha particle approaches gold ($Z = 79$) head-on. Find the electric-model turning distance; predict changes for twice the energy and for half the target charge. Explain the radius bound and why high-energy scattering can deviate from the model.
13. **AHL.** Use $R_0 = 1.20$ fm to find aluminium-27 radius and the radius ratio for $A = 216$ versus 27. Derive the approximate nuclear density using $m_u = 1.66\times10^{-27}$ kg and explain its limitations.
14. **AHL.** State Bohr's angular-momentum condition. Use hydrogen $E_n = -13.6/n^2$ eV to find the $4 \to 2$ photon wavelength with the given constants. Given $r_2 = 4r_1$, derive v_2/v_1. State one limit of the model.

Answers.

1. A narrow alpha beam strikes thin foil; a movable scintillation detector counts scattered particles at angles. Thin foil reduces repeated strong encounters and

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energy loss; evacuation reduces air scattering. The 1909 reflected fraction refers to specified platinum-reflector conditions, not every gold foil, energy or detector geometry.

2. Mostly small deflections support a central concentration rather than uniform mass/charge throughout the atom. Rare large-angle returns require a small, strong-field, sufficiently massive centre. The nuclear model fits this evidence; scattering alone did not establish neutrons or discrete electron levels.
3. The diffuse positive charge gives much weaker forces for the probe than a concentrated centre and predicts far too few very large deflections under these conditions.
4. 92 protons, $235 - 92 = 143$ neutrons and 92 electrons.
5. Emission has bright lines from excited atoms; absorption removes light from a background beam. For a particular level pair, matching gaps give matching photon energies and wavelengths. Different populations, temperature, ionisation and broadening change the line strengths and visible line lists.
6. $E = hc/\lambda = (6.63 \times 10^{-34})(3.00 \times 10^{8})/(434 \times 10^{-9}) = 4.583 \times 10^{-19}$ J $= 2.864$ eV, about 2.86 eV.
7. Gaps are 3.0, 1.5 and 4.5 eV. Highest frequency $= (4.5)(1.60 \times 10^{-19})/(6.63 \times 10^{-34}) = 1.086 \times 10^{15}$ Hz.
8. Several matched line positions, including the sodium pattern near 589 nm, support sodium in the line-forming material after calibration and shift checks. Strength also depends on excitation, ionisation, temperature and the radiation-transfer conditions.
9. 26 protons, $56 - 26 = 30$ neutrons and $26 - 3 = 23$ electrons. Isotope change alters neutron number at fixed Z; ordinary ionisation alters electrons.
10. A 3.0 eV photon excites the atom to -2.0 eV. A 6.0 eV photon can ionise from the -5.0 eV state, leaving about 1.0 eV kinetic energy; continuum absorption is not restricted to another bound-level gap.
11. Six different pair gaps can give six lines across the ensemble. One electron emits at most three photons in a descent through four levels, and may emit only one for a direct transition. Cascade photon energies sum to the same total drop.
12. **AHL.** $K = 6.40 \times 10^{-13}$ J; $d = k(79e)(2e)/K = 5.68 \times 10^{-14}$ m. Doubling K halves d; halving Q at fixed K also halves it. Agreement while the probe remains outside supplies an upper scale for nuclear extent, not its exact surface. Recoil, finite probe/target sizes and short-range nuclear interactions limit the electric point-charge model.
13. **AHL.** $R_{27} = 1.20(27^{1/3}) = 3.60$ fm; ratio $= (216/27)^{1/3} = 2$. $\rho = Am_u/[\frac{4}{3}\pi R_0^3 A] = 3m_u/(4\pi R_0^3) = 2.29 \times 10^{17}$ kg/m^3. This is an approximate spherical-size/mass model, with light-nucleus, shape, distribution and binding-mass limitations.
14. **AHL.** $mvr = nh/(2\pi)$, with positive integer n. $\Delta E = 13.6(1/4 - 1/16) = 2.55$ eV $= 4.08 \times 10^{-19}$ J; $\lambda = hc/\Delta E = 4.875 \times 10^{-7}$ m, about 488 nm with these rounded constants (measured line near 486 nm). $v_2/v_1 = (2/1)(r_1/r_2) = 1/2$. Modern

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states are not definite classical orbits; hydrogen-like ions need the Z^2 factor and many-electron spectra need a fuller model.

Exam tips

- Identify the evidence, initial state and model before using an equation.
- Convert photon energy to joules before using h in SI units; distinguish excitation, ionisation and a single cascade.
- **AHL.** State head-on/recoil assumptions, use cube-root radius scaling, and keep hydrogen-specific Bohr constants within their scope.

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Quantum physics

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches E.2 Quantum physics in full, for the course first assessed in 2025. The whole unit is additional higher level (AHL); there is no SL counterpart.

Prerequisites: photons, $E = hf$ and energy levels from E.1; the electronvolt and potential difference from B.5 and D.2; momentum and conservation of momentum and energy from A.2 and A.3; single-slit diffraction from C.3. Use $h = 6.63 \times 10^{-34}$ J s, $e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg and $c = 3.00 \times 10^{8}$ m s^{-1}.

AHL: The photoelectric effect

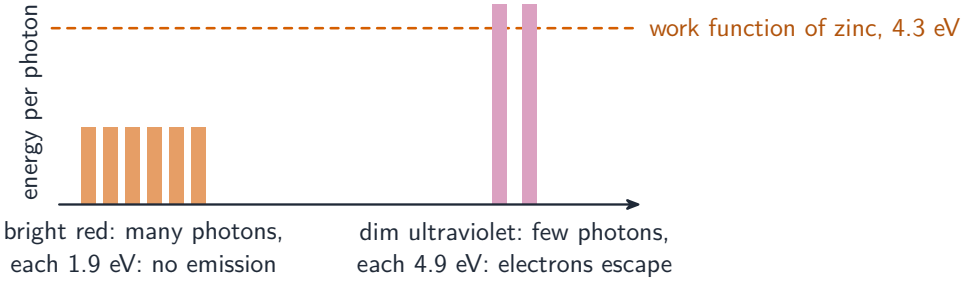
■ Why can a dim ultraviolet lamp do what a bright red lamp cannot?

Put a clean zinc plate on a negatively charged electroscope. Under ordinary single-photon conditions, suitable ultraviolet light can release electrons and discharge it, while bright red light cannot overcome the surface escape threshold. Brightness affects the rate when emission is allowed; it is not irrelevant. The example assumes a clean surface and negligible heating or other discharge mechanisms.



A photomultiplier tube: each photon striking its photocathode can free one electron, which the tube multiplies into a measurable pulse

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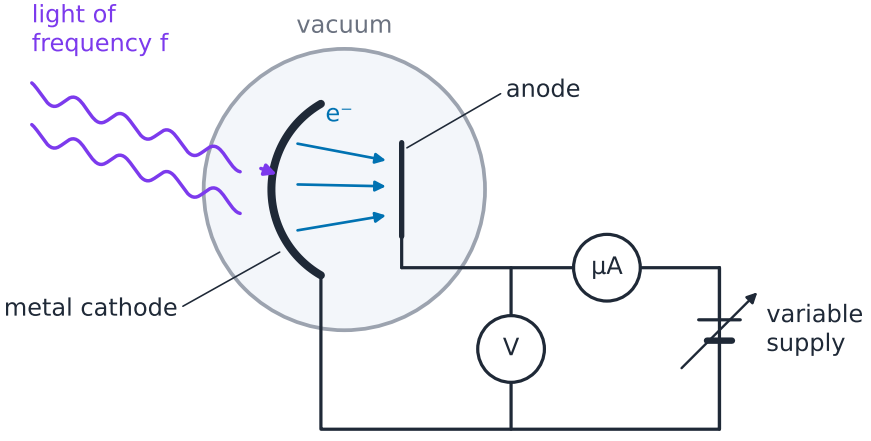


Energy per photon decides emission

A vacuum photocell collects electrons emitted from a surface. Solar cells and many solid-state sensors instead excite carriers within a semiconductor; the Department of Energy's solar-cell explanation describes this internal process. Both involve light transferring energy to electrons, but they are not the same apparatus or escape-barrier model.

What happens

The **photoelectric effect** 光电效应 is the emission of electrons from a metal surface when electromagnetic radiation of high enough frequency falls on it. The emitted electrons are called **photoelectrons** 光电子.



A photocell: light releases electrons from the cathode, and the microammeter measures them

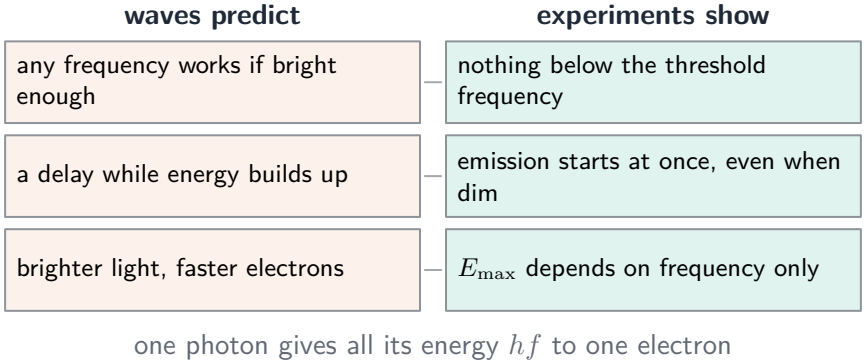
In the ordinary single-photon regime, the experiments show:

- For each metal there is a **threshold frequency** 截止频率 f_0. Below f_0, single-photon emission is absent below threshold. This statement does not cover arbitrarily strong fields or other emission processes.
- Above f_0, there is no measurable long energy-accumulation delay, even with dim light; individual absorption/emission events remain probabilistic.

- The **maximum kinetic energy** of the photoelectrons increases with the frequency of the light, but does **not** depend on its intensity.
- At fixed frequency and surface efficiency, emitted-electron rate scales with intensity. Saturation current is proportional to this rate when essentially all emitted electrons are collected. Intensity itself is energy per unit area per unit time, not a photon count.

AHL: Why the wave theory fails

If light were only a wave, its energy would arrive continuously and spread evenly over the surface. An electron would absorb energy gradually until it had enough to escape.



Wave predictions against the evidence

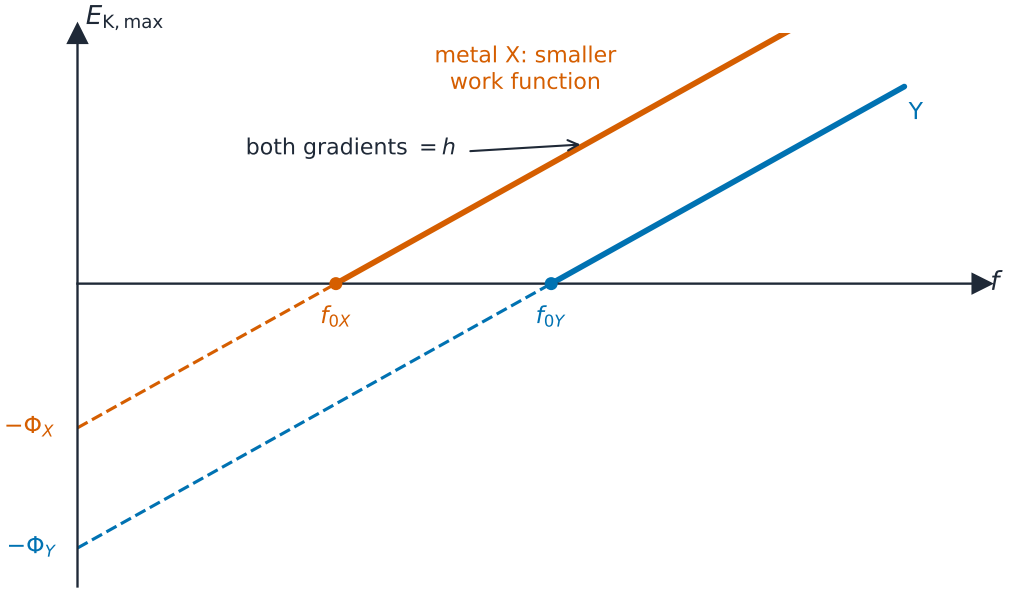
The wave theory predicts	The experiments show
Light of any frequency releases electrons if it is bright enough.	Below the threshold frequency no electrons are released, however bright the light.
With dim light, an electron needs time to absorb enough energy, so there is a delay.	Emission starts immediately, with no measurable delay, even for very dim light.
Brighter light gives the electrons more energy, so $E_{\max}$ increases with intensity.	$E_{\max}$ does not depend on intensity.
Frequency has no special effect on the energy of the electrons.	$E_{\max}$ increases linearly with frequency.

This classical continuous-energy accumulation account fails to explain these observations. Photon energy transfer explains them, while interference and diffraction still require a wave description.

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AHL: Einstein's explanation

In 1905 Einstein proposed that light consists of photons, each with energy $E = hf$, and that:



Two metals on one graph: parallel lines (the gradient is the Planck constant for both), each cutting the frequency axis at its own threshold and the energy axis at minus its own work function

- one photon gives **all** its energy to **one** electron, at once;
- the electron needs a minimum energy, the **work function** 逸出功 Φ of the metal, to escape from the surface;
- any energy left over becomes the kinetic energy of the electron.

For the electrons that escape most easily:

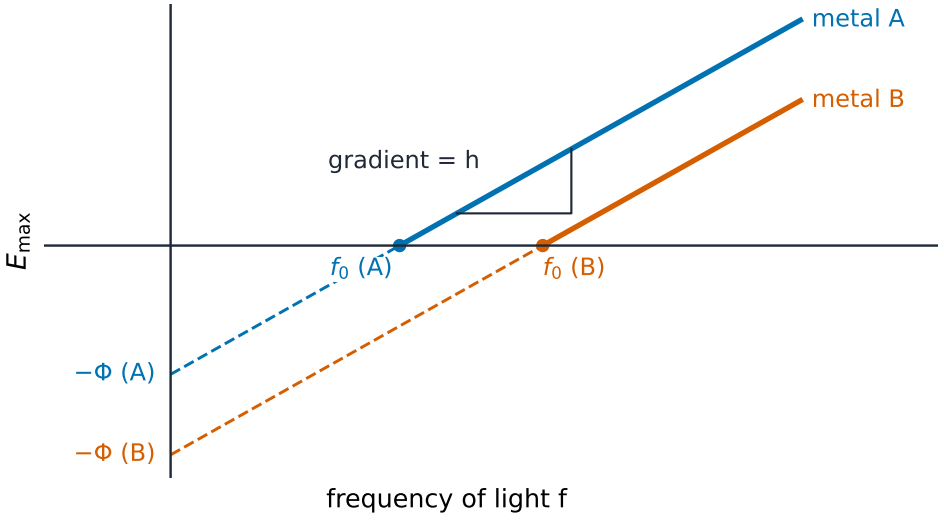
$$E_{\max} = hf - \Phi$$

This explains every observation:

- **Threshold frequency.** If $hf < \Phi$, a photon does not have enough energy to free an electron, in the elementary single-photon model. So this model requires $f \geq f_0$, where $hf_0 = \Phi$.
- **No delay.** Absorption supplies a packet without a long accumulation period; not every incident photon is absorbed or releases a collected electron.
- **Intensity.** At fixed frequency, brighter light means a greater photon arrival rate. With fixed emission efficiency and complete collection, saturation current increases. Individual photon energy stays fixed, so the ideal $E_{\max}$ is unchanged.

- **Frequency.** Each photon's energy hf rises with frequency, so $E_{\max}$ rises linearly with f.

Multiphoton photoemission is observed under specialised conditions; it lies outside these elementary single-photon assumptions. Electrons can start in different states or lose energy before escaping, which is why the equation gives the **maximum** kinetic energy.



Every metal gives a line of gradient h; the intercepts give f_0 and $-\Phi$

On a graph of $E_{\max}$ against f:

- the gradient is h when energy is in joules and frequency in hertz; with energy in eV the numerical gradient is h/e in eV s;
- the intercept on the frequency axis is the threshold frequency f_0;
- the line extended back meets the energy axis at $-\Phi$.

■ Worked example

For this illustrative zinc-surface calculation, take the work function as 4.3 eV. Actual work function depends on surface condition.

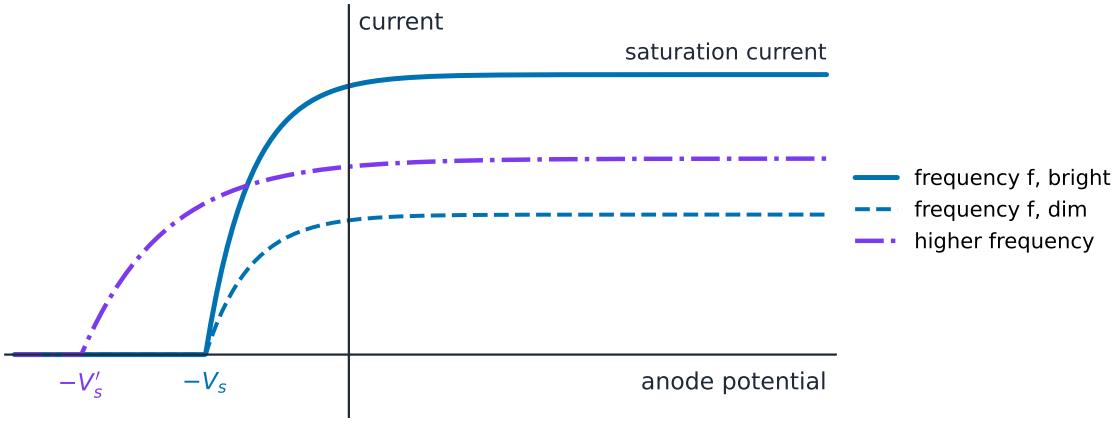
- Threshold frequency: $f_0 = \Phi/h = (4.3 \times 1.60 \times 10^{-19})/(6.63 \times 10^{-34}) = 1.0 \times 10^{15}$ Hz, a wavelength of about 290 nm (ultraviolet).
- Red light of 700 nm has photons of $hc/\lambda = 2.8 \times 10^{-19}$ J $= 1.8$ eV, less than 4.3 eV: no emission, however bright.
- Ultraviolet of 200 nm has photons of 6.2 eV, so $E_{\max} \approx 1.9$ eV. Carrying the unrounded photon energy gives $E_{\max} \approx 3.1 \times 10^{-19}$ J, and the fastest electrons have $v = \sqrt{2E_{\max}/m_e} = 8.2 \times 10^5$ m s^{-1}.

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AHL: Measuring the maximum kinetic energy

In the photocell circuit, the supply can be reversed so that the anode is **negative**. The electrons then slow down as they cross. The **stopping potential** 遏止电压 V_s is the smallest reverse potential difference that stops even the fastest electrons, so the current falls to zero:

$$eV_s = E_{\max}$$



Intensity changes the current; frequency changes the stopping potential

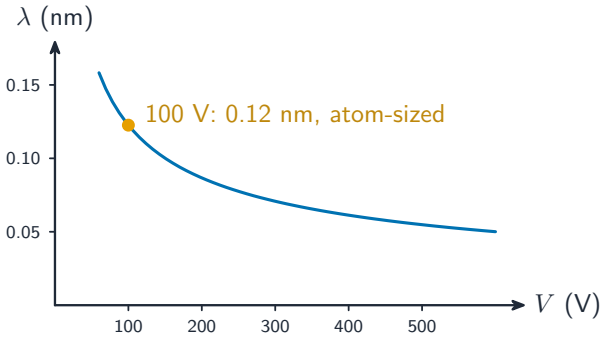
- **Brighter light, same frequency**: larger saturation current (more electrons per second), same stopping potential (same $E_{\max}$).
- **Higher frequency**: larger stopping potential.
- Combining the equations gives $V_s = \frac{h}{e}f - \frac{\Phi}{e}$, so a graph of V_s against f has gradient h/e. This is the principle of Millikan's 1916 photoelectric determination; real measurements require attention to surface and voltage offsets.

Worked example: for ultraviolet of 200 nm on zinc, $E_{\max} = 1.9$ eV, so the stopping potential is 1.9 V.

AHL: Matter waves and the de Broglie wavelength

In 1924 Louis de Broglie suggested that if light waves can behave as particles, then particles such as electrons might behave as waves. Every particle with momentum p has a **de Broglie wavelength** 德布罗意波长:

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$\lambda = h/\sqrt{2m_eeV}$: quadruple V to halve λ (non-relativistic)

Faster electrons, shorter waves

$$\lambda = \frac{h}{p}$$

For massive particles in the non-relativistic model, $p = mv$, giving $\lambda = h/(mv)$. High-energy electrons need relativistic momentum; the basic h/p relation remains applicable.

A photon also has momentum, $p = h/\lambda$, although its rest mass is zero, with $p = E/c$ in vacuum.

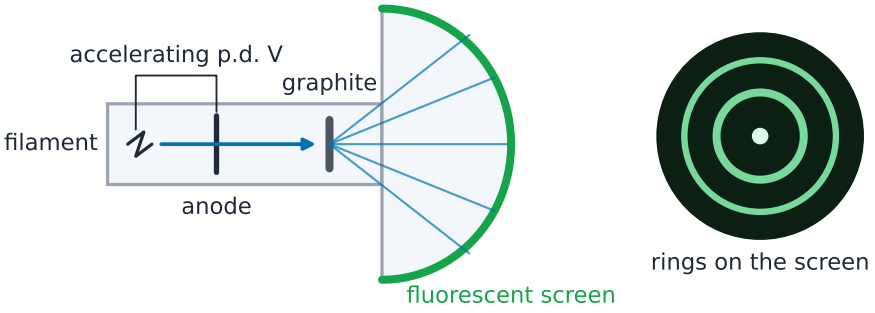
For a non-relativistic electron accelerated from rest through a positive voltage magnitude V: $\frac{1}{2}mv^2 = eV$, so $p = \sqrt{2m_eeV}$ and $\lambda = \dfrac{h}{\sqrt{2m_eeV}}$.

■ Worked examples

- An electron accelerated through 100 V: $p = \sqrt{2 \times 9.11 \times 10^{-31} \times 1.60 \times 10^{-19} \times 100} = 5.4 \times 10^{-24}$ kg m s^{-1}, so $\lambda = 6.63 \times 10^{-34}/5.4 \times 10^{-24} = 1.2 \times 10^{-10}$ m. This is about the spacing of atoms in a crystal, so a crystal can diffract these electrons.
- A tennis ball of mass 0.058 kg at 50 m s^{-1}: $\lambda = 6.63 \times 10^{-34}/(0.058 \times 50) = 2.3 \times 10^{-34}$ m. The wavelength is far too small for appreciable diffraction through an ordinary doorway; environmental decoherence adds a further practical limitation. This is not a fundamental absence of a wave description for macroscopic matter.

AHL: Diffraction of particles

Diffraction is a property of waves, so particle diffraction is evidence for the wave nature of matter. Electron diffraction was first seen in 1927, by Davisson and Germer in the United States and by G. P. Thomson in Britain. Neutrons, atoms and even large molecules have since been diffracted.



Electrons passing through graphite make rings, just as X-rays do

The electron diffraction tube:

- Electrons are accelerated through a few thousand volts and pass through a very thin layer of graphite.
- The screen shows **bright rings**, not just a spot. The rings arise from electron-wave scattering by regular atomic planes across many orientations in polycrystalline graphite. A suitably oriented single crystal can instead give spots.
- When the accelerating voltage is **increased**, the electrons have more momentum, so a **shorter** wavelength, and the rings get **smaller**. This is exactly what $\lambda = h/p$ predicts.

■ The position of a minimum

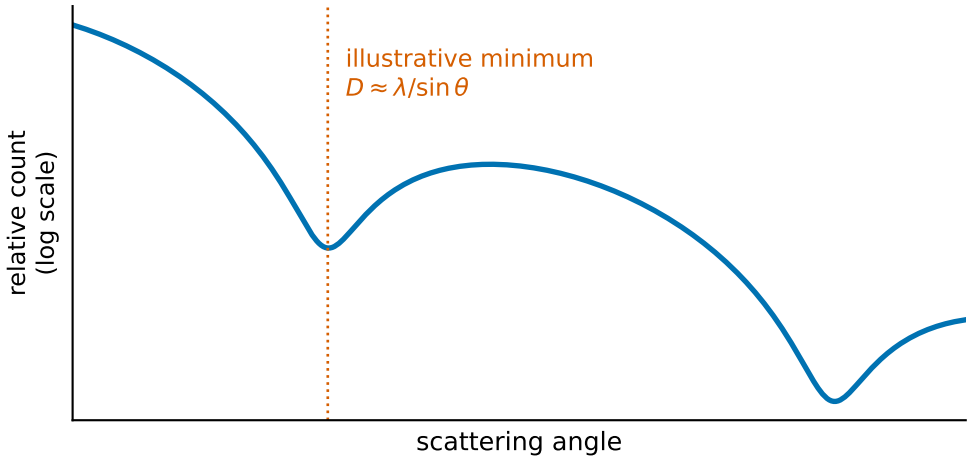
For a coherent beam through an ideal single slit of width b, the first-minimum condition is $b\sin\theta = \lambda$; for small angles, $\theta \approx \lambda/b$. A simplified nuclear-size estimate may be supplied in the analogous form

$$\sin\theta \approx \frac{\lambda}{b}$$

Use the stated geometry and angle convention. This nuclear-size analogy is approximate rather than an exact universal first-minimum law for every nuclear shape.

