

5.2 Galilean and special relativity (full A.5, HL)

Name: _____ Class: _____ Date: _____