High-energy electrons scattered by nuclei show the same effect. Scattering can show diffraction minima related to nuclear size and charge distribution. The estimate $D \approx \lambda/\sin\theta$ uses a stated simplified model; detailed analysis uses momentum transfer and a nuclear form factor. High-energy probe momentum must be treated relativistically or supplied as a wavelength.

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An invented scattering trend marks a first minimum for a stated approximate nuclear-size estimate

Worked example: electrons with a de Broglie wavelength of 3.0×10^{-15} m are scattered by carbon nuclei, and the first minimum is at 35°. $D \approx \lambda / \sin\theta = 3.0 \times 10^{-15} / \sin 35° = 5.2 \times 10^{-15}$ m, close to the value from $R = R_0 A^{1/3}$ for carbon-12 ($2R = 5.5 \times 10^{-15}$ m). Electrons avoid the strong nuclear interaction of an alpha probe but still interact electromagnetically with the nuclear charge.

AHL: Wave–particle duality

Both light and matter show wave behaviour and particle behaviour. This is **wave–particle duality** 波粒二象性.

	Wave behaviour	**Particle behaviour**
Light	Diffraction, interference, polarisation	Photoelectric effect, Compton scattering
Electrons	Electron diffraction	Deflection in fields, a charge and a mass, single dots on a screen

Which behaviour we see depends on the experiment. In an electron double-slit experiment with a very weak beam, each electron arrives at the screen as a single dot (a particle), but after many electrons the dots build up an interference pattern (a wave). Neither picture alone describes what an electron or a photon is.

AHL: Compton scattering

When X-rays are scattered by a material with loosely bound electrons, such as graphite, the scattered X-rays have a **longer wavelength** than the incident ones. This is **Compton scattering** 康普顿散射 (Arthur Compton, 1923).



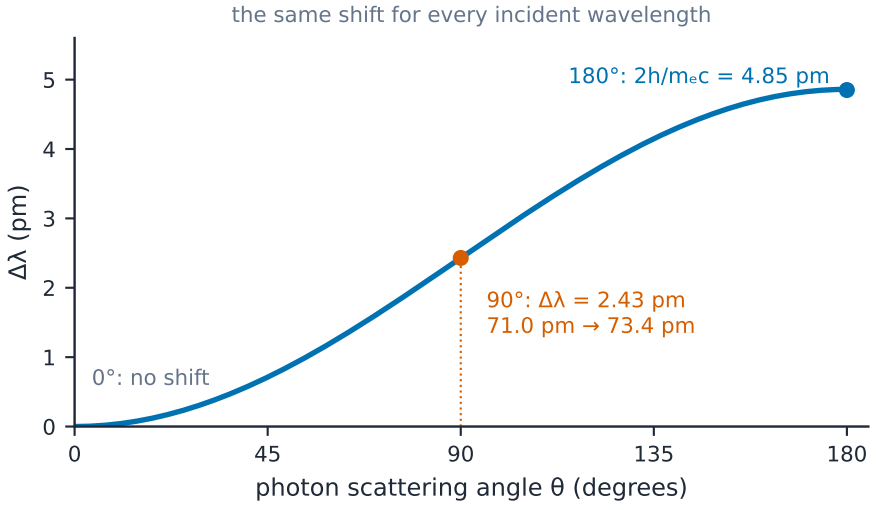
A photon collides with an electron, gives it energy, and leaves with a longer wavelength

Compton explained it as a collision between a photon and an electron, conserving both energy and momentum. The photon gives some of its energy and momentum to the electron, which recoils; the scattered photon has less energy, so a lower frequency and a longer wavelength. For an initially resting free electron, or negligible binding effects, the wavelength increase depends on the photon scattering angle θ:

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta)$$

Here θ is the photon deflection angle, not the electron recoil angle. The formula assumes an initially stationary electron and neglects binding effects.

- $\dfrac{h}{m_e c} = \dfrac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.00 \times 10^{8}} = 2.43 \times 10^{-12}$ m.
- At $\theta = 0$ there is no shift. The largest shift, $2h/m_e c = 4.85 \times 10^{-12}$ m, is when the photon bounces straight back ($\theta = 180°$).
- The shift is the same for every incident wavelength, so its fractional effect is much greater for short wavelengths such as X-rays and gamma rays. For visible light it is about one part in 10^5.



$\Delta\lambda$ rises from 0 to $2h/m_e c$ as the photon turns from 0° to 180°

Worked example

X-rays of wavelength 7.10×10^{-11} m are scattered through 90°.

- $\Delta\lambda = 2.43 \times 10^{-12} \times (1 - \cos 90°) = 2.43 \times 10^{-12}$ m, so $\lambda_f = 7.34 \times 10^{-11}$ m.
- Energy lost by the photon, which becomes electron recoil kinetic energy in the free-electron model: $\Delta E = hc(1/\lambda_i - 1/\lambda_f) \approx 9.26 \times 10^{-17}$ J, about 9.3×10^{-17} J or 580 eV. Keep the unrounded outgoing wavelength and photon energies until after subtraction: subtracting the displayed rounded energies 2.80×10^{-15} J and 2.71×10^{-15} J loses useful digits.

Why Compton scattering is strong evidence for photons

- A wave theory predicts that the electrons would oscillate at the frequency of the incident wave and re-emit radiation at the **same** wavelength. The measured shift cannot be explained this way.
- The shift is explained exactly by treating the photon as a particle with momentum $p = h/\lambda$ that collides with an electron like one ball hitting another. Unlike a ball, the photon does not slow down: it travels at c between interactions in vacuum, and loses energy by changing its wavelength.
- The photoelectric effect shows that light energy arrives in packets of hf. Compton scattering also shows that photons carry **momentum**, providing a quantitative momentum-transfer test in addition to photoelectric energy transfer. The quantum description must also explain wave interference and diffraction.

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Practice

1. **HL.** Describe the photoelectric effect and state what is meant by the threshold frequency. State three observations of the photoelectric effect that cannot be explained by the wave theory of light, and explain how the photon model explains each.
2. **HL.** Assume an illustrative sodium-surface work function of 2.3 eV. Find its threshold frequency and wavelength.
3. **HL.** Light of wavelength 400 nm falls on sodium. Find (a) the maximum kinetic energy of the photoelectrons in eV, (b) the stopping potential, (c) the maximum speed of the photoelectrons.
4. **HL.** A graph of $E_{\max}$ against frequency for a metal meets the frequency axis at 5.0×10^{14} Hz. Find the work function in J and in eV, and state what the gradient of the graph represents.
5. **HL.** The intensity of the light on a photocell is doubled while its frequency stays the same. State and explain the effect on (a) the saturation current, (b) the stopping potential. Assume single-photon emission, unchanged surface efficiency and complete collection.
6. **HL.** Find the de Broglie wavelength of an electron accelerated from rest through 150 V.
7. **HL.** Explain why a tennis ball moving at 50 m s^{-1} does not show diffraction when it passes through a doorway.
8. **HL.** An electron and a proton have the same kinetic energy. Find the ratio of their de Broglie wavelengths ($m_p = 1.67 \times 10^{-27}$ kg). Assume both particles are non-relativistic.
9. **HL.** Using the supplied approximate model $D \approx \lambda / \sin\theta$, estimate nuclear diameter for probe wavelength 3.0×10^{-15} m and a first minimum at 30°. State why this is a model-dependent estimate. In an electron diffraction tube, the accelerating voltage is increased. State and explain what happens to the rings on the screen.
10. **HL.** Gamma rays of wavelength 5.0×10^{-12} m are scattered through 60° by electrons. Find the wavelength of the scattered photons.
11. **HL.** Find the largest possible Compton shift, and explain why Compton scattering has a very small fractional shift in an ordinary demonstration for visible light of wavelength 500 nm.
12. **HL.** Find the momentum of a photon of wavelength 500 nm.
13. **HL.** Explain why Compton scattering is regarded as stronger evidence for the particle nature of light than the photoelectric effect.
14. **HL.** State what is meant by wave–particle duality, and give one piece of evidence for the wave nature and one for the particle nature of electrons.

Answers.

1. **HL.** Electrons are emitted from a metal surface when electromagnetic radiation

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falls on it. The threshold frequency is the minimum frequency of radiation that releases electrons from that metal ($hf_0 = \Phi$). Any three of: (i) no emission below f_0 however bright – one photon must have energy at least Φ, within the elementary single-photon model; multiphoton processes are outside that regime; (ii) emission is immediate – the energy arrives in one packet; (iii) $E_{\max}$ is independent of intensity – intensity changes the number of photons, not their energy; (iv) $E_{\max}$ increases with frequency – each photon's energy hf increases.

2. **HL.** $f_0 = (2.3 \times 1.60 \times 10^{-19})/(6.63 \times 10^{-34}) = 5.6 \times 10^{14}$ Hz; $\lambda_0 = c/f_0 = 5.4 \times 10^{-7}$ m (540 nm).
3. **HL.** (a) $hc/\lambda = 4.97 \times 10^{-19}$ J $= 3.1$ eV, so $E_{\max} = 3.1 - 2.3 = 0.8$ eV. (b) 0.8 V. (c) $v = \sqrt{2 \times 0.81 \times 1.60 \times 10^{-19}/9.11 \times 10^{-31}} = 5.3 \times 10^5$ m s^{-1}.
4. **HL.** $\Phi = hf_0 = 6.63 \times 10^{-34} \times 5.0 \times 10^{14} = 3.3 \times 10^{-19}$ J $= 2.1$ eV; the gradient is the Planck constant h.
5. **HL.** (a) Saturation current doubles under the stated assumptions: twice the fixed-frequency photon arrival rate gives twice the emitted and collected electron rate. (b) Stopping potential is unchanged because individual photon energy and maximum electron energy are unchanged.
6. **HL.** $p = \sqrt{2 \times 9.11 \times 10^{-31} \times 1.60 \times 10^{-19} \times 150} = 6.6 \times 10^{-24}$ kg m s^{-1}; $\lambda = h/p = 1.0 \times 10^{-10}$ m.
7. **HL.** Assuming a mass of about 0.06 kg, $\lambda = h/mv \approx 2 \times 10^{-34}$ m. For an ordinary metre-wide doorway, the characteristic diffraction angle would be of order 10^{-34} rad and unresolvable in that setup. This is an observability limit, not a fundamental absence of matter waves.
8. **HL.** $\lambda = h/\sqrt{2mE_k}$, so $\lambda_e/\lambda_p = \sqrt{m_p/m_e} = \sqrt{1.67 \times 10^{-27}/9.11 \times 10^{-31}} = 43$: the electron's wavelength is 43 times longer.
9. **HL.** $D \approx \lambda/\sin\theta = 3.0 \times 10^{-15}/0.50 = 6.0 \times 10^{-15}$ m. The coefficient and angle relation depend on the scattering geometry and charge distribution; this is not a universal exact nuclear formula. The rings become smaller. The electrons have more momentum, so their de Broglie wavelength is shorter, and they are diffracted through smaller angles.
10. **HL.** $\Delta\lambda = 2.43 \times 10^{-12} \times (1 - \cos 60°) = 1.2 \times 10^{-12}$ m; $\lambda_f = 6.2 \times 10^{-12}$ m.
11. **HL.** At 180°, $\Delta\lambda = 2h/m_e c = 4.85 \times 10^{-12}$ m. For 500 nm light this is a fractional change of about 10^{-5}, difficult to resolve in an ordinary visible-light demonstration, rather than fundamentally undetectable.
12. **HL.** $p = h/\lambda = 6.63 \times 10^{-34}/5.00 \times 10^{-7} = 1.33 \times 10^{-27}$ kg m s^{-1}.
13. **HL.** The wave theory predicts scattered radiation of unchanged wavelength, but the measured shift is explained by the stated initially resting free-electron collision model in which the photon carries momentum h/λ as well as energy, and both are conserved. The photoelectric effect shows that energy is transferred in packets; Compton scattering shows that photons also carry momentum, like particles.

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14. **HL.** Light and matter can each behave as waves or as particles, depending on the experiment. Wave: electron diffraction by graphite (or by nuclei). Particle: electrons have a definite charge and mass, are deflected by fields, and arrive as single dots on a screen.

Exam tips

- **HL.** In $E_{\max} = hf - \Phi$, put both energies in joules, or both in eV; $E = hf$ gives joules.
- **HL.** At fixed frequency and collection conditions, intensity controls photon rate; frequency controls individual photon energy.
- **HL.** State the single-photon regime and the relevant surface work function.
- **HL.** For an accelerated electron, find $p = \sqrt{2m_e eV}$ first, then $\lambda = h/p$.
- **HL.** Use the stated diffraction geometry and a momentum model appropriate to the electron energy.
- **HL.** Compton: the shift depends only on the angle, not on the wavelength; the scattered photon has a **longer** wavelength.

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Radioactive decay

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

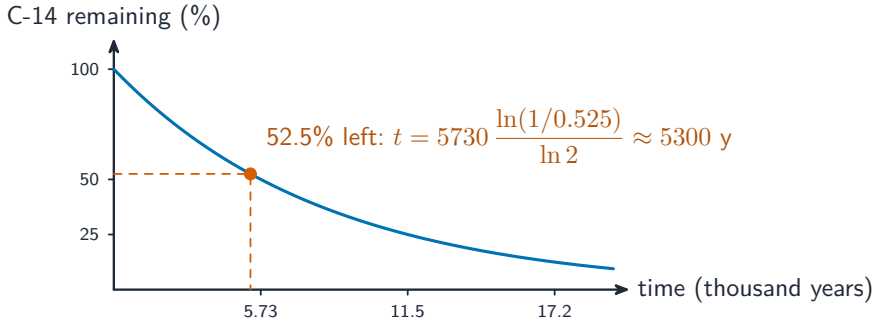
This handout teaches E.3 Radioactive decay in full, for first assessment 2025. Sections marked **AHL** are for Higher Level only.

Prerequisites: nuclear notation and the electronvolt from E.1, energy conservation from A.3, and Coulomb's law plus exponentials and logarithms for the additional higher-level work. Use $c = 3.00 \times 10^{8}$ m s^{-1}, 1 u $= 1.661 \times 10^{-27}$ kg $= 931.5$ MeV c^{-2}, proton mass 1.007276 u and neutron mass 1.008665 u.

Isotopes and the strong nuclear force

How old is the Iceman?

Ötzi, the body discovered in the Alps in 1991, illustrates how nuclear measurements can date once-living material. Real dating includes sample preparation, contamination tests and calibration for past atmospheric carbon-14 variation. A radiocarbon remeasurement study reports calibrated prehistoric date intervals; a classroom halving calculation is not the complete analysis.



Reading an age off the decay curve

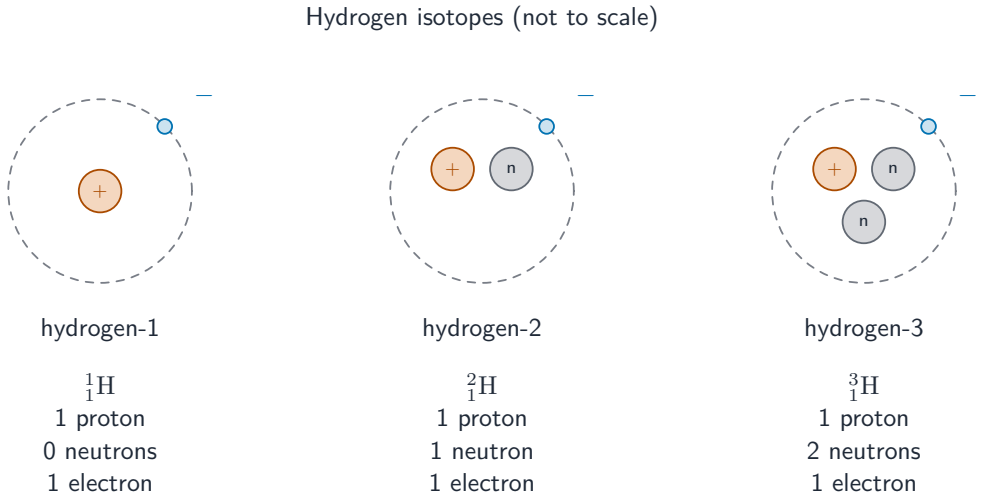
Our separate **invented dating example** has 52.5% of its assumed initial carbon-14 remaining. We will calculate its idealised decay age, without presenting that percentage as a measurement of Ötzi. Random nuclear events can support a statistical clock when the initial conditions and model are justified.

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Isotopes

Isotopes 同位素 are nuclides with the same number of protons (same element) but different numbers of neutrons. They have very similar chemical behaviour because their electronic structures are similar, but different masses and often different nuclear stability. Mass-dependent isotope effects mean their chemistry is not exactly identical.

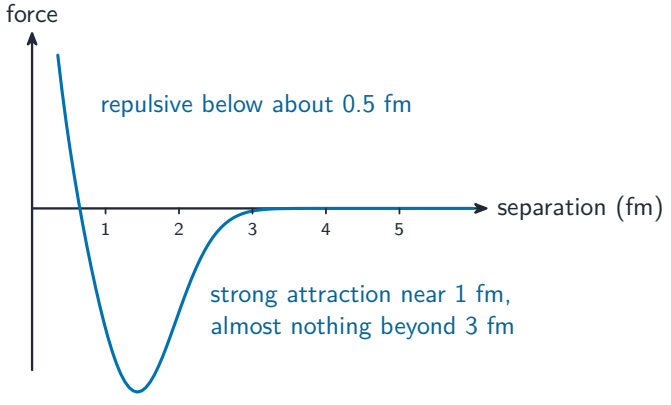
- Carbon-12 ($^{12}_{6}\mathrm{C}$: 6 protons, 6 neutrons) is stable; carbon-14 ($^{14}_{6}\mathrm{C}$: 6 protons, 8 neutrons) is radioactive.
- Chlorine-35 and chlorine-37 are both stable; natural chlorine is a mixture.



The three isotopes of hydrogen: the same one proton, different numbers of neutrons

The strong nuclear force

The protons in a nucleus repel each other strongly, yet the nucleus holds together. Something must attract the nucleons. This is the **strong nuclear force** 强核力:



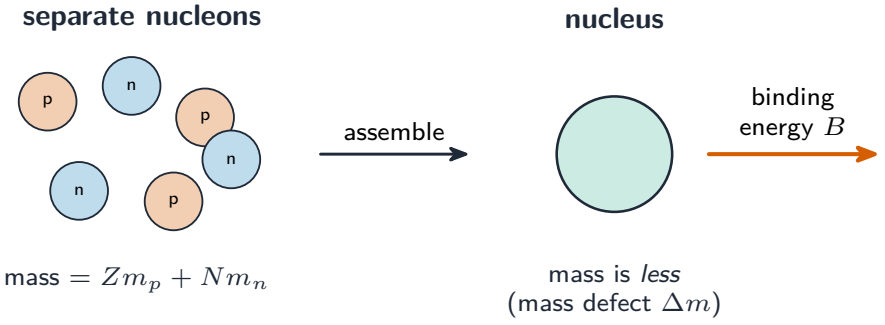
A short-range force

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- the residual nuclear interaction can provide **attraction** between nucleons of all three pair types at suitable nuclear separations;
- it is much **stronger** than the electric force at nuclear distances;
- it has a very **short range**, about 10^{-15} m, so its main binding contribution comes from nearby nucleons (at very small separations it becomes repulsive, which stops the nucleus collapsing).

Mass defect and binding energy

The mass of a **bound** nucleus is **less** than the total mass of its separate protons and neutrons. The difference is the **mass defect** 质量亏损 Δm.



The assembled nucleus has less mass than its separate nucleons; the missing mass is released as binding energy

The **binding energy** 结合能 of a nucleus is the energy needed to separate it completely into its individual nucleons (equally, the energy released when it is formed from them). Mass and energy are equivalent:

$$E = mc^2$$

so the binding energy is $\Delta m\, c^2$. Masses in nuclear physics are given in kg, in **unified atomic mass units** 原子质量单位 (u) or in MeV c^{-2}. Since 1 u = 931.5 MeV c^{-2}, a mass defect in u multiplied by 931.5 gives the binding energy in MeV directly.

Worked example: helium-4

The mass of a helium-4 nucleus is 4.001506 u.

- Separate nucleons: $2 \times 1.007276 + 2 \times 1.008665 = 4.031882$ u.
- Mass defect: $\Delta m = 4.031882 - 4.001506 = 0.030376$ u.
- Binding energy: $0.030376 \times 931.5 = 28.3$ MeV, or 7.07 MeV per nucleon.
- The same in SI units: $\Delta m = 0.030376 \times 1.661 \times 10^{-27} = 5.045 \times 10^{-29}$ kg, and $E = \Delta m\, c^2 = 4.54 \times 10^{-12}$ J.

Use **nuclear masses** with separate proton and neutron masses. If atomic masses are supplied, account consistently for electrons; one can often use hydrogen-atom mass instead of bare proton mass so electron masses cancel. Do not mix conventions silently.

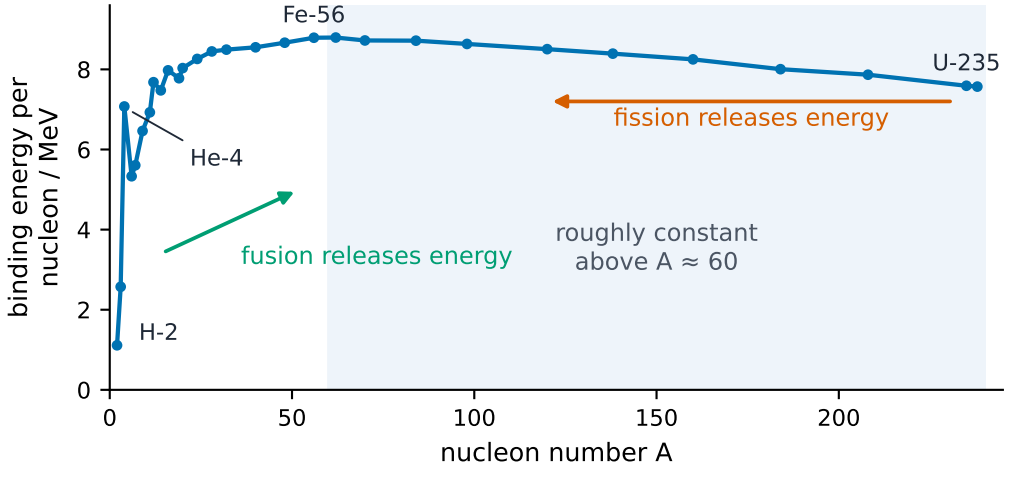
Energy released in a nuclear reaction

In any nuclear reaction or decay, if the sum of product rest masses is less than the initial rest-mass sum, the rest-energy decrease appears as kinetic energy and/or emitted radiation; total energy remains conserved: $E = \Delta m\, c^2$.

Worked example: radium-226 decays to radon-222 by alpha emission. Using atomic masses (the electrons balance on both sides): $226.025410 - (222.017578 + 4.002603) = 0.005229$ u, so the energy released is $0.005229 \times 931.5 = 4.87$ MeV.

Binding energy per nucleon

The **binding energy per nucleon** 比结合能 (binding energy ÷ nucleon number) measures how tightly each nucleon is held. A larger value means greater average binding against complete separation. This is not a guarantee against every possible radioactive decay; compare the total initial and final masses for the proposed process.



Selected binding energies: favourable fusion and fission products have greater total binding energy

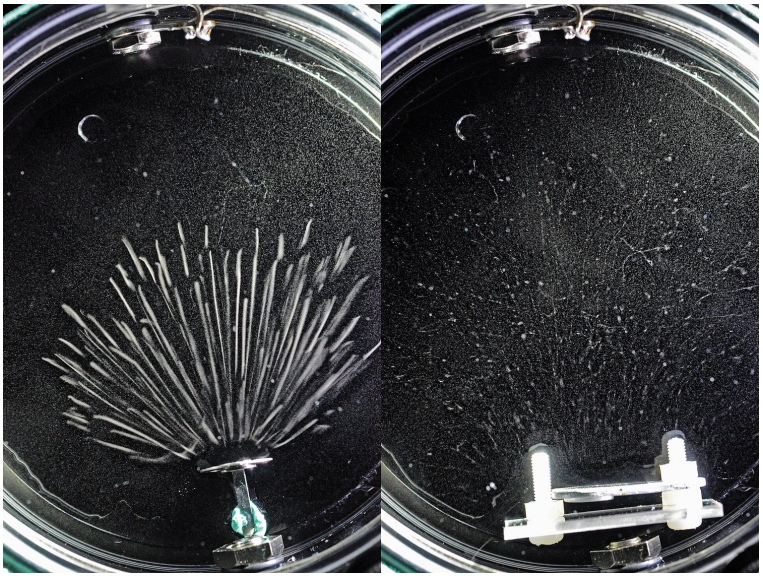
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Interpreting the curve:

- **Light nuclei** have low binding energy per nucleon. Helium-4 is unusually tightly bound for its size, which is why alpha particles are emitted as a unit.
- The **peak** is at about 8.8 MeV per nucleon, near iron-56 and nickel-62. These are among the most tightly bound nuclei per nucleon; nickel-62 is near the maximum.
- **Heavy nuclei** have a little less binding energy per nucleon (about 7.6 MeV for uranium).
- **Fusion** 核聚变: favourable reactions between light nuclei increase total binding energy and release energy; not every possible light-nucleus reaction is exothermic. This powers the stars (E.5).
- **Fission** 核裂变: favourable heavy-nucleus splits increase total binding energy and release energy. Compare all products, including free neutrons. This powers nuclear reactors (E.4).

The nature of radioactive decay

An unstable nucleus can change into a more stable one by emitting radiation. This is **radioactive decay** 放射性衰变. It is:



Tracks in a cloud chamber: each track marks a charged particle passing through the vapour, rather than necessarily one nuclear decay

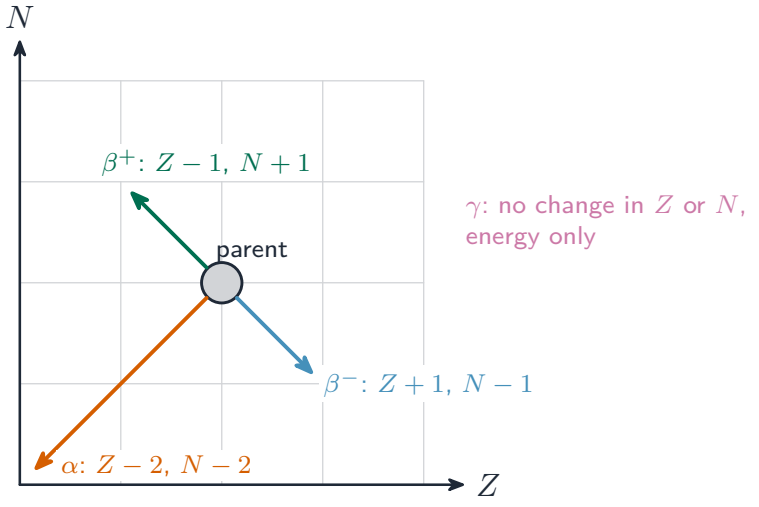
- **random** 随机的: it is impossible to predict when a particular nucleus will decay; each nucleus of a given nuclide has the same chance of decaying in the next second, however long it has existed;

- **spontaneous** 自发的: it occurs without a triggering collision. For the ordinary classroom decay model, its rate is approximately unaffected by temperature, pressure or chemical state; this is a model assumption rather than a universal statement about every decay mechanism.

Because a sample contains huge numbers of nuclei, the randomness averages out and the overall pattern is very predictable, in the same way that the total of many dice throws is predictable.

Alpha, beta and gamma decay

In every decay equation, the nucleon numbers (top) and the proton numbers (bottom) must balance.



Each decay moves the nucleus a fixed step on the N–Z chart; gamma emission only removes energy.

Alpha decay

An **alpha particle** 阿尔法粒子 is a helium nucleus, ${}^4_2\alpha$ (two protons and two neutrons). The nucleus loses 4 nucleons and 2 protons:

$$^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^4_2\alpha$$

Beta-minus decay

In **beta-minus decay** a neutron in the nucleus changes into a proton, emitting an electron (the beta-minus particle) and an **antineutrino** 反中微子 $\bar{\nu}$. The nucleon number stays the same and the proton number rises by 1:

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$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}$$

Beta-plus decay

In **beta-plus decay** a proton changes into a neutron, emitting a **positron** 正电子 (the antiparticle of the electron, charge $+e$) and a **neutrino** 中微子 ν. The proton number falls by 1:

$$^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne} + {}^{0}_{+1}\mathrm{e} + \nu$$

Neutrinos and antineutrinos have no charge and almost no mass, and they hardly interact with matter, so they are very hard to detect.

Gamma decay

After alpha or beta decay the new nucleus is often left in an **excited state**. It can emit a **gamma ray** 伽马射线, a high-energy photon of electromagnetic radiation, on moving to a lower nuclear energy level. This may be the ground state or another excited state; several successive emissions can occur. The nucleon and proton numbers do not change:

$$^{60}_{28}\mathrm{Ni}^{*} \rightarrow {}^{60}_{28}\mathrm{Ni} + \gamma$$

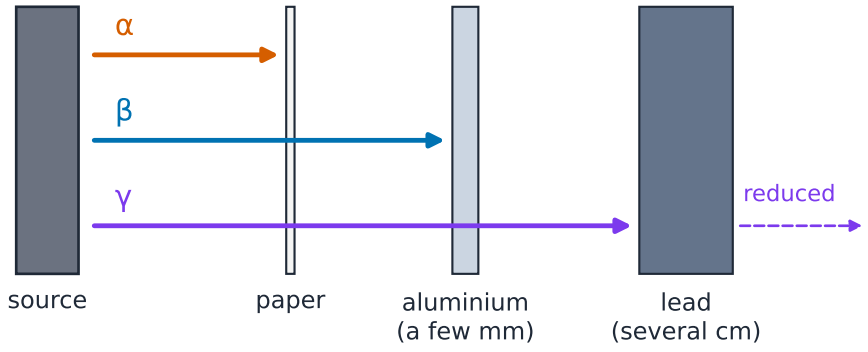
where the star marks the excited nucleus.

Decay	Particle emitted	Change in A	Change in Z
α	helium nucleus $^{4}_{2}\alpha$	-4	-2
β^{-}	electron and antineutrino	0	$+1$
β^{+}	positron and neutrino	0	-1
γ	photon	0	0

Penetration and ionisation

As the radiation passes through matter it can transfer energy that removes electrons from atoms, creating ions. Gamma photons cause ionisation indirectly through the charged particles they produce. This is **ionisation** 电离. The more strongly a radiation ionises, the faster it loses its energy, and so the less far it travels.

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Typical classroom comparison; actual ranges and attenuation depend on energy and material

	Alpha	**Beta**	**Gamma**
Nature	helium nucleus, charge $+2e$	electron or positron, charge $\mp e$	electromagnetic radiation, no charge
Ionising ability	strong	moderate	weak
Range in air	a few cm	about 1 m	very large (intensity falls gradually)
Stopped by	a sheet of paper or skin	a few mm of aluminium	reduced by several cm of lead or metres of concrete

For typical alpha energies, intact outer skin stops the alpha particles, while an internal alpha-emitting contaminant can deposit energy densely in tissue. The source may emit other radiation too. Risk depends on activity, energy, exposure route, time and shielding, not on a single label in this table.

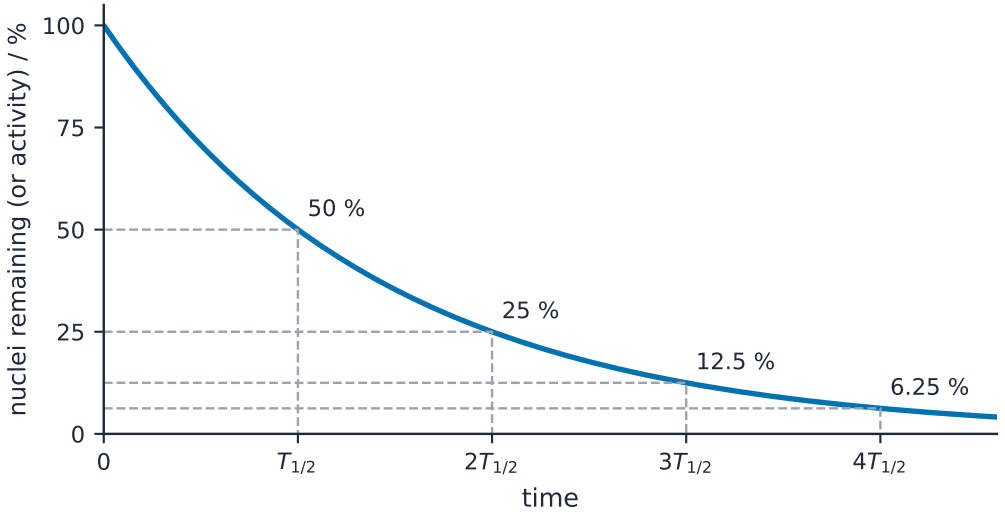
Irradiation means exposure to radiation from a source; **contamination** means radioactive material is present on or inside an object. Removing an external source ends that irradiation, but contamination can remain and keep emitting. Under the school radiation procedure, sources are teacher-managed: reduce exposure time, increase distance and use shielding appropriate to the radiation.

Activity and half-life

The **activity** 活度 A of a sample is the number of nuclei that decay per second. Its unit is the **becquerel** 贝克勒尔 (Bq): 1 Bq = 1 decay per second. A detector such as a Geiger–Müller tube measures a **count rate** 计数率, which is only a fraction of the activity, since radiation goes in all directions and not every particle is detected. Background-corrected count rate is proportional to activity when geometry, detection efficiency and radiation yield stay fixed and detector dead-time effects are negligible.

Counts are events recorded during an interval, not automatically the number of decays in the whole source.

Since the decay is random, the activity is proportional to the number of undecayed nuclei, and both fall in the same way. The **half-life** 半衰期 $T_{1/2}$ is the time for the expected undecayed population, activity, or background-corrected count rate under fixed conditions to halve. A large sample closely follows this statistical pattern; a small set need not lose exactly half its members each half-life.



Every half-life halves what is left: the same time each time

After n half-lives, the fraction remaining is $\left(\frac{1}{2}\right)^n$.

Half-lives	**0**	**1**	**2**	**3**	**4**	**5**
Fraction remaining	1	1/2	1/4	1/8	1/16	1/32

Half-lives range from fractions of a second to billions of years: technetium-99m has a half-life of 6.0 hours, carbon-14 5730 years, uranium-238 4.5×10^9 years.

■ Worked examples

- A source has an activity of 800 Bq and a half-life of 6.0 h. After 24 h (4 half-lives) its activity is $800 \times \frac{1}{16} = 50$ Bq.
- A sample's activity falls from 1200 Bq to 150 Bq in 15 days. $1200 \to 600 \to 300 \to 150$ is 3 half-lives, so $T_{1/2} = 5.0$ days.
- After 3 half-lives, $\frac{1}{8}$ of the nuclei remain and $\frac{7}{8}$ have decayed.

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Background radiation and measuring half-life

Background radiation 本底辐射 is always present: from radon gas in the air, rocks and building materials, cosmic rays, food and medical sources. A detector can record it when the investigated source is absent; the rate depends on detector and surroundings.



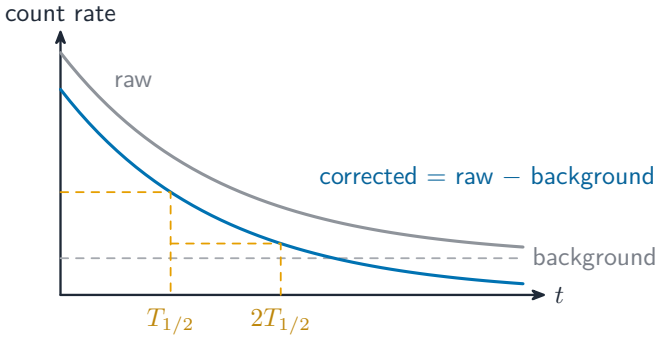
A Geiger counter counts ionising events; even with no source nearby it records a background count rate

- Measure the background count rate first, with the source far away, for a long time (to reduce the random uncertainty).
- **Subtract** the background from every reading to get the count rate due to the source.

Forgetting this matters most when the source count rate is small. For example, readings of 100 and 55 counts per minute, with a background of 10 counts per minute, are really 90 and 45 from the source: consistent with one half-life apart in this idealised example. The uncorrected ratio 0.55 is a count-rate fraction, not a number of half-lives.

Determining a half-life

1. Under the teacher-managed procedure, measure background over a sufficiently long interval and repeat it to check whether it is stable.
2. Keep detector geometry and settings fixed. Record source-plus-background counts, duration and elapsed time. Count intervals should be short compared with the half-life or the interval average must be modelled.
3. Subtract the background from each reading.
4. Plot corrected count rate against time and draw a smooth curve.
5. Estimate halving times across several suitable portions of the fitted trend and report spread or uncertainty. Random counts need not lie exactly on the curve; avoid low corrected rates dominated by background uncertainty.



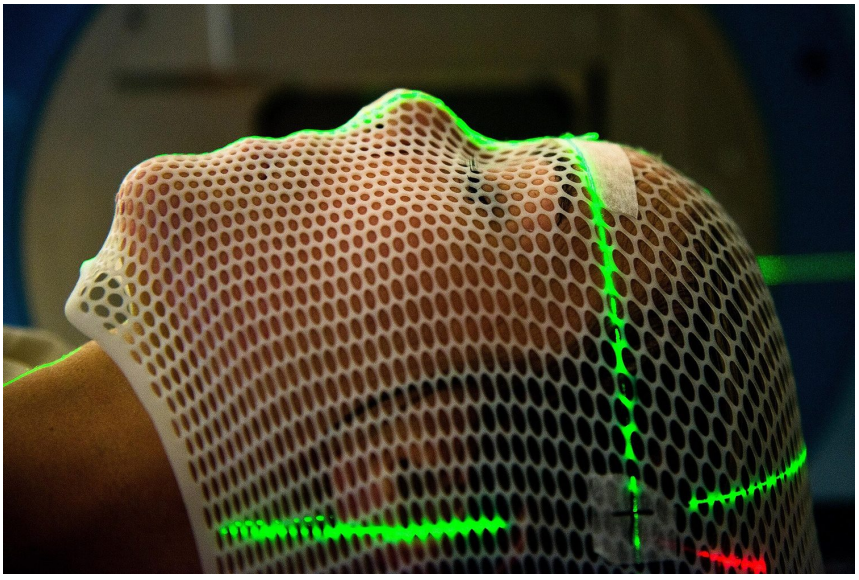
Subtract background, then read the half-life

For a very long half-life the activity hardly changes during an experiment, so other methods are needed.

Uses of radioactive isotopes

The choice of isotope depends on the radiation it emits and on its half-life.

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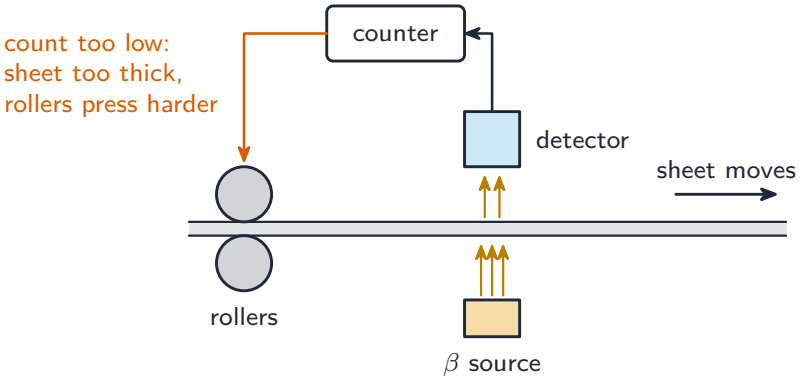
Radiotherapy: a mesh mask holds the patient still, and alignment lasers position the patient so the treatment beam reaches the tumour precisely

These are physics comparisons, not instructions to handle or administer sources.

- **Medical imaging.** A common technetium-99m imaging principle uses escaping gamma photons and an approximately 6 h physical half-life. Suitable chemical targeting and the time needed for measurement matter. Biological removal and administered activity also affect dose; a short physical half-life alone does not guarantee a low dose. External gamma imaging does not use the alpha particle itself as its escaping signal.
- **Radiotherapy.** A cobalt-60 source can supply gamma photons for controlled irradiation. Its roughly 5.3 year half-life lets a source operate for a long time, but activity declines and treatment needs professional planning and shielding.
- **Industrial tracers.** A suitable tracer can reveal flow or leaks because radiation reaching an external detector changes with position. Penetration, chemical behaviour, detection sensitivity and managed containment all matter. A short half-life reduces later activity; it does not by itself make released material safe.
- **Thickness control.** Choose radiation partly attenuated by the intended material: the count must change measurably across the working thickness range. A beta source can suit thin paper, while alpha may be stopped completely and gamma may change too little. Energy and thickness determine suitability; the ranking is not universal. Fixed geometry and a long half-life reduce unwanted drift, which calibration can monitor.
- **Radioactive dating.** Once carbon exchange stops, the carbon-14 ratio decreases in a closed sample. Compare equivalent carbon quantities, justify the initial ratio, correct contamination and calibrate real dates. Potassium–argon dating instead uses appropriate minerals and accounts for branching and retention. A useful clock must leave a measurable parent/daughter signal over the age range; choosing

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a half-life equal to the age is not a universal optimum.



Thickness control: if the sheet thickens, fewer beta particles get through and the rollers press harder.

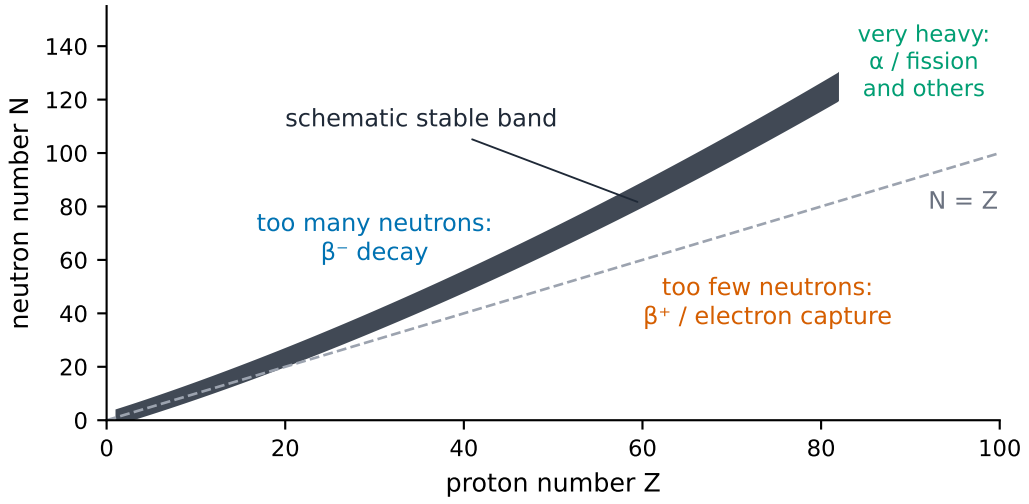
AHL: Evidence for the strong nuclear force

- **Nuclei exist.** Two protons 2.0 fm apart repel with an electric force of $\frac{8.99 \times 10^{9} \times (1.60 \times 10^{-19})^{2}}{(2.0 \times 10^{-15})^{2}} = 58$ N, a huge force on such small particles. The gravitational attraction is about 5×10^{-35} N, around 10^{36} times too small. A much stronger attractive force must act.
- **Short range.** The binding energy per nucleon is roughly constant for all but the lightest nuclei, and average nuclear density is approximately similar across many nuclei (E.1). These trends support short-range saturation: each nucleon gains most of its binding from nearby neighbours. They do not make real nuclei identical or remove surface and shell effects.
- **Scattering.** When alpha particles or protons have enough energy to reach a nucleus, scattering no longer follows Rutherford's electric-force prediction (E.1): another force acts at very short range.

AHL: Neutrons, protons and stability

The **ratio of neutrons to protons** helps organise stability, together with total size, energy differences and nuclear structure. It cannot alone determine every allowed decay.

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Heavier stable nuclei need more neutrons than protons

- **Light nuclei** are stable with $N \approx Z$, for example ${}^{12}_{6}\mathrm{C}$ and ${}^{16}_{8}\mathrm{O}$.
- **Heavy nuclei** need more neutrons than protons: lead-208 has $N/Z = 126/82 = 1.54$. Every proton repels every other proton (the electric force has a long range), but each nucleon is attracted only by its neighbours. Extra neutrons add strong-force attraction without adding repulsion.
- Lead ($Z = 82$) is the highest-proton-number element with stable nuclides. Even bismuth-209 is radioactive: its extremely slow alpha decay was detected experimentally. Long-lived is not the same as stable.

The band suggests likely directions of change, subject to energetics and other available modes:

- **too many neutrons** (above the band): β^- decay turns a neutron into a proton, moving the nucleus towards the band;
- **too few neutrons** (below the band): β^+ decay or electron capture turns a proton into a neutron;
- **very heavy** nuclei: α decay removes 2 protons and 2 neutrons, reducing the size; spontaneous fission and other decay modes can also occur.

AHL: The binding energy curve above A = 60

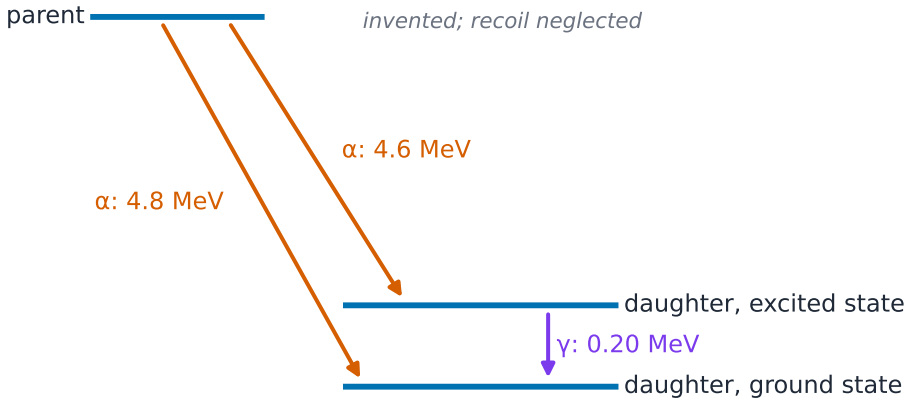
Above a nucleon number of about 60, the binding energy per nucleon stays roughly constant, between about 7.6 and 8.8 MeV.

- Because the strong force has a short range, each nucleon is bound mainly to a small number of nearby nucleons. Once a nucleus is large enough that most nucleons are surrounded by neighbours, adding more nucleons adds about the same binding energy for each one.

- The slow fall towards uranium is caused by the electric repulsion, which acts between **all** pairs of protons and grows faster than the number of nucleons.
- For light nuclei, a large fraction of the nucleons are at the surface with fewer neighbours, so the binding energy per nucleon is smaller.

AHL: Nuclear energy levels

Alpha particles from a particular nuclide have one, or a small number of, **definite** energies. Gamma rays from a particular nuclide also have definite energies: a **line spectrum**, just like the photons emitted by atoms (E.1).



Invented level scheme, not to scale; neglect daughter recoil when comparing the alpha-energy gap with the gamma energy

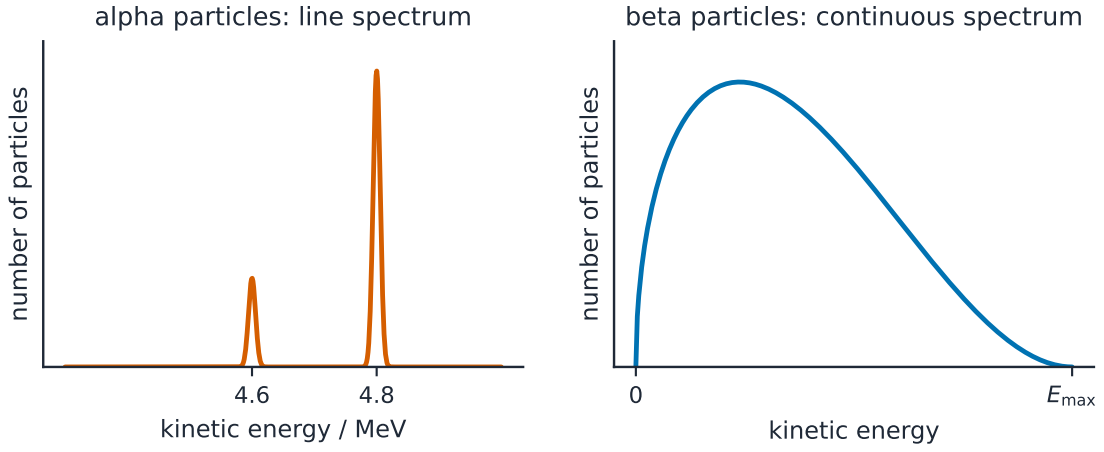
This is evidence that nuclei have **discrete energy levels**. In this illustrative heavy-nucleus approximation, neglect daughter recoil: the alpha-energy difference ($4.8 - 4.6 = 0.2$ MeV) estimates the gamma energy. More precisely, the branch-energy difference includes changes in daughter recoil. In the approximation, the nucleus left in the excited state emits the difference as a gamma ray. Nuclear transitions often have keV-to-MeV gaps, far greater than ordinary visible atomic transitions. The exact gap depends on the nucleus; the diagram is not data for a named isotope.

Invented two-step example. Nuclear levels are at 0.50, 0.20 and 0 MeV. Neglecting daughter recoil, a transition from 0.50 to 0.20 MeV emits a photon of $0.50 - 0.20 = 0.30$ MeV; the next transition to the ground state emits $0.20 - 0 = 0.20$ MeV. Together the photons carry 0.50 MeV. Each photon corresponds to one level difference, not necessarily the whole initial excitation energy.

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AHL: The beta spectrum and the neutrino

Unlike alpha particles, beta particles from one nuclide have a **continuous** range of kinetic energies, from small values up to an endpoint. The available energy is shared among beta particle, (anti)neutrino and daughter recoil; the endpoint is close to the available kinetic energy when recoil and neutrino mass are neglected.



Illustrative spectra, not measured data: narrow alpha branches and continuous beta energies

- If only two particles (the daughter nucleus and the electron) were produced, conservation of energy and momentum would give the electron one definite energy, as for alpha decay.
- Most beta particles have much less than E_{max}, so some energy seemed to be missing. In 1930 Wolfgang Pauli proposed that a third particle, neutral and very light, carries off the rest of the energy and some momentum. This is the (anti)neutrino.
- The energy shared among the beta particle, (anti)neutrino and recoiling daughter varies from decay to decay, which gives the continuous spectrum. Neutrinos were finally detected in 1956.

AHL: The radioactive decay law

The **decay constant** 衰变常数 λ is the constant of proportionality between the rate of decay and the number of undecayed nuclei. For one independently decaying parent nuclide with constant λ and no replenishment, the expected number left after time t is

$$N = N_0 e^{-\lambda t}$$

and the activity, the rate of decay, is

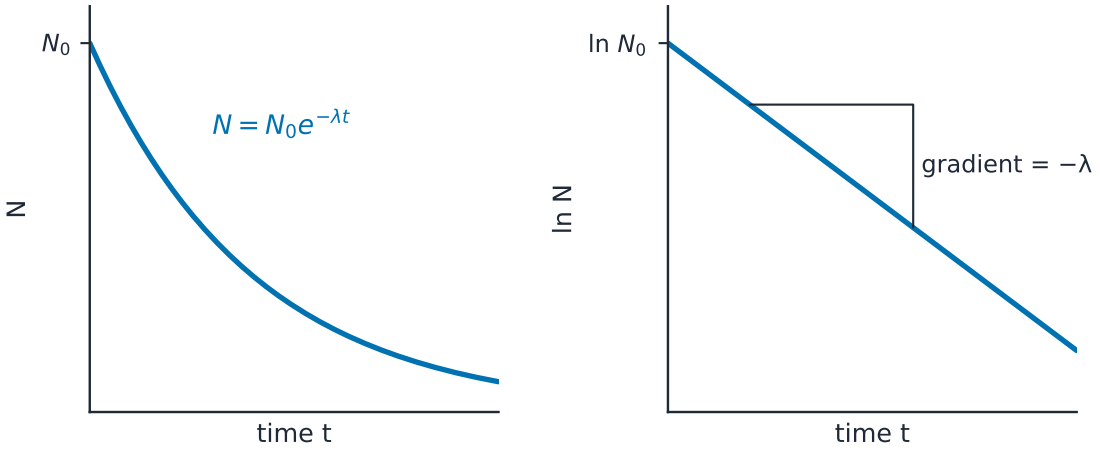
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$$A = \lambda N = \lambda N_0 e^{-\lambda t}$$

Setting $N = N_0/2$ at $t = T_{1/2}$ gives the link with the half-life:

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

These equations work for **any** time, not just whole numbers of half-lives. The unit of λ is s^{-1} (or day^{-1}, year^{-1}, as long as the time units match).



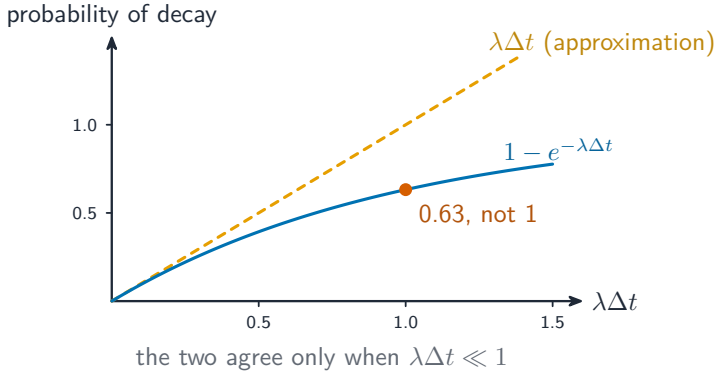
Taking logarithms turns the decay curve into a straight line of gradient $-\lambda$

Taking logarithms: $\ln N = \ln N_0 - \lambda t$ (and similarly $\ln A = \ln A_0 - \lambda t$). A graph of $\ln A$ (or ln of the corrected count rate) against t is a straight line of gradient $-\lambda$: the best way to find λ, and so $T_{1/2}$, from data when the one-exponential model, fixed efficiency and background treatment are justified. With dimensionless logarithms, plot $\ln(R/R_{\mathrm{ref}})$; the fixed reference changes the intercept, not the slope.

The decay constant as a probability

The probability that a nucleus decays in a time Δt is $1 - e^{-\lambda \Delta t}$. When $\lambda \Delta t$ is small, this is approximately $\lambda \Delta t$, so λ **is approximately the probability of decay per unit time**, but only when $\lambda \Delta t \ll 1$.

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lambda is a probability only for short times

For $\lambda = 0.010\ \text{s}^{-1}$: in 1.0 s the probability is $1 - e^{-0.010} = 0.00995 \approx 0.010$; but in 100 s it is $1 - e^{-1} = 0.63$, not $\lambda t = 1$.

■ Worked examples

- Iodine-131 has a half-life of 8.0 days. $\lambda = \ln 2/(8.0 \times 86400) \approx 1.003 \times 10^{-6}\ \text{s}^{-1}$. A sample of 5.0×10^{15} nuclei initially has activity $A_0 = \lambda N_0 \approx 5.014 \times 10^{9}$ Bq. Keep the full calculator values until the final answer. After 20 days, $A = A_0\, 2^{-20/8} \approx 8.9 \times 10^{8}$ Bq to two significant figures.
- Time for that activity to fall to 1.0×10^{8} Bq: $t = \ln(A_0/A)/\lambda$. Using the unrounded initial activity and decay constant gives about 45 days.
- Invented closed-sample dating calculation: carbon-14 has $\lambda = 0.693/5730 = 1.21 \times 10^{-4}\ \text{year}^{-1}$. With 52.5 % remaining, $t = \ln(1/0.525)/(1.21 \times 10^{-4}) = 5.3 \times 10^{3}$ years. This idealised result is not a calibrated date for Ötzi.

Practice

1. (a) Define isotopes, and state the numbers of protons and neutrons in ${}^{35}_{17}\text{Cl}$ and ${}^{37}_{17}\text{Cl}$. (b) State three properties of the strong nuclear force.
2. A carbon-12 nucleus has a mass of 11.996706 u. Find its mass defect, its binding energy in MeV and its binding energy per nucleon.
3. (a) Use total binding energies to explain why suitable fusion and fission reactions release energy. Must every reaction of either type release energy? (b) In an alpha decay the total mass decreases by 0.0059 u. Find the energy released in MeV and in J.
4. Explain what is meant by saying that radioactive decay is random and spontaneous.
5. Complete the equations: (a) ${}^{210}_{84}\text{Po}$ decays by α emission; (b) ${}^{90}_{38}\text{Sr}$ decays by β^- emission; (c) ${}^{18}_{9}\text{F}$ decays by β^+ emission.
6. With no absorber, a detector near a source reads 600 counts per minute; with

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paper in between, 450; with 5 mm of aluminium, 30. The background is 30 counts per minute. State which radiations the source emits. Assume the usual classroom energies, matched geometry, negligible absorber scattering into the detector and sufficient repeated counts to resolve the reductions.

7. A source has an activity of 640 Bq and a half-life of 2.0 h. Find its activity after 8.0 h.
8. A detector reads 245 counts per minute from a source; the background is 25 counts per minute. Find the reading expected after three half-lives.
9. In an invented closed-sample dating model, wood has one eighth of its assumed initial carbon-14 per equivalent carbon quantity. With half-life 5730 years, find its ideal decay age and state one limitation of applying it to a real archaeological date.
10. Explain the physical radiation and half-life choices for external technetium-99m gamma imaging, using an approximately 6 h physical half-life. Why does half-life alone not determine dose?
11. Explain why a beta source is used to monitor the thickness of paper.
12. **AHL.** (a) Two protons in a nucleus are 2.0 fm apart. Calculate the electric force between them, and explain what this shows about nuclear forces. (b) Explain why heavy stable nuclei have more neutrons than protons, and predict how a nuclide lying above the band of stability will decay. (c) Explain why the binding energy per nucleon is roughly constant above $A \approx 60$.
13. **AHL.** (a) A nuclide emits alpha particles of 5.30 MeV and 5.12 MeV, and gamma photons of one energy. Explain this, and deduce the gamma photon energy. Treat the numbers as illustrative and neglect daughter recoil. (b) Explain how the energy spectrum of beta particles led to the prediction of the neutrino.
14. **AHL.** (i) Iodine-131 has a half-life of 8.0 days. Find (a) its decay constant in s^{-1}, (b) the activity of 2.0×10^{15} nuclei, (c) the time for the activity to fall to 10 % of its starting value. (ii) A source has a decay constant of $0.010\ \mathrm{s}^{-1}$. Find the probability that a given nucleus decays (a) in the next 1.0 s, (b) in the next 100 s, and comment. (iii) A graph of ln(corrected count rate) against time falls from 8.0 to 5.0 in 10 minutes. Find the decay constant and the half-life. (iv) Find the fraction of a sample remaining after 3.5 half-lives.

Answers.

1. (a) Nuclides with the same number of protons but different numbers of neutrons. Chlorine-35: 17 protons, 18 neutrons; chlorine-37: 17 protons, 20 neutrons. (b) Any three: residual nuclear attraction between nearby nucleons at suitable separations; strong compared with electrical repulsion there; short range of order femtometres; repulsion at sufficiently short separation. Do not describe it as attractive at every distance.
2. Nucleons: $6 \times 1.007276 + 6 \times 1.008665 = 12.095646$ u; $\Delta m = 0.098940$ u; binding energy $= 0.098940 \times 931.5 = 92.2$ MeV; per nucleon $92.2/12 = 7.68$ MeV.

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3. (a) For an exothermic reaction the products have greater total binding energy, so their total rest mass is lower. Many light-nucleus fusion and heavy-nucleus fission reactions do this. Include all products and compare actual totals; the broad curve alone does not prove every proposed reaction releases energy. (b) $0.0059 \times 931.5 = 5.5$ MeV $= 5.5 \times 10^6 \times 1.60 \times 10^{-19} = 8.8 \times 10^{-13}$ J.
4. Random: an individual decay time cannot be predicted and the constant-rate model gives equal conditional probabilities in equal future intervals. Spontaneous: no triggering collision is needed; the ordinary classroom rate is approximately insensitive to temperature, pressure and chemistry.
5. (a) ${}^{210}_{84}\mathrm{Po} \rightarrow {}^{206}_{82}\mathrm{Pb} + {}^{4}_{2}\alpha$. (b) ${}^{90}_{38}\mathrm{Sr} \rightarrow {}^{90}_{39}\mathrm{Y} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}$. (c) ${}^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + {}^{0}_{+1}\mathrm{e} + \nu$.
6. Within the stated model the paper-sensitive component indicates alpha and the aluminium-sensitive component indicates beta. The final count is consistent with background, so no gamma component is resolved; this is not proof that an arbitrarily weak gamma component is absent.
7. 8.0 h is 4 half-lives: $640/16 = 40$ Bq.
8. Corrected: 220 counts per minute; after 3 half-lives, $220/8 = 27.5$; add back the background: about 53 counts per minute.
9. $\frac{1}{8}$ is 3 half-lives: $3 \times 5730 = 17\,190$ years, about 1.7×10^4 years. Real dates require a justified initial ratio, contamination checks and atmospheric calibration; this classroom age is not automatically a calibrated calendar age.
10. Gamma photons can escape to the camera. A physical half-life near 6 h provides a measurement window while activity subsequently declines. Administered activity, photon energy, chemical distribution and biological removal also affect dose; a short half-life alone is not a guarantee of low dose.
11. Alpha particles would all be stopped by the paper, and gamma rays would pass through almost unaffected; the number of beta particles getting through changes noticeably with small changes in thickness.
12. **AHL.** (a) $F = 8.99 \times 10^9 \times (1.60 \times 10^{-19})^2/(2.0 \times 10^{-15})^2 = 58$ N, repulsive. Gravity is far too weak to balance it, so there must be a very strong attractive force between nucleons at these distances: the strong nuclear force. (b) The electric repulsion acts between all pairs of protons, while the strong attraction acts mainly between near neighbours, so extra neutrons are needed to add attraction without repulsion. A nuclide above the band has too many neutrons, so beta-minus decay moves it towards the band when that decay is energetically allowed; the ratio is a guide rather than a full decay prediction. (c) The strong force is short range, so each nucleon is bound mainly to nearby neighbours; in a large nucleus each nucleon has about the same number of neighbours, so each adds about the same binding energy.
13. **AHL.** (a) The daughter nucleus has discrete energy levels. Alpha decay leads either to the ground state or to an excited state; the excited nucleus then emits a gamma photon. Gamma energy $= 5.30 - 5.12 = 0.18$ MeV. This equality is approximate: changing daughter recoil also contributes to the exact branch-

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energy balance. (b) With only two products, beta particles would all have the same energy. They have a continuous range up to a maximum, so a third, undetected particle must share the energy (and momentum): the neutrino. Include daughter recoil in exact energy and momentum accounting; beta-minus emits an electron antineutrino, beta-plus an electron neutrino.

14. **AHL.** (i) (a) $\lambda = 0.693/(8.0\times 86\,400) = 1.0\times 10^{-6}\ \text{s}^{-1}$. (b) $A = \lambda N = 2.0\times 10^{9}$ Bq. (c) $t = \ln 10/\lambda = 2.303/0.0866\ \text{day}^{-1} = 27$ days. (ii) (a) $1 - e^{-0.010} = 0.00995$, almost exactly $\lambda t = 0.010$. (b) $1 - e^{-1.0} = 0.63$. λt is a good estimate of the probability only when λt is small; for 100 s it would wrongly give 1.0. (iii) Gradient $= -3.0/10 = -0.30\ \text{min}^{-1}$, so $\lambda = 0.30\ \text{min}^{-1}$; $T_{1/2} = 0.693/0.30 = 2.3$ min. (iv) $\left(\tfrac{1}{2}\right)^{3.5} = e^{-3.5\ln 2} = 0.088$, about 8.8 %.

Exam tips

- Binding energy is the energy needed to **separate** a nucleus into nucleons, not the energy "stored" in it. Higher binding energy per nucleon means tighter average binding, not immunity to all decay modes.
- Check that nucleon and proton numbers balance in every decay equation, and include the neutrino or antineutrino in beta decay.
- Subtract background radiation before calculating with count rates; add it back if asked for a detector reading.
- For a whole number of half-lives use $(\tfrac{1}{2})^{n}$; show each halving step.
- When explaining a choice of isotope, discuss **both** the type of radiation and the half-life.
- **AHL.** Use $N = N_0 e^{-\lambda t}$ for any time; keep the units of λ and t consistent, and use $T_{1/2} = \ln 2/\lambda$.

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Fission

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches E.4 Fission in full, for the course first assessed in 2025. There is no additional higher level content in E.4, so SL and HL students study the same material.

Prerequisites: binding energy, mass defect, $E = mc^2$, the binding energy curve and decay equations from E.3; half-life from E.3; energy transfers, power and efficiency from A.3. Use 1 u = 931.5 MeV c^{-2} = 1.661×10^{-27} kg, 1 MeV = 1.60×10^{-13} J and the neutron mass 1.008665 u.

Nuclear fission

■ Why does a shut-down reactor still need cooling?

Control rods can suppress a self-sustaining fission chain, yet radioactive products already made in the fuel continue decaying and releasing heat. Stopping the chain and removing the remaining heat are different tasks. This unit follows the energy from nuclear rest-mass differences to electricity, explains neutron control, and connects spent-fuel properties to storage and disposal.



A nuclear power station: the visible cooling-tower cloud consists of tiny liquid water droplets, not smoke

Our numerical fuel comparisons are **assigned idealised models**, not measured fuel consumption or pellet performance for a named plant. They assume a stated energy per fission and distinguish the uranium-235 actually fissioned from the total mass of

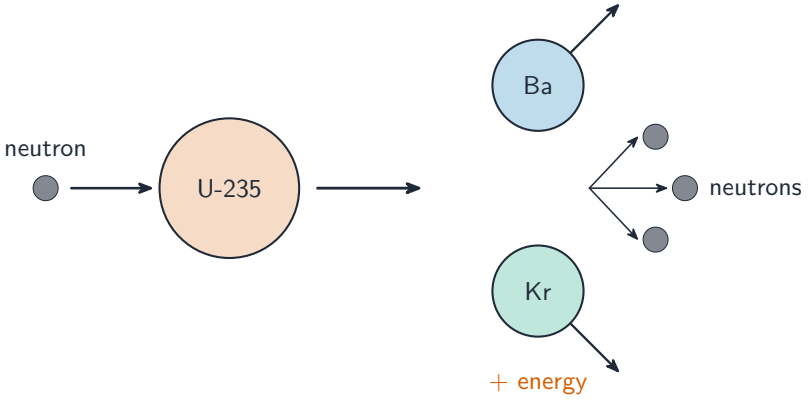
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a real fuel assembly.

Splitting a heavy nucleus

Nuclear fission 核裂变 is the splitting of a heavy nucleus into two medium-mass fragments, which need not have equal masses (the **fission fragments** 裂变碎片), usually with emitted neutrons and released energy. Neutron numbers and fragment identities vary between events; two or three emitted neutrons are common examples rather than an exact universal count.

- **Spontaneous fission** 自发裂变: some very heavy nuclei, such as californium-252, split by themselves, randomly, like other radioactive decay. For most nuclei it is extremely rare.
- **Neutron-induced fission** 诱发裂变: neutron absorption can leave a nucleus sufficiently excited to split; some neutron captures instead lead to other outcomes. This is the reaction used in reactors. Uranium-235 absorbs a slow neutron, becomes uranium-236 in a highly excited state, and splits almost at once.



Neutron-induced fission of uranium-235

A typical reaction:

$$ {}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n} $$

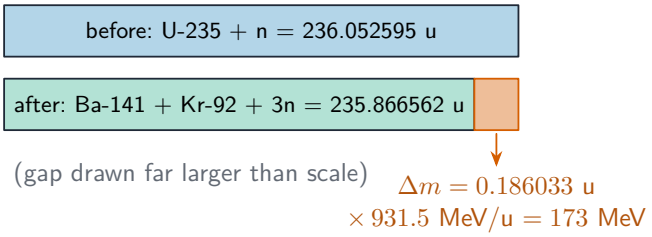
Check the balance: nucleon numbers $1 + 235 = 141 + 92 + 3$; proton numbers $0 + 92 = 56 + 36 + 0$. The fragments are not always barium and krypton: a uranium nucleus can split in many different ways.

Energy released in fission

For an exothermic fission reaction, the sum of product rest masses is less than the initial rest-mass sum. The corresponding rest-energy decrease is released, $E = \Delta m\, c^2$, mainly as the kinetic energy of the fragments and neutrons. As the fast fragments collide with surrounding atoms, this becomes thermal energy in the fuel.

Method 1: from masses

For the reaction above, using atomic masses (the electrons balance):



The missing mass becomes energy

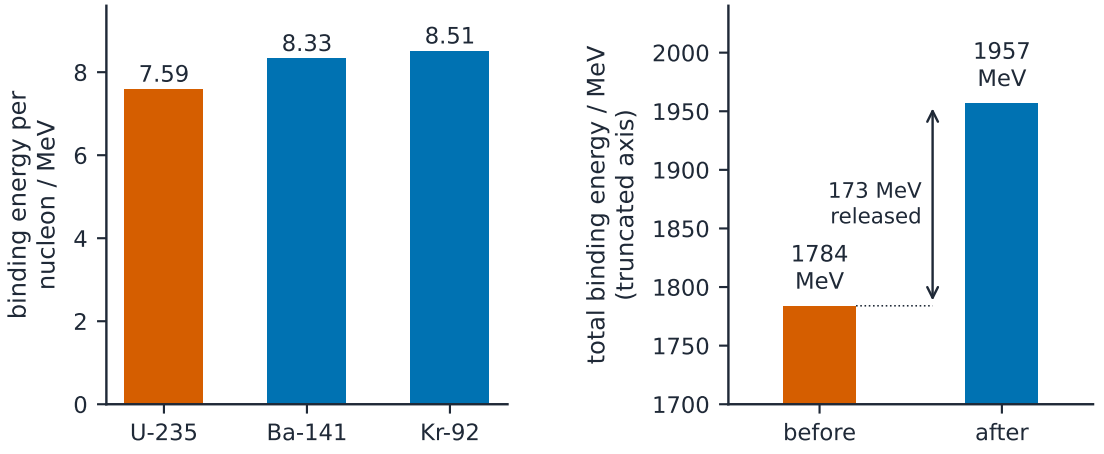
	Mass / u
uranium-235	235.043930
barium-141	140.914411
krypton-92	91.926156
neutron	1.008665

- Before: $235.043930 + 1.008665 = 236.052595$ u.
- After: $140.914411 + 91.926156 + 3 \times 1.008665 = 235.866562$ u.
- $\Delta m = 0.186033$ u, so $E = 0.186033 \times 931.5 = 173$ MeV.
- In joules: $173 \times 1.60 \times 10^{-13} = 2.8 \times 10^{-11}$ J (or $\Delta m = 3.09 \times 10^{-28}$ kg, $E = \Delta m\, c^2 = 2.8 \times 10^{-11}$ J).

Later fragment decays add energy. A rounded value near 200 MeV per fission is useful when explicitly assigned for a reactor-energy estimate; the specified immediate channel above gives about 173 MeV. Energy carried away by weakly interacting particles is not all deposited as useful reactor heat, so do not interchange detailed reaction energy and a chosen deposited-energy model silently.

Method 2: from binding energies

The fragments are nearer the peak of the binding energy curve than uranium, so they are more tightly bound. The energy released equals the **increase** in total binding energy.



Rounded per-nucleon values show the trend; total values are rounded from a mass-consistent calculation

- From the displayed **rounded** per-nucleon inputs: $B_{\text{before}} = 235 \times 7.59 = 1783.65$ MeV; the free incoming neutron has zero nuclear binding energy.
- $B_{\text{after}} = 141 \times 8.33 + 92 \times 8.51 = 1957.45$ MeV; free outgoing neutrons again contribute zero.
- $Q = B_{\text{after}} - B_{\text{before}} = 173.80$ MeV ≈ 174 MeV. Rounding the per-nucleon inputs explains the small difference from the more precise mass result, about 173 MeV. Do not truncate each intermediate product and claim exact agreement.

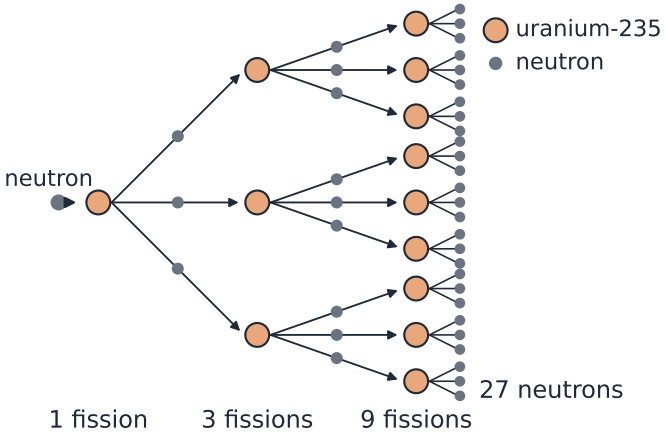
For a specified reaction, count only **bound** nucleons in each binding-energy total. Emitted free neutrons are included in mass accounting but do not receive the fragments' average binding energy per nucleon.

■ Energy from a kilogram of fuel

One kilogram of uranium-235 contains $\dfrac{1}{235 \times 1.661 \times 10^{-27}} = 2.56 \times 10^{24}$ nuclei. At 200 MeV (3.2×10^{-11} J) each, complete fission releases 8.2×10^{13} J, about 2.7×10^{6} times an assigned coal energy of 3.0×10^{7} J per kilogram. This compares ideal complete fission of pure uranium-235 with that assigned chemical energy, not real fuel-pellet burnup or whole-plant efficiency.

Chain reactions

In the induced-fission chain model, a fission initiated by one neutron releases further neutrons. If these go on to cause more fissions, the reaction can keep itself going: a **chain reaction** 链式反应.



If every neutron caused a fission, the number of fissions would grow very quickly

- If, on average, **more than one** neutron from each fission causes another fission, the expected fission rate grows. Growth alone is not a nuclear explosion: reactors can increase power in a controlled way, and the time response and feedback matter.
- If, on average, **exactly one** does, the expected rate stays steady in this idealised generation model. This is the balance required at constant reactor power.
- If **fewer than one** does, the reaction dies away.

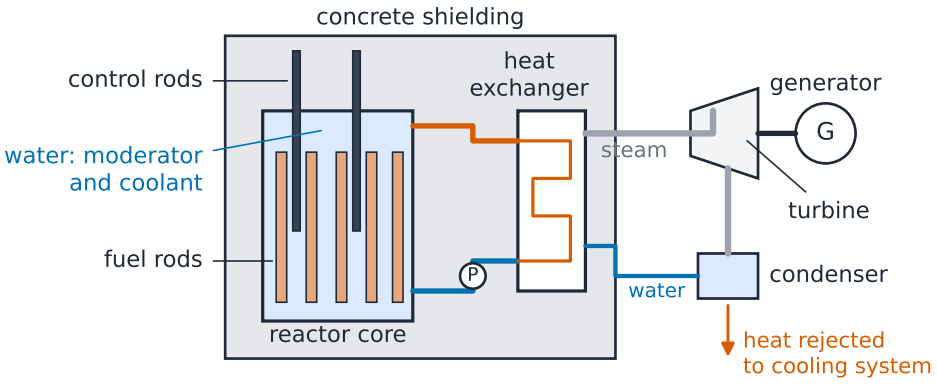
Neutrons can be lost by escaping from the surface of the fuel or by being absorbed without causing fission. In a small piece of uranium too many escape. The **critical mass** 临界质量 is the minimum mass that can sustain a chain reaction **for specified material and physical conditions**. It is not a universal number attached only to an element; composition, geometry, surroundings and neutron losses affect the balance.

Natural uranium is mostly uranium-238 with roughly 0.7% uranium-235. Uranium-238 does not readily fission with thermal neutrons, although fast-neutron fission is possible and capture matters. Many light-water reactors use **enriched** uranium with a few percent uranium-235. Other designs can use natural uranium; neutron economy and reactor type matter.

Inside a nuclear power station

The energy chain: nuclear energy → kinetic energy of fragments → thermal energy of fuel and coolant → steam → kinetic energy of turbine → electrical energy from the generator.

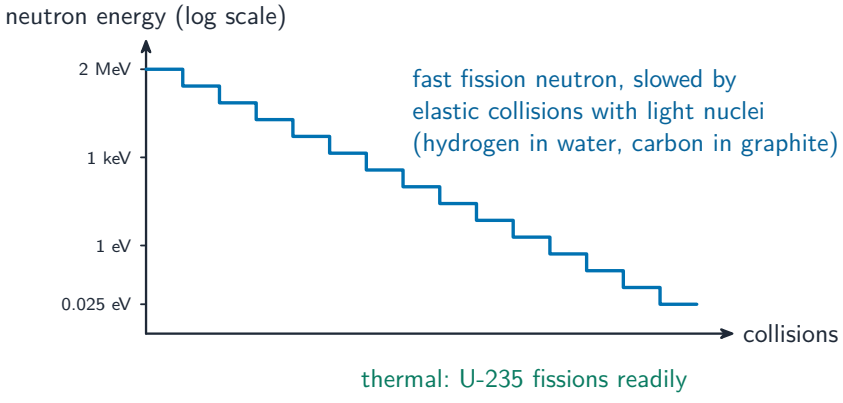
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Fission heats the coolant; the heat exchanger makes steam to turn the turbine

The moderator

Neutrons released in fission are fast (about 2 MeV). In a thermal reactor, uranium-235 has a much larger fission probability per target nucleus for **slow** neutrons than for typical fast fission neutrons. Around room temperature, thermal-neutron energies are of order 0.025 eV; the characteristic energy changes with moderator temperature. The **moderator** 慢化剂 slows the neutrons down.



Slowing a neutron collision by collision

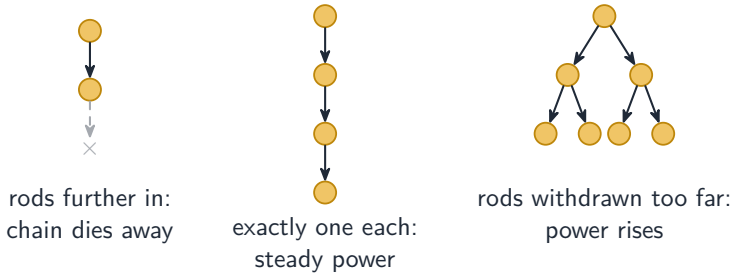
- Neutrons lose kinetic energy in elastic collisions with the nuclei of the moderator.
- A neutron loses the most energy when it hits a nucleus of similar mass (think of a snooker ball hitting another snooker ball, rather than bouncing off a wall). So moderators contain light nuclei: water (hydrogen), heavy water or graphite (carbon).
- Many collisions usually distribute energy to the moderator. Collision count depends on the material, scattering and initial/final energies, rather than one fixed count for all reactors.

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- The moderator must not absorb many neutrons itself.

The control rods

Control rods 控制棒 are made of a material such as boron or cadmium that absorbs neutrons strongly.



Control rods set the chain

- Lowering them further into the core absorbs more neutrons, so fewer fissions occur and the power falls.
- Raising them lets the rate rise.
- They are adjusted so that, on average, exactly one neutron from each fission causes another. Shutdown systems insert neutron absorbers to suppress the self-sustaining chain; the insertion arrangement depends on the reactor. **Decay heat remains**, so cooling is still necessary. Moderation and absorption are distinct roles; a fast reactor does not use a moderator to thermalise its chain neutrons.

The heat exchanger

The **coolant** 冷却剂 carries thermal energy away from the core. In the **pressurised-water reactor** shown, high pressure keeps primary water liquid. A **heat exchanger** 热交换器 transfers heat through a separating wall to a secondary circuit, where water boils for the turbine. With intact barriers, bulk primary coolant and secondary water do not mix; the primary system may contain radioactive material.

This is one reactor design, not a universal circuit. The Department of Energy's reactor comparison distinguishes this from a boiling-water reactor, where steam is produced in the reactor vessel and goes to the turbine. The condenser returns steam to liquid water and transfers rejected heat to a cooling system; heat rejection is part of the energy balance, not an optional missing output.

Shielding

The core emits intense neutrons and gamma rays. A thick steel **pressure vessel** and a concrete **shield** 屏蔽层, several metres thick, attenuate radiation to reduce exposure. Shielding reduces radiation transmission; containment and fuel barriers limit escape of radioactive material. These are different functions, and no single wall guarantees every accident outcome. The building is also designed to contain radioactive material

if there is an accident.

Worked example: fuel use

In an assigned idealised model, a station produces 1000 MW electrical at efficiency 33%. Take **deposited** energy per fission as 200 MeV, approximate each uranium-235 nucleus as 235 u, and assume steady operation.

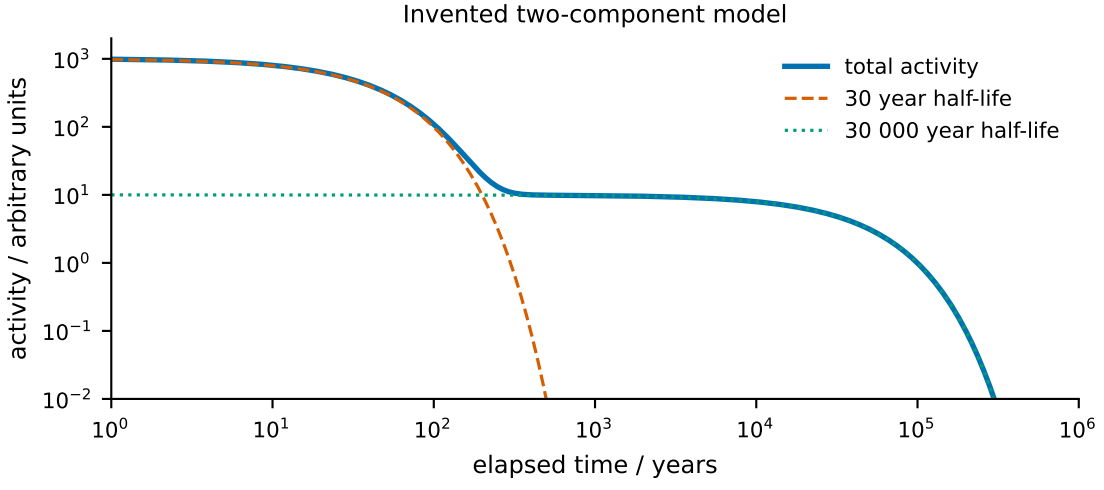
- Thermal input: $P_{\text{th}} = P_{\text{el}}/\eta = (1.00 \times 10^9)/0.33 \approx 3.03 \times 10^9$ W.
- Fission rate: $\dot{N} = P_{\text{th}}/E_f \approx (3.0303 \times 10^9)/(3.20 \times 10^{-11}) = 9.47 \times 10^{19}$ s^{-1}.
- Daily fissioned mass: $m = \dot{N}\Delta t\, m_U \approx 9.4697{\times}10^{19}{\times}86400{\times}235{\times}1.661{\times}10^{-27} = 3.19$ kg ≈ 3.2 kg.

This is uranium-235 fissioned, not total fuel loaded. Rejected thermal power is $1000/0.33 - 1000 \approx 2030$ MW. In a separate model with the **same efficiency** and coal energy 3.0×10^7 J per kilogram, required coal is about 8.7×10^3 tonnes per day.

Fission products and their management

The properties of fission products

- Many primary fragments have too many neutrons for their size (heavy nuclei need a higher neutron–proton ratio than medium ones), so they are **radioactive**, mostly beta-minus and gamma emitters.
- They have a wide range of half-lives, from seconds to millions of years.
- Spent fuel is so radioactive that it is **hot**: it keeps producing thermal energy from decay after the reactor stops, and must be cooled.
- Neutron capture followed by beta decays can convert some uranium-238 into **plutonium-239** (half-life 24 000 years), which is radioactive, toxic, and can be used in nuclear weapons.



Invented mixture in arbitrary activity units, not a spent-fuel inventory, uranium-ore comparison or prediction of a safe disposal time

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Managing nuclear waste

Nuclear waste 核废料 management considers activity, half-lives, heat generation and the physical/chemical form. Categories and acceptance rules are not determined by activity alone; ARPANSA's explanation illustrates this distinction.

- Suitable **low-level waste**, such as some contaminated clothing and tools, may go to engineered near-surface facilities after conditioning. Not every used object belongs in that category.
- **Intermediate-level waste** needs appropriate shielding and isolation; packages, depth and disposal routes depend on long-lived content and the waste acceptance criteria.
- **High-level waste**, including spent fuel where declared waste, needs heat management and long-term isolation. Water pools initially remove decay heat and provide shielding. Later storage can use licensed dry casks; reprocessing is an alternative fuel-cycle route, not disposal and not elimination of all radioactive waste.
- Some liquid high-level residues from reprocessing can be immobilised in glass, called vitrification. Intact spent-fuel assemblies need not be melted into glass. Geological disposal uses engineered packages and suitable rock to limit release and transport over long times.

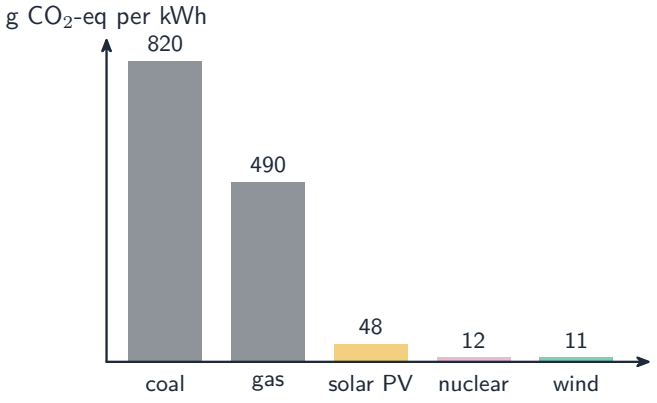
The impact of long-term storage

- Storage is monitored and can require continuing management. Geological disposal aims for long-term passive isolation using several barriers, rather than relying solely on containers remaining perfect or on future people actively operating a cooling pond forever.
- A technical IAEA account describes geological disposal for high-activity and/or long-lived waste, including spent fuel. Site-specific groundwater, heat, material degradation and long-term geological change must be considered.
- Security, records, safeguards, costs and community participation matter. No one generic activity curve establishes a universal time at which every spent fuel becomes safe or matches uranium ore; activity, heat and exposure risk are different quantities.

Fission and climate change

Fission is a debated part of the response to climate change.

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whole life cycle: mining, building, running, decommissioning; medians from the IPCC (2014), rounded

Life-cycle emissions per kWh

- **For**: almost no carbon dioxide is released while generating electricity; it can provide sustained output with a high fuel energy density. Mining, fuel preparation, construction and decommissioning still have environmental impacts, and cooling constraints or maintenance can limit output.
- **Against**: long-lived radioactive waste; the risk of serious accidents (Chernobyl 1986, Fukushima 2011); high building costs and long construction times; links to nuclear weapons.

Practice

1. Distinguish between spontaneous fission and neutron-induced fission.
2. Complete the reaction ${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + x\,{}^{1}_{0}\mathrm{n}$ by finding x, and check that charge is conserved.
3. For the reaction in question 2, the atomic masses are uranium-235 235.043930 u, xenon-140 139.921641 u and strontium-94 93.915361 u. Find the energy released in MeV and in J.
4. For the balanced reaction in question 2, use uranium-235 binding energy per nucleon 7.6 MeV and the fragments' mean 8.5 MeV. Estimate released energy, accounting for free neutrons.
5. Explain why the products of fission have less total mass than the original nucleus and neutron.
6. (a) Explain what is meant by a chain reaction and by critical mass. (b) State what must be true about the neutrons from each fission for a reactor to run at a steady power.
7. Explain the role of the moderator, and why light nuclei make good moderators.

8. Explain how the control rods control the power of a reactor, and what happens if they are raised.
9. (a) For the pressurised-water design shown, explain the heat exchanger and separate primary loop. Is that circuit universal for every reactor? (b) State the purpose of the shielding, and what it must absorb.
10. A power station has an electrical output of 600 MW and an efficiency of 32 %. Each fission releases 200 MeV. Find the mass of uranium-235 used in one year (3.16×10^7 s). Treat 200 MeV as deposited energy, operation as steady, and each uranium-235 nucleus as 235 u. Distinguish this mass from total fuel loaded.
11. Explain why fission fragments are radioactive, and state the type of decay they usually undergo.
12. Describe how high-level nuclear waste is managed, and discuss two problems of storing it for a long time.
13. Why do many light-water reactors use enriched uranium, and why should this not be stated as a requirement for every reactor?
14. Discuss one advantage and one disadvantage of nuclear fission as a way of reducing carbon dioxide emissions.

Answers.

1. Spontaneous fission happens by itself, randomly, without any outside cause; neutron-induced fission happens when a nucleus absorbs a neutron and becomes unstable.
2. Nucleons: $1+235 = 140+94+x$, so $x = 2$. Charge: $0+92 = 54+38+0 = 92$. ✓
3. Before: 236.052595 u. After: $139.921641+93.915361+2\times1.008665 = 235.854332$ u. $\Delta m = 0.198263$ u, $E = 0.198263 \times 931.5 = 185$ MeV. Using the unrounded 184.6819845 MeV gives 2.95×10^{-11} J.
4. $B_{\text{before}} = 235 \times 7.6 = 1786$ MeV. The two fragments have $140 + 94 = 234$ bound nucleons, so $B_{\text{after}} = 234 \times 8.5 = 1989$ MeV; $Q = 1989 - 1786 = 203$ MeV, about 2.0×10^2 MeV. Free neutrons have zero nuclear binding energy; the rounded assigned inputs give an estimate rather than the precise mass result.
5. The fragments are more tightly bound (higher binding energy per nucleon), so energy is released when they form; by $E = mc^2$ this energy corresponds to a decrease in mass.
6. (a) A chain reaction is one in which neutrons released by each fission go on to cause further fissions, so the reaction keeps itself going. Critical mass is the smallest mass of fuel in which this can be sustained, because in a smaller mass too many neutrons escape. The critical mass depends on specified composition and physical conditions, not the element alone. (b) On average, exactly one neutron from each fission must cause another fission.

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7. It slows fast neutrons down by elastic collisions, so that they are much more likely to be absorbed by uranium-235 and cause fission. A neutron loses most energy in a collision with a nucleus of similar mass, so light nuclei such as hydrogen or carbon are used.
8. They absorb neutrons, so fewer neutrons are available to cause fission. Raising them absorbs fewer neutrons, so the rate of fission and the power increase. This can be a controlled increase, not necessarily an explosion. Shutdown does not remove existing radioactive products or their decay heat.
9. (a) In a pressurised-water reactor, primary coolant transfers heat through a wall to secondary water, producing turbine steam without normal bulk mixing. This limits transfer of primary radioactive material. A boiling-water reactor instead makes steam in the reactor vessel; the two-loop diagram is not universal. (b) To protect workers and the environment by absorbing the neutrons and gamma rays from the core.
10. Thermal power = $600/0.32 = 1875$ MW. Fissions per second = $(1.875\times10^{9})/(3.2\times10^{-11}) = 5.9\times10^{19}$. Per year: 1.85×10^{27} nuclei, mass = $1.85\times10^{27}\times235\times1.661\times10^{-27} = 7.2\times10^{2}$ kg. This is the uranium-235 actually fissioned under the assigned model, not total fuel-assembly replacement mass.
11. They have too many neutrons for their size (a neutron–proton ratio too high for medium nuclei), so they are unstable; they usually decay by beta-minus emission, often with gamma rays.
12. Initially remove decay heat and provide shielding, commonly in water pools; later storage may use dry casks. Reprocessing recovers materials but leaves radioactive waste. Some residues can be vitrified; intact spent fuel need not be. Geological disposal aims at passive isolation. Long-term problems include heat and long-lived activity, groundwater transport/barrier degradation, security/safeguards, costs/records and site acceptance. A generic activity curve gives no universal safe-time threshold.
13. Natural uranium has only roughly .7% uranium-235. Many light-water reactors need a higher fissile fraction to offset neutron losses and sustain their intended chain. Other reactor designs can use natural uranium; moderation, absorption, leakage and fuel properties differ.
14. Low direct operational carbon emissions and sustained output are advantages; long-lived waste/decay heat, accident risk, cost/construction time and security concerns are disadvantages. This does not imply zero lifecycle impact or uninterrupted output.

Exam tips

- Balance A, Z and every neutron in the reaction.
- Calculate Q from precise rest masses or **total** binding energies.

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Fusion and stars

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

This handout teaches E.5 Fusion and stars in full, for the course first assessed in 2025. There is no additional higher level content in E.5, so SL and HL students study the same material.

Prerequisites: binding energy, the binding energy curve and $E = mc^2$ from E.3; fission from E.4; black-body radiation, the Stefan–Boltzmann law, Wien's law and apparent brightness from B.1; Coulomb repulsion from D.2 (the barrier-energy estimate is introduced here); emission and absorption spectra from E.1. Use 1 u = 931.5 MeV c^{-2}, the Sun's luminosity $L_\odot = 3.83\times10^{26}$ W, mass $M_\odot = 1.99\times10^{30}$ kg, radius $R_\odot = 6.96\times10^{8}$ m and surface temperature $T_\odot = 5800$ K.

Stars in equilibrium

How can a hot star be faint?

Temperature alone does not tell us a star's total power. A very hot white dwarf can have a small emitting area and low luminosity; a cooler giant can radiate much more because its area is enormous. Distance adds a second ambiguity: a luminous distant star can look as faint as a less luminous nearby one. We need several measurements and a physical model to distinguish these cases.



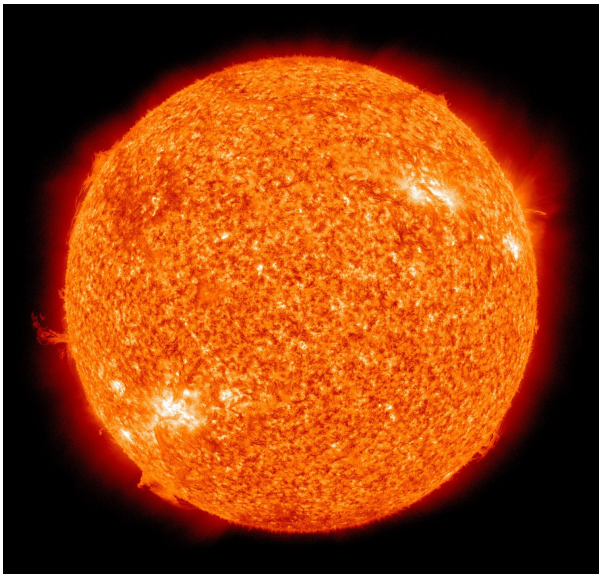
Area matters as much as temperature

Worked stellar calculations below use **assigned illustrative data**, not reported measurements of named stars, except the explicitly stated rounded solar reference values. The model links fusion and pressure support to luminosity, distance, radius and stellar evolution.

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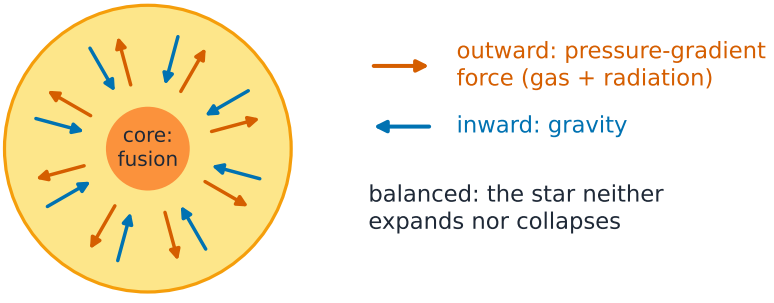
A balance of forces

A star is a huge ball of hot gas, mostly hydrogen and helium. Two effects act on every layer of it:



The Sun, seen in ultraviolet light: a stable main-sequence star in which gravity and outward pressure balance

- **Gravity** pulls all the gas inwards, towards the centre.
- The outward force comes from a **pressure gradient**: gas pressure and **radiation pressure** 辐射压 are greater at smaller radius, so the inner face of a layer feels more pressure than the outer face. Absolute pressure is not itself a net outward force.



Hydrostatic support: the outward pressure-gradient force balances gravity in each layer

In a stable star these two effects balance, so the star is in **equilibrium** 平衡: it neither collapses nor expands. The balance controls itself:

- If fusion speeds up, the pressure rises and the star expands slightly. The core cools, fusion slows down again, and the star settles back.

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- If fusion slows down, gravity squeezes the core. It heats up, and fusion speeds up again.

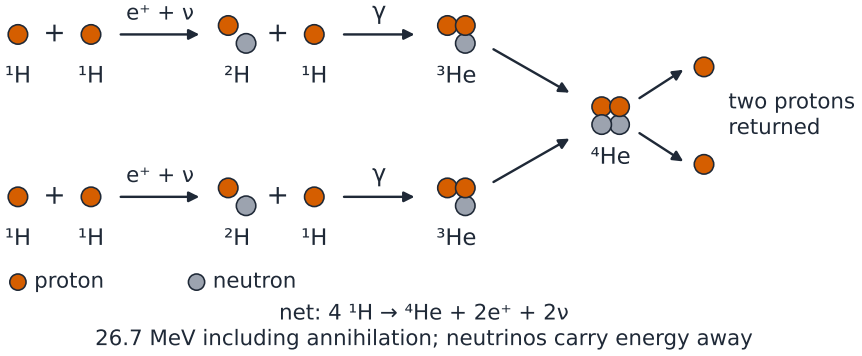
This is hydrostatic equilibrium, not motionlessness of every particle and not simply equality of the nuclear and gravitational energy totals. Main-sequence thermal regulation links temperature-sensitive fusion to expansion/contraction. Exhausting a core fuel changes the structure; later quasi-equilibrium stages can still exist.

Fusion: the energy source of stars

Nuclear fusion 核聚变 is the joining of two light nuclei to form a heavier nucleus. Suitable light-nucleus reactions increase total binding energy and release energy. Compare the actual initial and final masses; not every possible fusion reaction is exothermic.

The proton–proton chain

In the Sun, hydrogen becomes helium in the **proton–proton chain** 质子质子链:



Four protons become one helium-4 nucleus

$$ {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + {}^{0}_{+1}\mathrm{e} + \nu $$

$$ {}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + \gamma $$

$$ {}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H} $$

The first step makes a **positron** 正电子 and a **neutrino** 中微子, through a weak-interaction conversion of a proton into a neutron during fusion. This is not spontaneous beta-plus decay of an isolated free proton. No stable two-proton nucleus is implied. The overall result is

$$ 4\,{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{0}_{+1}\mathrm{e} + 2\nu $$

Worked example: the energy released

Atomic masses: hydrogen-1 1.007825 u, helium-4 4.002603 u.

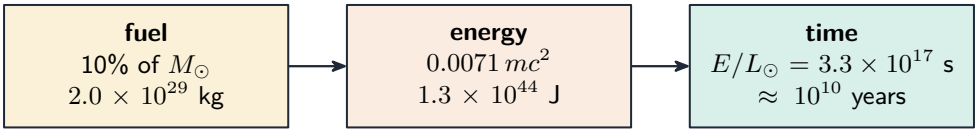
- Before: $4 \times 1.007825 = 4.031300$ u. After: 4.002603 u.
- $\Delta m = 0.028697$ u, so $E = 0.028697 \times 931.5 = 26.7$ MeV.

Using atomic masses also counts the energy released when the two positrons annihilate with two electrons. A small part of the energy is carried away by the neutrinos, which escape from the Sun.

Only $0.028697/4.031300 = 0.71\,\%$ of the initial hydrogen mass is released as energy; most remains in helium. For these hydrogen-to-helium and uranium-fission examples, the fusion energy per kilogram is greater. That comparison is reaction-specific.

How long will the Sun shine?

- Using photon luminosity alone, the radiated mass-equivalent rate is $\frac{L_\odot}{c^2} = \frac{3.83 \times 10^{26}}{(3.00 \times 10^8)^2} = 4.3 \times 10^9$ kg s^{-1}: about four million tonnes each second.
- In an assigned photon-only model with mass-conversion fraction 0.0071, hydrogen consumption is about $(4.26 \times 10^9)/0.0071 = 6.0 \times 10^{11}$ kg s^{-1}. This is much larger than the converted mass; most fuel mass remains in products.
- For a rough lifetime model, **assume an effective available hydrogen fuel mass** of 10% of the solar mass. This is not a claim that exactly 10% of the star is pure hydrogen at one temperature. Available energy: $E_{\text{avail}} = 0.10 \times 1.99 \times 10^{30} \times 0.0071 \times (3.00 \times 10^8)^2 = 1.27161 \times 10^{44}$ J before final rounding.
- Time: $t = E_{\text{avail}}/L_\odot$. Using the unrounded energy, $t = (1.27161 \times 10^{44})/(3.83 \times 10^{26}) \approx 3.32 \times 10^{17}$ s $\approx 1 \times 10^{10}$ years. This estimates the **total** main-sequence lifetime, not time remaining. The model treats luminosity and available fuel approximately; photon luminosity excludes escaping neutrino energy, and solar-wind mass loss is a separate contribution.



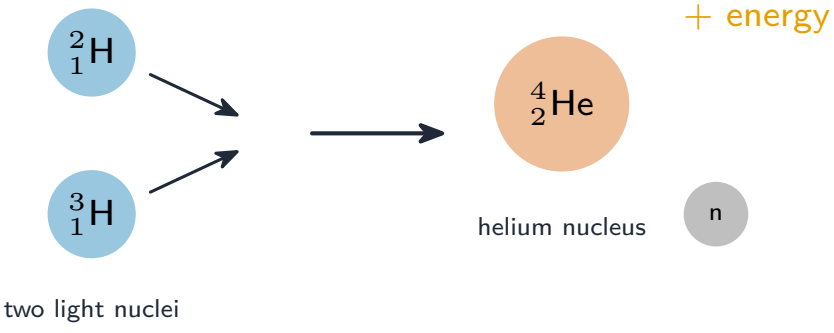
a rough model: $L_\odot = 3.83 \times 10^{26}$ W held constant

A budget for the Sun's lifetime

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Fusion and fission compared

	Fission	Fusion
Nuclei	one heavy nucleus splits	two light nuclei join
Where on the binding energy curve	from the right, towards the peak	from the left, towards the peak
Energy per kilogram of fuel	about 8×10^{13} J (uranium-235)	about 6×10^{14} J (hydrogen to helium)
Conditions	a suitable neutron balance; thermal and fast reactors differ	suitable temperature, density and confinement time for the reaction
Products	many radioactive fragments and emitted neutrons	depend on the reaction: helium and neutrons for D–T; tritium is radioactive

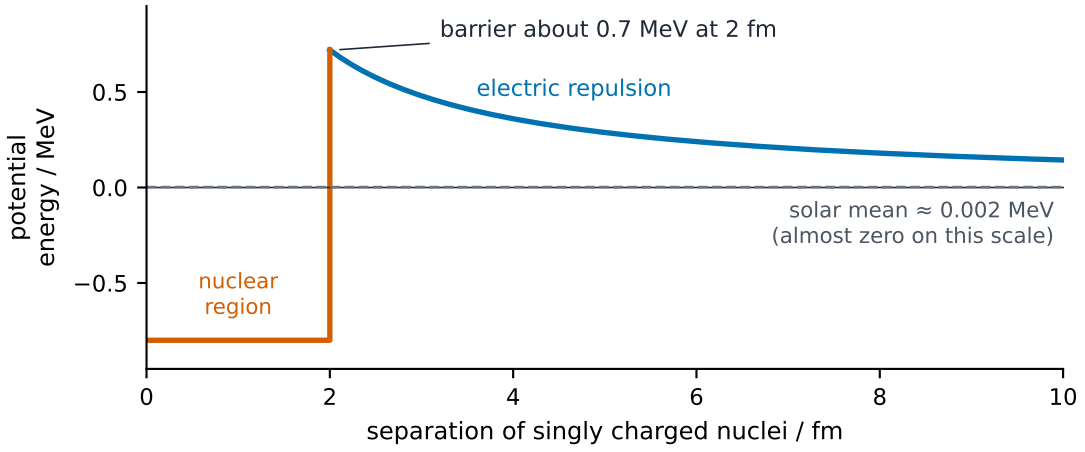


In fusion two light nuclei join into a heavier one, releasing a neutron and energy – the deuterium–tritium reaction used in fusion reactors

One studied terrestrial fusion reaction uses deuterium and tritium, $^{2}_{1}\mathrm{H}+^{3}_{1}\mathrm{H} \rightarrow ^{4}_{2}\mathrm{He}+^{1}_{0}\mathrm{n}$, which releases 17.6 MeV. Confinement, energy balance and materials are engineering challenges. D–T neutrons can activate surrounding materials, while tritium itself is radioactive; fusion is not automatically free of radioactive waste.

Conditions for fusion

Nuclei are positive, so they repel each other. Nuclear interactions become important at femtometre separations. Reaching that region requires penetration of the electric repulsion, called the **Coulomb barrier** 库仑势垒.



Schematic barrier penetration; tunnelling does not require classical energy above the barrier, and the well does not represent a stable diproton

■ Worked example: how hot?

- The electric potential energy of two protons 2.0×10^{-15} m apart: $E = \dfrac{ke^2}{r} = \dfrac{8.99 \times 10^9 \times (1.60 \times 10^{-19})^2}{2.0 \times 10^{-15}} = 1.2 \times 10^{-13}$ J ≈ 0.7 MeV.
- If the average kinetic energy $\frac{3}{2}kT$ had to equal this: $T = \dfrac{2E}{3k_\mathrm{B}} = \dfrac{2 \times 1.2 \times 10^{-13}}{3 \times 1.38 \times 10^{-23}} = 6 \times 10^9$ K.

The Sun's core is only about 1.5×10^7 K, four hundred times cooler. Fusion still happens because:

- the particles have a range of speeds, and a very small fraction move much faster than average;
- quantum physics lets a proton sometimes pass through the barrier without enough energy to go over it (**quantum tunnelling** 量子隧穿).

Only a small fraction of encounters fuse. In the proton–proton chain, the weak-interaction first step further limits the rate; tunnelling plus a high-energy tail is not a claim that any close proton pair instantly becomes stable helium.

The conditions for fusion in a star are therefore:

- **high temperature** for the relevant stellar reaction, giving a distribution of collision energies and appreciable barrier penetration;
- **very high density**, so that collisions happen often enough. Density and a long confinement time supplied by gravity sustain the stellar reaction rate. Temperature and density alone do not specify a universal rate or ignition threshold for every fuel.

Gravity provides both. As a cloud of gas collapses, it becomes denser, and gravita-

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tional potential energy is transferred to thermal energy until the core is hot enough for fusion.

Distances to stars

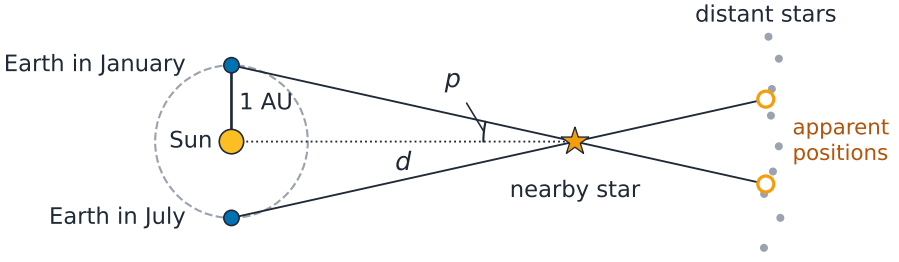
Stars are so far away that metres are inconvenient. Three larger units are used.

Unit	Definition	Value
astronomical unit 天文单位 (AU)	a defined length scale close to the Earth–Sun orbital semimajor axis	1.50×10^{11} m
light year 光年 (ly)	the distance light travels in one year	9.46×10^{15} m
parsec 秒差距 (pc)	the distance at which 1 AU makes an angle of one arc-second	3.09×10^{16} m $= 3.26$ ly

One arc-second is $\frac{1}{3600}$ of a degree. The nearest star other than the Sun, Proxima Centauri, is about 1.30 pc (4.24 ly) away; the centre of our galaxy is about 8000 pc away.

Stellar parallax

As the Earth moves round the Sun, a nearby star seems to move slightly against the background of much more distant stars. This is **stellar parallax** 恒星视差. You see the same effect if you hold up a finger and look at it with each eye in turn.



The parallax angle p is half the total apparent shift (angle exaggerated)

- Measure the star's position against the background, then again six months later.
- The **parallax angle** p is half the angle through which the star appears to move. It is the angle at the star made by one astronomical unit.
- The distance is

$$d\,(\text{parsec}) = \frac{1}{p\,(\text{arc-second})}$$

Invented example: $p = 0.050''$ gives $d = 20$ pc $= 65.2$ ly $= 6.18 \times 10^{17}$ m with the stated conversions. In the ideal geometry shown the total shift is $2p$; in real observations the

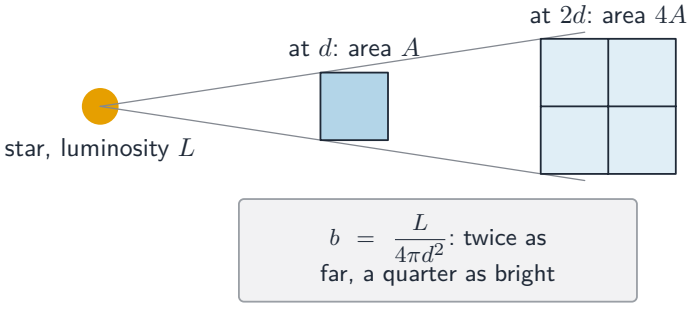
annual parallax pattern is fitted separately from proper motion.

As distance grows, the angle becomes harder to distinguish from instrumental uncertainty. There is no universal 100 pc cutoff for every ground instrument or one distance limit for all stars. Space astrometry improves precision, but distance estimates still need uncertainties and an appropriate fit. For small relative error, $\Delta d/d \approx \Delta p/p$; if $p = 0.050 \pm 0.002$ arc-seconds, the approximate distance uncertainty is 4%, or 0.8 pc. Very uncertain parallaxes should not be inverted as though exact.

Luminosity, brightness and temperature

Luminosity and apparent brightness

- The **luminosity** 光度 L of a star is the total power it emits, in W. For a black body, $L = \sigma A T^4 = 4\pi R^2 \sigma T^4$.
- The **apparent brightness** 视亮度 b is the power received per unit area at the observer, in W m^{-2}. For approximately isotropic emission, **bolometric** brightness includes radiation across all wavelengths. Correct for extinction before using the unattenuated model; the power spreads over a sphere, so



The same power spreads over a sphere of area $4\pi d^2$

$$b = \frac{L}{4\pi d^2}$$

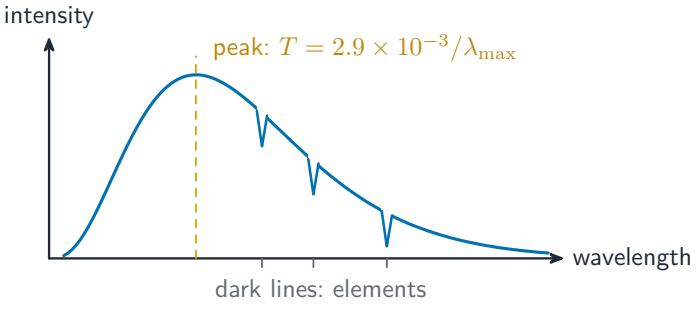
A star can look bright because it is very luminous or because it is close. Knowing b and the distance d from parallax gives L.

What the spectrum tells us

- **Surface temperature**: a stellar continuum can be approximated by a black-body wavelength spectrum, with effective temperature and atmospheric departures kept in mind. Wien's law, $\lambda_{\max} T = 2.9 \times 10^{-3}$ m K, gives the temperature from the peak wavelength. The Sun peaks at about 500 nm; hot stars peak in the ultraviolet and look blue-white, cool stars peak in the infrared and look red.
- **Composition**: the cooler outer gas absorbs particular wavelengths, giving dark absorption lines. Several calibrated line positions can indicate elements and ionisa-

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tion states. Line visibility and strength also depend on temperature, populations and atmospheric conditions; one weak line does not by itself prove low abundance.



Reading a stellar spectrum

■ **Worked example: an assigned hot star**

An invented hot star has a parallax of $0.379''$, a bolometric apparent brightness of 1.1×10^{-7} W m^{-2}, and an ideal wavelength-continuum peak at 293 nm. Neglect extinction and retain unrounded intermediate values.

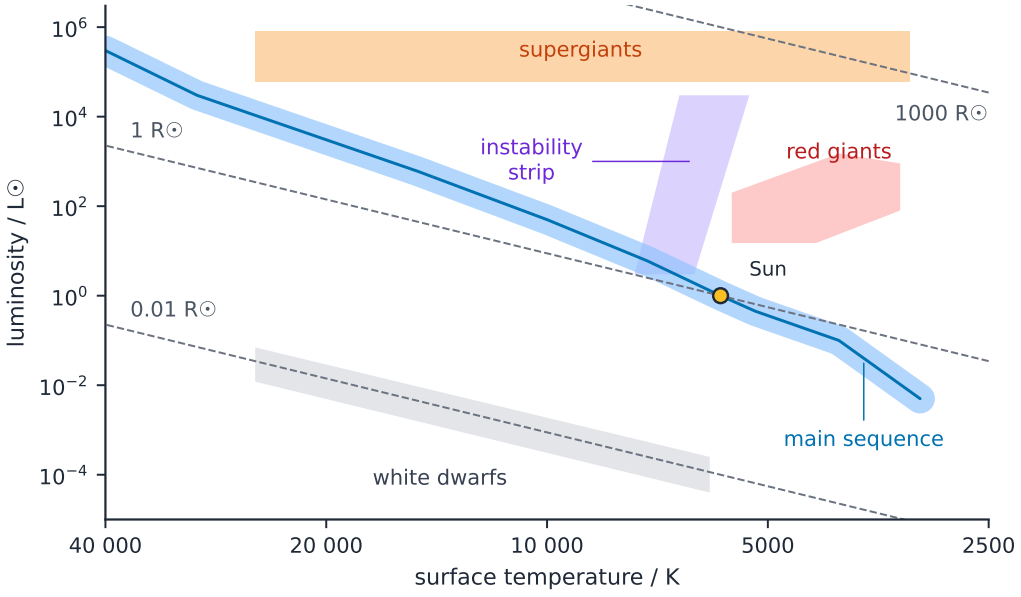
- Distance: $d = 1/0.379 = 2.6385$ pc $\approx 2.6385 \times 3.09 \times 10^{16} = 8.153 \times 10^{16}$ m.
- Luminosity: $L = 4\pi d^2 b = 4\pi \times (8.153 \times 10^{16})^2 \times 1.1 \times 10^{-7} = 9.2 \times 10^{27}$ W, about $24\,L_\odot$.
- Temperature: $T = (2.9 \times 10^{-3})/(293 \times 10^{-9}) = 9900$ K.

The Hertzsprung–Russell diagram

A **Hertzsprung–Russell diagram** 赫罗图 (HR diagram) plots the luminosity of stars against their surface temperature.

- Luminosity is on the vertical axis, often as a multiple of the Sun's luminosity, on a logarithmic scale.
- Temperature is on the horizontal axis, also logarithmic, and it **decreases** to the right.

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Stars fall into a few clear groups on the HR diagram

Region	Where	Properties
main sequence 主序	a band from top left to bottom right	stars fusing hydrogen in their core; more massive ones are hotter and more luminous
red giants 红巨星	above the main sequence, on the right	cool (3000–5000 K) but luminous, so very large
supergiants 超巨星	across the top	the most luminous stars, up to a million times the Sun's luminosity; red supergiants are up to about 1000 times the Sun's radius
white dwarfs 白矮星	below the main sequence, on the left	hot but dim, so very small, about the size of the Earth
instability strip 不稳定带	a narrow band crossing the main sequence upwards	a region containing many pulsating variable stars; being drawn inside a schematic strip does not guarantee every object has the same variability

■ Lines of constant radius

From $L = 4\pi R^2 \sigma T^4$, stars with the same radius lie on a straight line on the logarithmic axes. Radius increases towards the top right: a cool star can only be very luminous if it is very large. This is how we know that red giants are giant and that white dwarfs are small, without seeing their size.

Stellar radii

Rearranging $L = 4\pi R^2 \sigma T^4$:

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

so the radius comes from the luminosity (from brightness and distance) and the temperature (from the spectrum). Most stars are unresolved by ordinary images, but suitable interferometry can measure angular diameters of some stars; ESO describes this technique. Combining angular size with distance provides another radius estimate.

■ Worked example: the same assigned hot star

$$R = \sqrt{\frac{9.2 \times 10^{27}}{4\pi \times 5.67 \times 10^{-8} \times 9900^4}} = 1.2 \times 10^9 \text{ m} = 1.7\, R_\odot$$

■ Comparing with the Sun

Dividing by the same equation for the Sun cancels the constants:

$$\frac{R}{R_\odot} = \sqrt{\frac{L}{L_\odot}} \times \left(\frac{T_\odot}{T}\right)^2$$

- Invented red-supergiant example: $L = 1.2 \times 10^5\, L_\odot$, $T = 3500$ K. $R = \sqrt{1.2 \times 10^5} \times (5800/3500)^2\, R_\odot = 346 \times 2.75\, R_\odot = 950\, R_\odot$, a red supergiant larger than the orbit of Mars.
- Invented hot-white-dwarf example: $L = 0.026\, L_\odot$, $T = 25\,000$ K. $R = \sqrt{0.026} \times (5800/25\,000)^2\, R_\odot = 0.0087\, R_\odot = 6 \times 10^6$ m, a white dwarf about the size of the Earth.

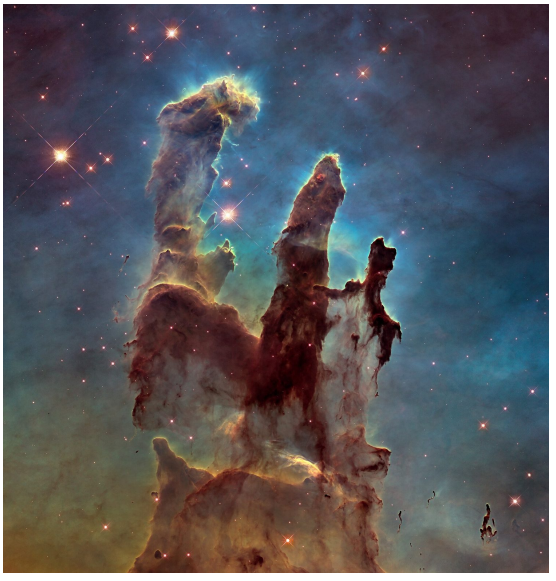
How mass decides a star's life

■ Birth and the main sequence

- A cloud of gas and dust, a **nebula** 星云, collapses under gravity. It heats up and becomes a **protostar** 原恒星.
- When the core reaches about 10^7 K, hydrogen fusion starts and the star joins the main sequence, in equilibrium.

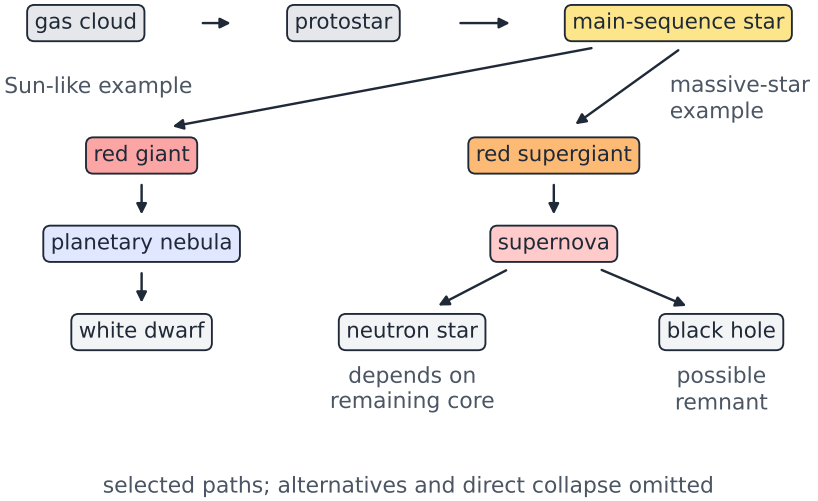
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- Hydrogen-core burning is usually the longest luminous nuclear-burning stage; the fraction and duration depend on mass and composition. Where it sits depends on its mass: more massive stars are hotter and more luminous.



The Pillars of Creation: dense columns of gas and dust where new stars are forming; each star's mass will decide its life

More massive stars have more fuel, but they use it much faster. A star of 10 solar masses is thousands of times as luminous as the Sun, so it lives only tens of millions of years, not ten billion.



Initial mass strongly affects evolution; these selected paths omit binary effects and alternatives such as direct collapse

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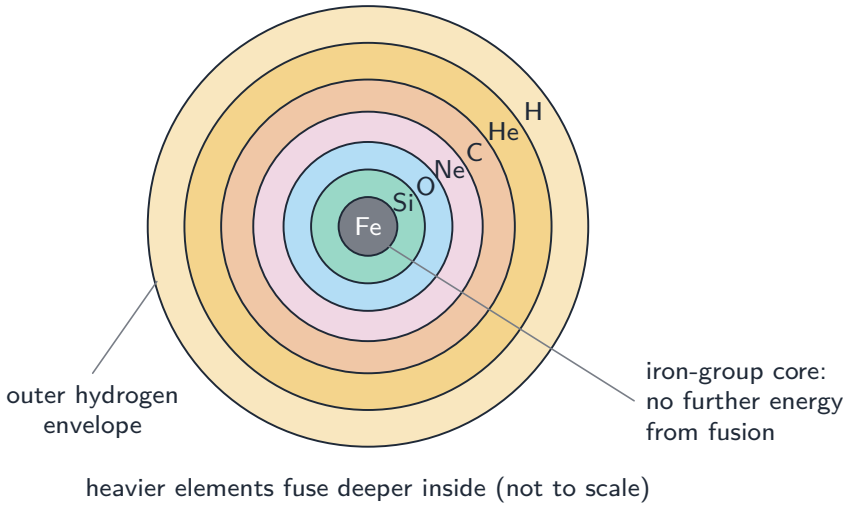
A Sun-like single-star path

- When the hydrogen in the core runs out, fusion there stops. The core contracts and heats up; hydrogen fuses in a shell around it, and the outer layers expand and cool. The star becomes a **red giant**: it moves up and to the right on the HR diagram.
- The core becomes hot enough (about 10^8 K) for helium to fuse into carbon and oxygen.
- When this fuel runs out, the outer layers drift away as a glowing shell of gas, a **planetary nebula** 行星状星云.
- The hot core is left as a **white dwarf**: no sustained core fusion, made mostly of carbon and oxygen, about the size of the Earth. It is held up by the pressure of electrons packed closely together, which is possible only if its mass is less than about 1.4 solar masses. It cools slowly and generally moves down and to the right on the HR diagram. The approximate 1.4 solar-mass limit concerns the electron-degenerate remnant, not the initial star. Very low-mass stars can have different, much slower paths and may never ignite core helium.

Massive stars (more than about 8 solar masses)

- A massive star can pass through a **red supergiant** phase; not every massive star follows the same red path.
- Its core is so hot that fusion continues to heavier and heavier elements: carbon, neon, oxygen, silicon and finally iron, in layers like an onion.
- The iron-group region is tightly bound, so continued fusion to substantially heavier products does not provide the same exothermic support. An iron-group core can temporarily be supported without further core burning; continued evolution can destabilise that support and cause rapid collapse.
- The outer layers are blown off in a huge explosion, a **supernova** 超新星, which can for a while be as luminous as a whole galaxy.
- A remaining compact core can become a **neutron star** 中子星 or a **black hole** 黑洞. Neutron stars have masses of order a few solar masses or less and sizes of order tens of kilometres. A black-hole event horizon prevents outward light signals escaping.
- Initial mass and final core mass are different. Approximate classroom mass boundaries are not exact universal thresholds: mass loss, composition, rotation, binary interaction and the high-density equation of state affect outcomes. Some massive stars can collapse to black holes without a bright visible supernova.

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Onion layers in a late massive star: heavier elements fuse deeper inside, around an iron-group core (not to scale).

How elements are made

- The early universe made hydrogen, much of the helium and some lithium. Stellar burning produces many heavier nuclei, including carbon and oxygen and iron-group material; the origin of every element is not the same single fusion sequence.
- **Slow neutron capture** in evolved stars can build nuclei heavier than iron, as illustrated by ESO's lead-star observations. Rapid neutron capture in extreme neutron-rich events, including neutron-star mergers, makes other heavy nuclei. The relative contributions of different sites are a research question; not all heavy nuclei must come from one type of explosion.
- Explosions and stellar winds disperse enriched material, from which later stars and planets form. This explains why a later-generation system can contain material made by several earlier processes.

Practice

1. State the two effects that are balanced in a main-sequence star, and explain what happens if the rate of fusion in the core increases.
2. In the last step of the proton–proton chain, two helium-3 nuclei (3.016029 u each) fuse to form helium-4 (4.002603 u) and two hydrogen-1 atoms (1.007825 u each). Find the energy released in MeV.
3. For the illustrative D–T fusion reaction, use ${}_1^2\mathrm{H} + {}_1^3\mathrm{H} \rightarrow {}_2^4\mathrm{He} + {}_0^1\mathrm{n}$. The masses are hydrogen-2 2.014102 u, hydrogen-3 3.016049 u, helium-4 4.002603 u and neutron 1.008665 u. Find the energy released.

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4. Using photon luminosity alone, calculate the Sun's radiated mass-equivalent rate. Explain why this is not its hydrogen consumption rate or its complete mass-loss budget.
5. (a) Explain why fusion in a star needs both a very high temperature and a very high density. (b) Estimate the temperature at which the average kinetic energy of protons would let them reach a separation of 1.0×10^{-15} m. Explain why fusion in the Sun happens at a much lower temperature.
6. Convert 4.2 ly into parsecs and into metres.
7. (a) A star has a parallax angle of $0.050''$. Find its distance in parsecs, light years and metres. (b) Explain why stellar parallax can be used only for relatively nearby stars.
8. The spectrum of a star peaks at 350 nm. Find its surface temperature, and state one other property of the star that its spectrum can show.
9. A star of luminosity 3.0×10^{28} W is 50 pc from the Earth. Find its apparent brightness.
10. The star in question 9 has a surface temperature of 4000 K. Find its radius in metres and as a multiple of the Sun's radius, and suggest which region of the HR diagram it is in.
11. Star A has the same luminosity as star B but twice its surface temperature. Find the ratio of their radii, R_A/R_B.
12. State the region of the HR diagram for each star: (a) $T = 20\,000$ K, $L = 0.01\,L_\odot$; (b) $T = 3500$ K, $L = 10^5\,L_\odot$; (c) $T = 6000$ K, $L = 1\,L_\odot$.
13. Describe what happens to a star like the Sun after it leaves the main sequence, including its path on the HR diagram.
14. Explain why fusion in a massive star stops at iron, and state where elements heavier than iron are made. State the possible remnants of a star of 20 solar masses.

Answers.

1. The outward pressure-gradient force balances inward gravity. In the stable main-sequence feedback picture, a higher energy-generation rate can cause expansion and cooling, reducing fusion; hydrostatic equilibrium does not imply that every gas particle is stationary.
2. Before: $2 \times 3.016029 = 6.032058$ u. After: $4.002603 + 2 \times 1.007825 = 6.018253$ u. $\Delta m = 0.013805$ u, $E = 12.9$ MeV.
3. Before: 5.030151 u. After: 5.011268 u. $\Delta m = 0.018883$ u, $E = 17.6$ MeV $= 2.8 \times 10^{-12}$ J.
4. $\dot{m}_\gamma = L_\odot/c^2 = (3.83 \times 10^{26})/(3.00 \times 10^8)^2 = 4.26 \times 10^9$ kg s^{-1}. Photon energy escaping reduces solar mass-energy by that equivalent amount. Most consumed hydrogen mass remains as helium; neutrino energy and solar-wind mass loss are separate from the given photon-luminosity budget.
5. (a) High temperature gives a distribution of collision energies; tunnelling allows

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barrier penetration even when typical kinetic energy lies below the classical barrier. High stellar density and long gravitational confinement sustain enough encounters. The proton–proton weak-interaction step also limits its rate; temperature alone is not a universal ignition rule. (b) $E = ke^2/r = [8.99 \times 10^9 \times (1.60 \times 10^{-19})^2]/(1.0 \times 10^{-15}) = 2.3 \times 10^{-13}$ J; $T = 2E/(3k_B) = 1.1 \times 10^{10}$ K. In the Sun a small fraction of protons move much faster than average, and quantum tunnelling lets some pass through the barrier.

6. $4.2/3.26 = 1.3$ pc; $4.2 \times 9.46 \times 10^{15} = 4.0 \times 10^{16}$ m.
7. (a) $d = 1/0.050 = 20$ pc $= 65$ ly $= 6.2 \times 10^{17}$ m. (b) The angular amplitude decreases as distance grows. For a fixed absolute angular uncertainty, relative distance uncertainty grows roughly as $\Delta p/p$. The useful range depends on the instrument and observations, not one universal ground-based cutoff.
8. $T = (2.9 \times 10^{-3})/(350 \times 10^{-9}) = 8300$ K. Its absorption lines show which elements are in its outer layers.
9. $d = 50 \times 3.09 \times 10^{16} = 1.545 \times 10^{18}$ m, retained for the next step; $b = L/(4\pi d^2) = 3.0 \times 10^{28}/(4\pi \times (1.545 \times 10^{18})^2) \approx 1.0 \times 10^{-9}$ W m^{-2}.
10. $R = \sqrt{3.0 \times 10^{28}/(4\pi \times 5.67 \times 10^{-8} \times 4000^4)} = 1.3 \times 10^{10}$ m $= 18\,R_\odot$. Cool and large: a red giant.
11. $R \propto \sqrt{L}/T^2$, so $R_A/R_B = 1/2^2 = 1/4$.
12. (a) white dwarf; (b) red supergiant (or the supergiant region); (c) main sequence.
13. The core runs out of hydrogen and contracts and heats; hydrogen fuses in a shell, and the star expands and cools into a red giant (moving up and right on the HR diagram). Helium fuses to carbon and oxygen. The outer layers are lost as a planetary nebula, and the core becomes a white dwarf (moving to the lower left), which slowly cools.
14. Iron-group material is near the broad binding-energy maximum, so continued fusion to substantially heavier products does not supply the earlier exothermic support. Heavier nuclei can form through slow neutron capture in evolved stars and rapid capture in extreme neutron-rich events such as neutron-star mergers. A 20-solar-mass initial star can leave a neutron star or black hole; the actual outcome depends on the remaining core and evolution, not an exact initial-mass-only threshold.

Exam tips

- Use parallax semi-amplitude p in arc-seconds for $d = 1/p$ in parsecs; convert distance units afterwards.
- Square d in $b = L/(4\pi d^2)$ and use T^4 in $L = 4\pi R^2 \sigma T^4$.

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Experimental programme

International Baccalaureate · IB Diploma · Physics · HL

Scope and prerequisites

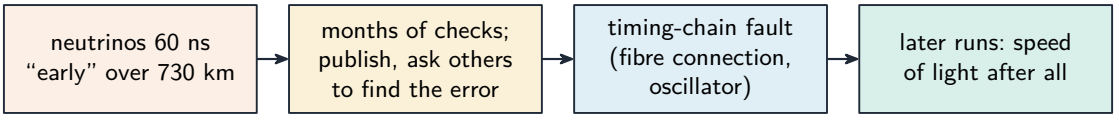
This handout teaches the experimental programme of the course first assessed in 2025: the tools (experimental techniques, technology and mathematics), the inquiry process, and the internal assessment. They are the same for SL and HL students, and they are tested in every paper as well as in the scientific investigation.

Prerequisites: SI units and prefixes, rearranging equations, straight-line graphs ($y = mx + c$), and logarithms. All numerical measurement datasets below are invented teaching examples, not claimed experimental records. The examples use physics from earlier units, especially the simple pendulum, $T = 2\pi\sqrt{L/g}$.

Measurement and uncertainty

Faster than light?

In 2011 the OPERA experiment timed neutrinos travelling 730 km from Geneva to Italy and found that they arrived 60 nanoseconds earlier than light would. If true, this would have broken special relativity. The physicists checked their method for months, then published the result and asked others to find the mistake. The timing chain was investigated, including its fibre-optic connection and oscillator; later measurements agreed with the speed of light. CERN reports the timing-system fault. An offset can survive many repeat measurements, so precision alone cannot establish accuracy. Every measurement has an uncertainty, and good physics depends on knowing how large it is and where it comes from.



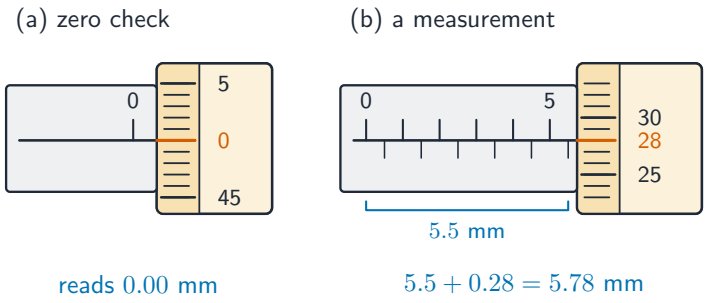
repeating a measurement cannot remove a systematic offset

Precision is not accuracy

Every measurement has an uncertainty

No measurement is exact. The **uncertainty** 不确定度 quantifies the limitation on a measured result. For 25.4 ± 0.1 cm, the stated interval is 25.3–25.5 cm. Its interpretation depends on how uncertainty was estimated; it is not automatically a guaranteed bound or a stated statistical confidence interval.

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Reading a micrometer: add the main scale reading to the thimble reading

How to estimate the uncertainty of a single reading:

- **Analogue scale** (ruler, thermometer, meter with a needle): half the smallest division is a common reading estimate when interpolation is reasonable. Include calibration, parallax and reading difficulties where relevant.
- **Digital scale**: the last displayed digit is a simple resolution estimate, not proof of accuracy. Manufacturer specifications, calibration and instability may give a larger uncertainty; a display of 85.23 g alone does not establish ±0.01 g accuracy.
- **Measurements by hand**: a stopwatch shows 0.01 s, but human reaction time adds about ±0.1 to 0.2 s.
- **A length** measured with a ruler needs two readings (both ends), so its uncertainty is twice the reading uncertainty if the two endpoint reading uncertainties are treated separately. Aligning an end to a mark does not make the alignment exact; a common additive scale offset can cancel in a difference, while a scale-factor error does not.



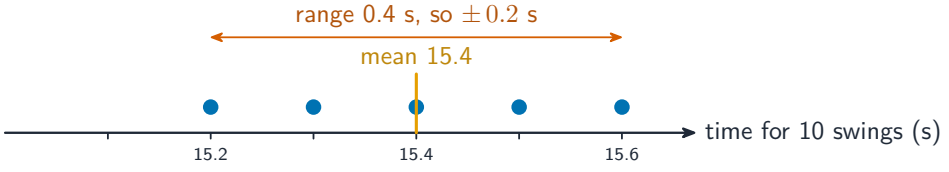
An analogue scale: the smallest division on this micrometer's thimble is 0.01 mm, which sets the reading uncertainty

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Quantity	Typical instrument	Typical uncertainty
length	metre rule; vernier caliper; micrometer	±1 mm; ±0.1 mm; ±0.01 mm
mass	electronic balance	±0.01 g
time	stopwatch; light gate and timer	±0.1–0.2 s (by hand); ±0.001 s
temperature	thermometer; temperature sensor	±0.5 °C; ±0.1 °C
current, potential difference	digital ammeter, voltmeter	resolution estimate; check calibration/specification
angle	protractor	±1°
volume	measuring cylinder	± half a division
force	newton meter; force sensor	± half a division
light or sound intensity	light sensor; sound level meter	resolution estimate; check calibration/specification

Repeated measurements

For stable repeated measurements, the **mean** is a useful estimate. **Half the range** is a simple classroom estimate of repeat scatter, used in the examples here. It is not a universal confidence interval or the standard uncertainty of the mean; include instrument limits and shared calibration errors rather than assuming the range captures everything.



Mean and half the range

Example: the time for 10 oscillations of a pendulum is measured five times: 15.2, 15.5, 15.3, 15.6 and 15.4 s.

- Mean $= 77.0/5 = 15.4$ s. Range $= 15.6 - 15.2 = 0.4$ s, so the uncertainty is ±0.2 s.
- Time for 10 oscillations: 15.4 ± 0.2 s, so the period is $T = 1.54 \pm 0.02$ s.

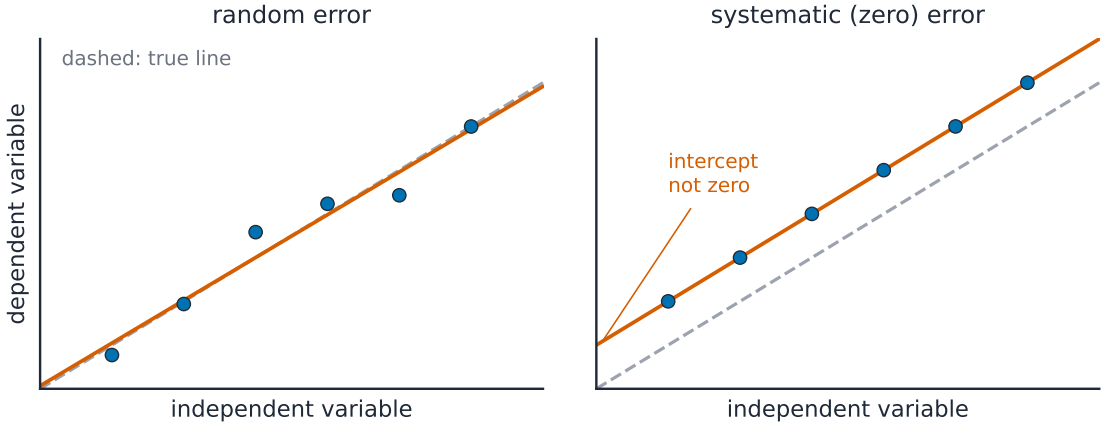
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For a similar absolute start–stop timing uncertainty, timing ten oscillations and dividing reduces its contribution to the period uncertainty by about ten. There are still start and stop reactions; miscounting oscillations, changing amplitude or a clock scale error are different limitations.

Errors, accuracy and precision

Random and systematic errors

- **Random errors** 随机误差 make readings scatter unpredictably about an underlying mean, which may itself be biased. Causes: reaction time, reading a scale, small changes in conditions. Independent repeats reduce the random contribution to uncertainty in the mean; they do not make the scatter of individual readings disappear. Extra graph points help only when the model, sampling and uncertainty treatment are suitable.
- **Systematic errors** 系统误差 produce a consistent bias under the measurement conditions; the bias can depend on the measured value. Causes: an instrument that is wrongly calibrated, a **zero error** 零点误差 (a meter reading 0.02 A with no current), reading a scale from an angle (parallax), heat loss in every trial. Repeating does **not** reduce them. They are reduced by calibrating, checking zero readings, and improving the method.

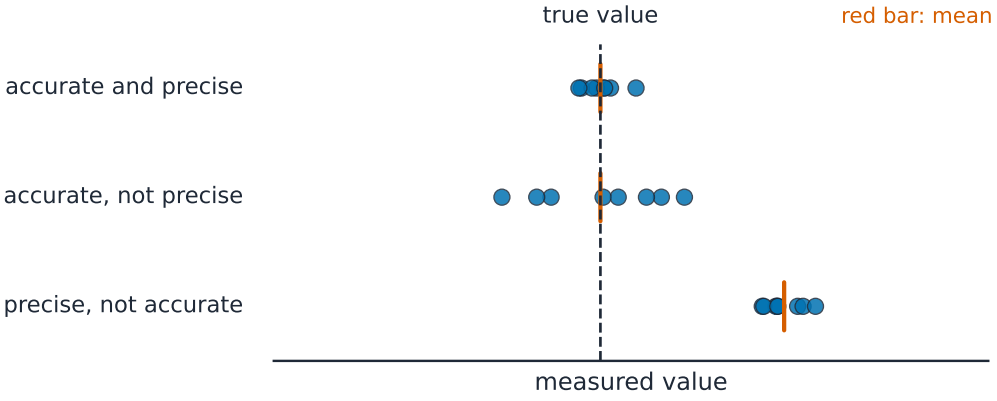


A systematic error often shows up as an unexpected intercept

A constant additive offset in the dependent variable changes a straight-line intercept without changing its gradient. A multiplicative calibration error changes the gradient; an independent-variable offset changes the inferred intercept too. A zero-compatible intercept therefore does not rule out systematic errors.

Accuracy and precision

- A measurement is **accurate** 准确 if it is close to the true value. Systematic errors reduce accuracy.
- Measurements are **precise** 精密 if repeated readings are close to each other. Random errors reduce precision.



Precise is not the same as accurate

Two more words are used when evaluating:

- **Reliability**: the method gives consistent results when repeated, by you or by someone else.
- **Validity**: the method really measures what the research question asks, with the other variables controlled.

Expressing uncertainties

An uncertainty can be written three ways. For $L = 2.50 \pm 0.05$ m:

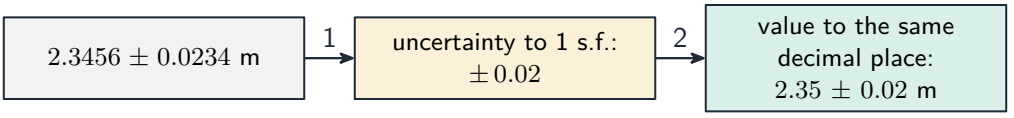
Form	Calculation	Value
absolute uncertainty	ΔL	± 0.05 m
fractional uncertainty	$\Delta L / L = 0.05/2.50$	± 0.02
percentage uncertainty	$\Delta L / L \times 100\,\%$	$\pm 2\,\%$

Significant figures

- Give the uncertainty to **one** significant figure (two at most), and give the value to the **same number of decimal places**.
- 2.3456 ± 0.0234 m becomes 2.35 ± 0.02 m. 158.2 ± 7 s becomes 158 ± 7 s.
- In a table of raw data, every value in a column has the same number of decimal places, matching the instrument.

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- Carry extra digits during calculations, then round the final value to the decimal place of its reported uncertainty. A simple significant-figure rule is a useful check, not a substitute for propagation.



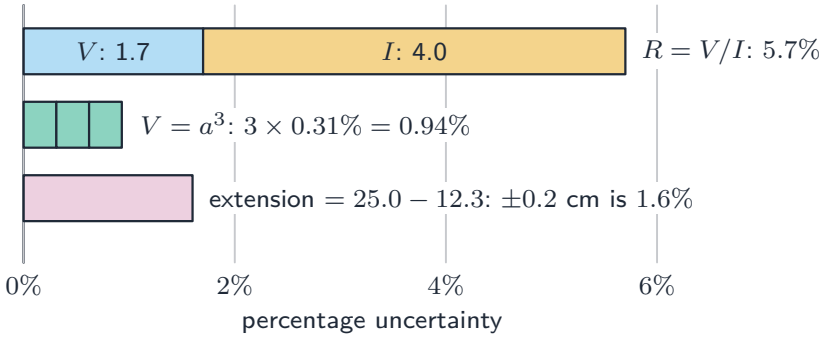
158.2 ± 7 s $\rightarrow 158 \pm 7$ s; carry extra digits until the end

Round the uncertainty first

For the following first-order maximum estimates, treat small uncertainties as separate bounds. These are not universal statistical rules. Shared errors can cancel: the same reading $x - x$ is exactly zero. Independent standard uncertainties combine differently. Use positive magnitudes $|y|$ and $|a|$ for signed quantities.

Propagating uncertainties

When measured values are combined in a calculation, their uncertainties combine too. The data booklet gives the rules:



The worked examples as bars: percentages add for a quotient, multiply by the power, and grow when similar values are subtracted.

Calculation	Rule
$y = a \pm b$	add the **absolute** uncertainties: $\Delta y = \Delta a + \Delta b$
$y = \dfrac{ab}{c}$	add the **fractional** uncertainties: $\dfrac{\Delta y}{y} = \dfrac{\Delta a}{a} + \dfrac{\Delta b}{b} + \dfrac{\Delta c}{c}$
$y = a^n$	multiply the fractional uncertainty by the power: $\dfrac{\Delta y}{y} = \left\lvert n \dfrac{\Delta a}{a} \right\rvert$

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■ Worked example: subtraction

A spring is 12.3 ± 0.1 cm long unloaded and 25.0 ± 0.1 cm loaded. Extension = 12.7 ± 0.2 cm. The percentage uncertainty has grown from under 1 % to 1.6 %: subtracting two similar values gives a large percentage uncertainty.

■ Worked example: a ratio

A resistor has $V = 6.0 \pm 0.1$ V and $I = 0.25 \pm 0.01$ A.

- $R = V/I = 24\ \Omega$.
- $\Delta R/R = 0.1/6.0 + 0.01/0.25 = 0.017 + 0.040 = 0.057$.
- $\Delta R = 0.057 \times 24 = 1.4\ \Omega$, so $R = 24.0 \pm 1.4\ \Omega$ if retaining two significant figures in the uncertainty, or $24 \pm 1\ \Omega$ with one.

■ Worked example: a power

A cube has mass $m = 85.2 \pm 0.1$ g and side $a = 3.20 \pm 0.01$ cm.

- $V = a^3 = 32.8\ \text{cm}^3$, with $\Delta V/V = 3 \times 0.01/3.20 = 0.94\,\%$.
- $\rho = m/V = 85.2/32.77 = 2.60\ \text{g cm}^{-3}$.
- $\Delta\rho/\rho = 0.12\,\% + 0.94\,\% = 1.06\,\%$, so $\Delta\rho = 0.03$ and $\rho = 2.60 \pm 0.03\ \text{g cm}^{-3}$.

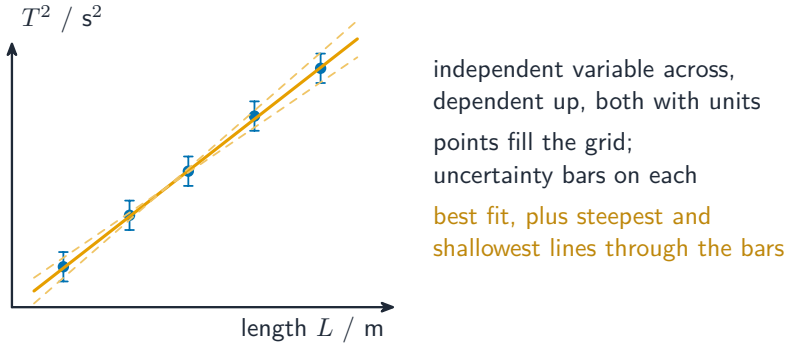
Notice which measurement matters most: the side length, because it is cubed. To improve the experiment, measure the side more precisely.

Graphs and their uncertainties

■ Drawing a good graph

- Put the independent variable on the horizontal axis and the dependent variable on the vertical axis. Label each with the quantity and unit, for example "length L / m".
- Choose scales so that the points fill most of the grid.
- Plot **uncertainty bars** 误差棒 showing the absolute uncertainty of each point. If a bar is too small to draw, say so.
- Draw a smooth **line or curve of best fit** with points balanced either side. It does not have to pass through the origin or through every point, and need not pass through every statistical uncertainty bar. For the classroom maximum/minimum-line method, test compatibility with all stated bars; if no line can do so, examine the model, uncertainties and possible outliers rather than forcing a line.
- A point far from the line may be an **outlier** 异常值. Check it, and repeat the measurement if you can. Leave it out of the line only with a reason, and say so.

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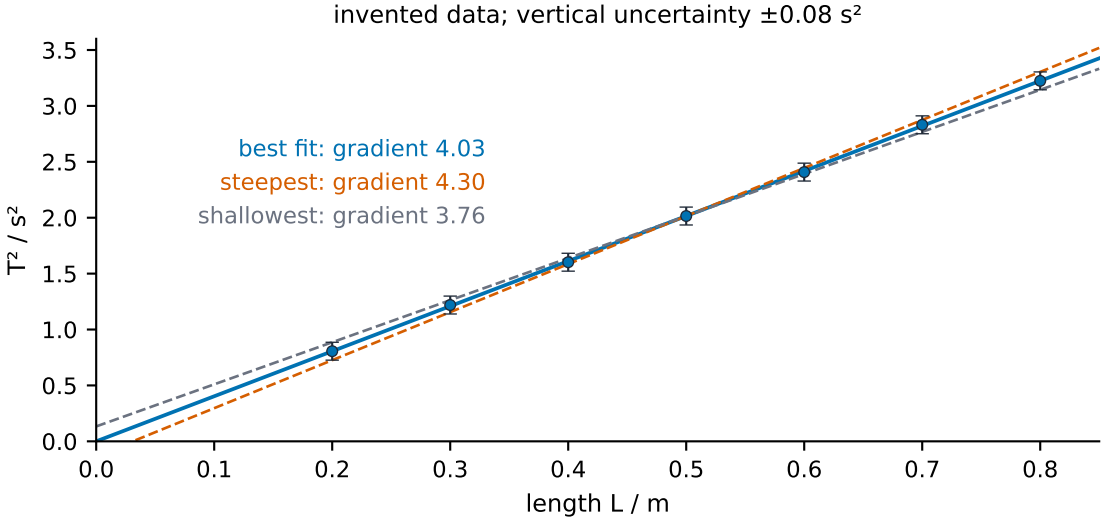
What a good graph shows

A **sketch graph** has labelled axes but no scales. It shows the shape of a relationship, for example "I increases with V, more slowly at high V".

Gradient and intercept

- The **gradient** 斜率 is $\dfrac{\Delta y}{\Delta x}$, found from a large triangle on the best-fit line, not from two data points. Its unit is the y unit divided by the x unit.
- The **intercept** 截距 is where the line crosses the vertical axis.
- The **area under** a graph can also mean something: under a velocity–time graph it is displacement, under a force–extension graph it is work done.

Uncertainty in the gradient



Invented data: both limiting lines are checked against every vertical bar, not only the endpoints

- Draw the **steepest** and the **shallowest** straight lines that still pass through all

the uncertainty bars. Check every intermediate bar as well: endpoint constructions can miss one. Include horizontal uncertainties as rectangles where significant; the pictured example neglects horizontal uncertainty.

- Uncertainty in the gradient $= \dfrac{\text{steepest gradient} - \text{shallowest gradient}}{2}$.
- In a roughly symmetric example, half the intercept span provides an approximate intercept uncertainty. In general, the lines that give extreme gradients need not give the extreme intercepts; determine the intercept range separately if required.

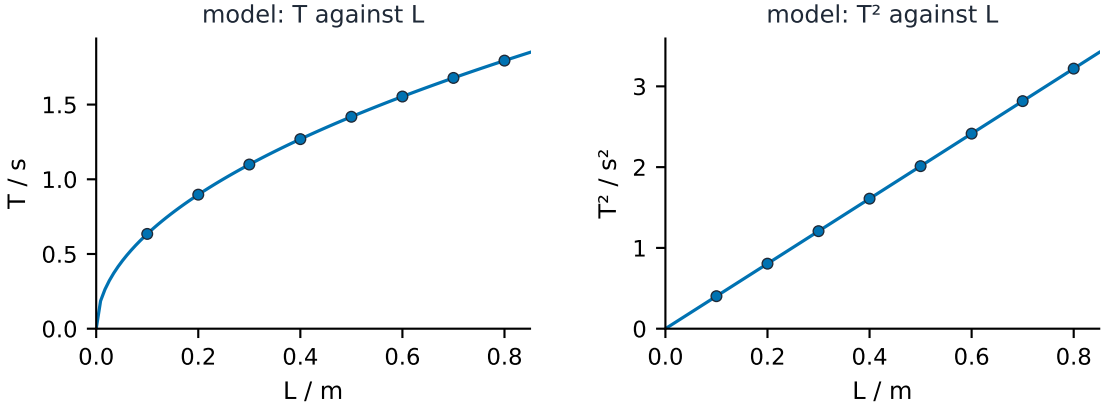
■ **Worked example: finding g with a pendulum**

Because $T = 2\pi\sqrt{L/g}$, $T^2 = \dfrac{4\pi^2}{g}L$, so a graph of T^2 against L should be a straight line through the origin with gradient $4\pi^2/g$.

- Best-fit gradient $= 4.03\ \text{s}^2\,\text{m}^{-1}$; steepest 4.30, shallowest 3.76.
- Uncertainty $= (4.30 - 3.76)/2 = 0.27$, giving an unrounded relative estimate $0.27/4.03 = 6.7\,\%$. Reporting the gradient to one uncertainty figure gives $4.0 \pm 0.3\ \text{s}^2\,\text{m}^{-1}$; use unrounded values for the following calculation.
- $g = 4\pi^2/4.03 = 9.80\ \text{m s}^{-2}$, with the same percentage uncertainty: $\Delta g = 0.067 \times 9.80 = 0.66$. So $g = 9.8 \pm 0.7\ \text{m s}^{-2}$.
- Here the limiting lines have intercepts about -0.133 and $+0.133\ \text{s}^2$, giving a symmetric estimate $c = 0.00 \pm 0.13\ \text{s}^2$. Zero is compatible with the data, supporting proportionality within the tested range; this does not exclude a scale-factor error or prove the model at every length.

Linearising graphs

A straight line is much easier to test than a curve: you can see at once whether the points lie on it, and the gradient and intercept give the constants. To **linearise** 线性化 a relationship, choose what to plot so that the equation has the form $y = mx + c$.



Plotting T^2 instead of T turns the curve into a straight line

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Relationship	Plot	Gradient	Intercept
$T = 2\pi\sqrt{L/g}$	T^2 against L	$4\pi^2/g$	0
$s = \frac{1}{2}at^2$	s against t^2	$a/2$	0
$F = \dfrac{k}{r^2}$	F against $1/r^2$	k	0
$A = A_0e^{-\lambda t}$	$\ln(A/A_{\text{ref}})$ against t	$-\lambda$	$\ln(A_0/A_{\text{ref}})$
$y = kx^n$ (n unknown)	$\ln(y/y_{\text{ref}})$ against $\ln(x/x_{\text{ref}})$	n	depends on k and the reference units

The last two use logarithms. Take logarithms of dimensionless ratios to fixed positive reference quantities. For $y = kx^n$, the gradient is n, while the intercept is $\ln(kx_{\text{ref}}^n/y_{\text{ref}})$. All logged values must be positive. The common shorthand "ln A" means the log of its numerical value in stated units; changing units changes the intercept, not the fitted exponent or decay constant. This is how to find an unknown power: for a pendulum, a graph of $\ln T$ against $\ln L$ has gradient 0.5.

A **logarithmic scale** goes up in equal factors (1, 10, 100, …) rather than equal steps. It is useful when data cover a very large range, such as the luminosities on an HR diagram.

Technology in practical work

- **Sensors and data loggers** measure quantities such as position, force, temperature, light intensity and current many times a second, without a human start–stop reaction contribution. Sensor response time, sampling rate, timing calibration and triggering can still limit results. Calibrate a sensor first, for example against a known mass or temperature.
- **Light gates** time a moving object precisely: the time for a card of known length to pass gives its speed.
- **Video analysis** (for example with free software such as Tracker) records motion and gives position against time frame by frame.
- **Spreadsheets** process data, calculate uncertainties and plot graphs. A spreadsheet can also give the gradient, but you should still check the steepest and shallowest lines.
- **Simulations and models**, such as PhET, generate data and let you test ideas that are hard or dangerous to measure directly.
- **Databases**, such as astronomical star catalogues or weather records, give large sets of real data. You must explain how you selected and sampled the data.

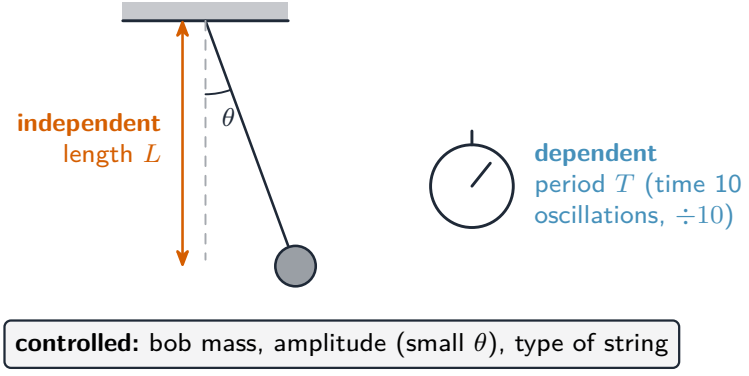
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Designing an investigation

Research question and variables

A good **research question** 研究问题 names the variables and the system: "How does the length of a simple pendulum (0.20–0.80 m) affect its period?"

- The **independent variable** 自变量 is the one you change on purpose (the length).
- The **dependent variable** 因变量 is the one you measure (the period).
- The **control variables** 控制变量 must stay the same so that the test is fair (the mass of the bob, the amplitude, the type of string). State how you keep each one constant, and why it matters.



The pendulum investigation: change L, measure T, keep the rest the same.

Planning the measurements

- Choose enough distinct values and repeats to reveal the predicted trend and estimate uncertainty. Five or more settings is a useful starting plan for many school experiments, not a universal assessment minimum. Justify the spacing and range within safe conditions and model validity.
- Repeat each measurement (often three to five times) to reduce random error.
- Try the method first (a pilot) to find problems early.
- Choose instruments with an uncertainty small enough for the change you want to see.

Controlling variables in physics

Know when and how to:

- calibrate measuring apparatus, including sensors;
- keep environmental conditions constant;
- insulate against heat loss or gain;
- reduce friction;
- reduce electrical resistance (short, thick connecting wires, clean contacts);

- take background radiation into account (measure it and subtract it).

Safety, ethics and the environment

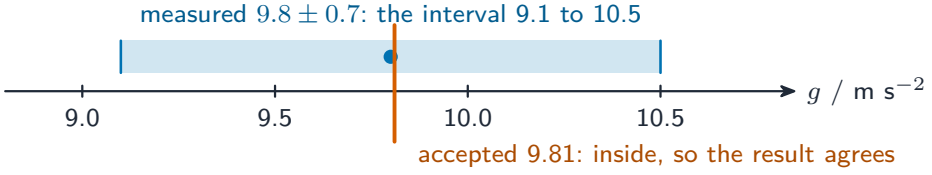
Identify the hazards and how you will reduce them: heavy masses falling, hot liquids, high voltages, lasers (never look along the beam), radioactive sources (teacher-controlled approved sources, handling tools, short exposure time, distance and source-appropriate shielding/storage under the school protocol). Lead is not a universal shielding instruction for every radiation type. Dispose of materials responsibly, and use energy and resources carefully.

Concluding and evaluating

Comparing with an accepted value

- **Percentage error** $= \dfrac{|\text{experimental value} - \text{accepted value}|}{\text{accepted value}} \times 100\,\%.$
- **Percentage change** $= \dfrac{\text{new value} - \text{old value}}{\text{old value}} \times 100\,\%.$
- If reference uncertainty is negligible here, a value inside the stated measurement interval is **compatible** under that uncertainty model. Otherwise include both uncertainties. Compatibility is not proof of correctness; outside the interval, check uncertainty estimates, statistical variation, assumptions and possible systematic errors.

Example: $g = 9.8 \pm 0.7$ m s^{-2} against the accepted 9.81. From the displayed values, the percentage error is $100|9.8-9.81|/9.81 \approx 0.10\,\%$, smaller than the 7 % uncertainty, and 9.81 is inside the range 9.1–10.5, so the result agrees.



percentage error $\approx 0.10\,\%$, far smaller than the 7 % uncertainty; agreement is not proof

Agreement means the accepted value lies inside the measured interval

Writing a conclusion

- Answer the research question directly, using the processed data: "T^2 is proportional to L within the uncertainties, because…".
- Quote values with their uncertainties.
- Compare with theory or with a published value, and explain any difference.

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- Discuss how large the uncertainties are, and whether they are small enough to support the conclusion.

Evaluating the method

- Identify the **main sources of error**, saying whether each is random or systematic, and estimate its effect: does it make the result too large or too small?
- Discuss **weaknesses and assumptions**: for example, the small-angle approximation in the pendulum equation, or heat loss that was ignored.
- Suggest **realistic improvements** linked to each weakness: "Use a light gate at the bottom of the swing to remove reaction-time error", not "be more careful".

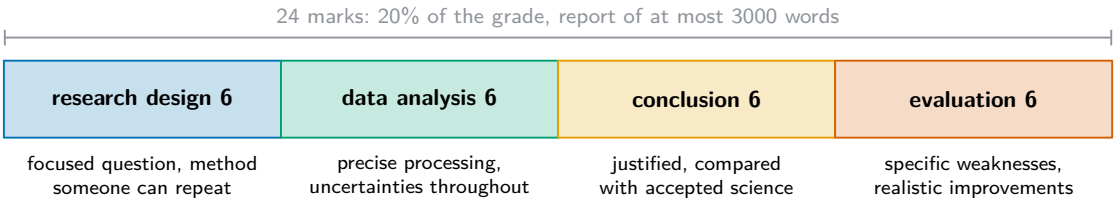
The internal assessment

The internal assessment (IA) is a **scientific investigation** that counts for **20 %** of the final grade, at both SL and HL.

- About 10 hours of class time; you choose your own research question.
- The investigation must analyse quantitative data; hands-on work, fieldwork, database extraction, simulations and spreadsheet modelling are possible approaches. Explain limitations of the chosen evidence.
- You write a report of at most **3000 words** (tables, graphs, equations, calculations and references do not count).
- Optional collaboration is limited to groups of at most three; each student needs a unique research question and distinct raw data or an appropriate distinct subset of shared data. Every report, including the method description, is individually authored.

Criterion	Marks	What gets top marks
Research design	6	a focused research question in a specific context, with relevant theory; methodological choices explained; a method detailed enough for someone else to repeat
Data analysis	6	clear and precise recording and processing of data, with uncertainties considered throughout, relevant to the question
Conclusion	6	a conclusion justified by the analysis, related to the research question and compared with accepted science
Evaluation	6	the impact of specific weaknesses and limitations discussed, with realistic improvements

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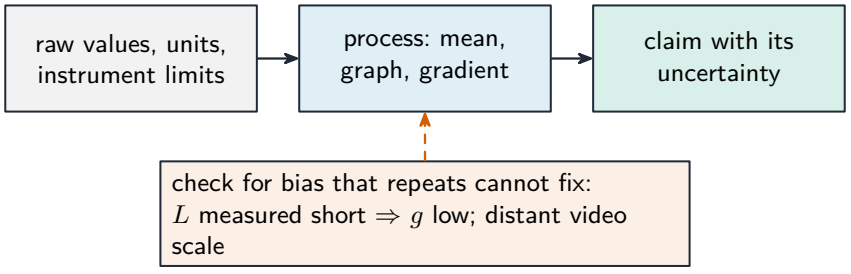


Four criteria of 6 marks: 20% of the grade in a report of at most 3000 words

Reference data, theory, methods and software where used, and distinguish your own observations from simulated or invented examples. The IB Physics guide gives the assessment requirements. The collaborative sciences project is a separate interdisciplinary activity, not a group-authored replacement for the individual investigation.

From data to a justified claim

Record raw values, units, instrument limits and qualitative observations before processing; keep rejected values visible with a reason. For a pendulum, measure pivot to bob centre and keep amplitude small. A length measured too short biases the inferred g low in $g = 4\pi^2 L/T^2$. More stopwatch repeats cannot correct that length definition.



From data to a claim

For motion video, place a scale in the plane of motion, view square-on and check frame timing: a distant scale can cause a perspective error. A light-gate card gives average speed over its passage, approximating instantaneous speed only when its speed changes little in that time. For sound levels, decibels are logarithmic, so doubling a displayed dB reading does not double intensity.

Use interpolation inside the observed range cautiously; extrapolation beyond it also assumes the model remains valid. A high correlation does not establish causation if another variable changes. Residuals (measured minus predicted values) reveal curvature, drift or changing scatter that a high correlation coefficient can hide. Report which limitations dominate and connect each proposed improvement to its predicted effect.

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Practice

1. A length is 2.50 ± 0.05 m. State its absolute, fractional and percentage uncertainty.
2. (a) Distinguish between random and systematic errors, and state how each can be reduced. (b) An ammeter reads 0.02 A when no current flows. State the type of error, and its effect on a graph of current against potential difference.
3. Five times are measured: 2.31, 2.35, 2.28, 2.40 and 2.33 s. Find the mean and its uncertainty. Use half the range as the repeat-scatter estimate.
4. Write each to an appropriate number of significant figures: (a) 2.3456 ± 0.0234 m; (b) 0.004563 ± 0.00012 A.
5. A rectangle is 4.0 ± 0.1 cm by 2.5 ± 0.1 cm. Find its area with its uncertainty. Use the first-order maximum-uncertainty rule.
6. The radius of a sphere is 2.00 ± 0.02 cm. Find its volume with its uncertainty.
7. Two temperatures are 25.0 ± 0.5 °C and 18.5 ± 0.5 °C. Find the temperature rise with its absolute and percentage uncertainty, and comment.
8. A trolley travels 1.50 ± 0.01 m in 0.62 ± 0.02 s. Find its average speed with its uncertainty.
9. A mass of 0.500 ± 0.005 kg moves at 3.0 ± 0.1 m s^{-1}. Find its kinetic energy with its uncertainty.
10. State what to plot to give a straight line, and what the gradient represents: (a) $s = \frac{1}{2}at^2$; (b) $A = A_0 e^{-\lambda t}$; (c) $P = kV^n$ with k and n unknown.
11. (a) A best-fit gradient is 2.50, the steepest line through the bars has gradient 2.75 and the shallowest 2.20. State the gradient with its uncertainty. (b) A graph of T^2 against L has gradient 4.0 ± 0.3 s^2 m^{-1}. Find g with its uncertainty.
12. A student finds $g = 9.2 \pm 0.3$ m s^{-2}. Find the percentage error from 9.81, state whether the result agrees with the accepted value, and suggest a reason.
13. Write a research question for an investigation into how the length of a wire affects its resistance, and state the independent, dependent and two control variables.
14. In a pendulum experiment the period was timed for one oscillation using a hand-held stopwatch. Identify the main weakness, and suggest a realistic improvement.

Answers.

1. ± 0.05 m; ± 0.02; $\pm 2\,\%$.
2. (a) Random effects scatter readings about an underlying mean, which may be biased. Independent repeats reduce the random uncertainty in that mean. Systematic effects produce a consistent measurement bias; calibrate, correct the cause or improve the model rather than relying on more repeats. (b) A systematic (zero) error. Every reading is 0.02 A too high, so the line is shifted: the intercept changes, but the gradient does not.
3. Mean $= 11.67/5 = 2.334$ s; range $= 0.12$ s; so 2.33 ± 0.06 s.
4. (a) 2.35 ± 0.02 m. (b) 0.0046 ± 0.0001 A with one uncertainty figure, or 0.00456 ± 0.00012 A with two; the quoted value matches its uncertainty decimal place.

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5. $A = 10.0\ \text{cm}^2$; $\Delta A/A = 0.1/4.0 + 0.1/2.5 = 6.5\,\%$; $A = 10.0 \pm 0.7\ \text{cm}^2$.
6. $V = \frac{4}{3}\pi r^3 = 33.5\ \text{cm}^3$; $\Delta V/V = 3 \times 1\,\% = 3\,\%$; $V = 34 \pm 1\ \text{cm}^3$.
7. 6.5 ± 1.0 °C, which is 15 %. Subtracting two similar values gives a much larger percentage uncertainty than either reading.
8. $v = 1.50/0.62 = 2.41935\ \text{m s}^{-1}$. The fractional bound is $0.01/1.50 + 0.02/0.62 = 0.0389247$, giving $\Delta v = 0.0941728\ \text{m s}^{-1}$. Report $2.42 \pm 0.09\ \text{m s}^{-1}$; do not round the fractions before multiplying.
9. $E = \frac{1}{2}mv^2 = 2.25$ J; $\Delta E/E = 1\,\% + 2 \times 3.3\,\% = 7.7\,\%$; $E = 2.3 \pm 0.2$ J.
10. (a) s against t^2, gradient $a/2$. (b) $\ln(A/A_{\text{ref}})$ against t, gradient $-\lambda$. (c) $\ln(P/P_{\text{ref}})$ against $\ln(V/V_{\text{ref}})$, gradient n, intercept $\ln(kV_{\text{ref}}^n/P_{\text{ref}})$. Reference quantities are fixed and positive.
11. (a) $(2.75 - 2.20)/2 = 0.275$, so 2.5 ± 0.3. (b) $g = 4\pi^2/4.0 = 9.9\ \text{m s}^{-2}$; percentage uncertainty $0.3/4.0 = 7.5\,\%$; $g = 9.9 \pm 0.7\ \text{m s}^{-2}$.
12. $100|9.2 - 9.81|/9.81 = 6.22\,\%$, about 6%. The interval 8.9–9.5 excludes 9.81 if reference uncertainty is negligible. An underestimated pivot-to-centre length would bias g low; underestimated uncertainty or a model problem should also be considered. This one mismatch does not uniquely identify the cause.
13. For example: "How does the length of a constantan wire (0.10–1.00 m) affect its resistance?" Independent: length. Dependent: resistance (from V and I). Control (any two): material, diameter, temperature of the wire (small current, switch off between readings).
14. Timing a single oscillation by hand gives a large percentage uncertainty from reaction time. Time 10 or 20 oscillations and divide, or use a light gate at the lowest point.

Exam tips

- Give uncertainties to one significant figure and the value to the same decimal place.
- State the uncertainty model; for separate small bounds add fractional contributions in products and absolute contributions in differences.
- Find gradients from a large triangle on the line, never from two data points, and include the unit.
- The uncertainty of a gradient is half the difference between the steepest and shallowest lines through all the bars.
- When evaluating, name the error, say whether it is random or systematic, say how it affected the result, and give a specific improvement.

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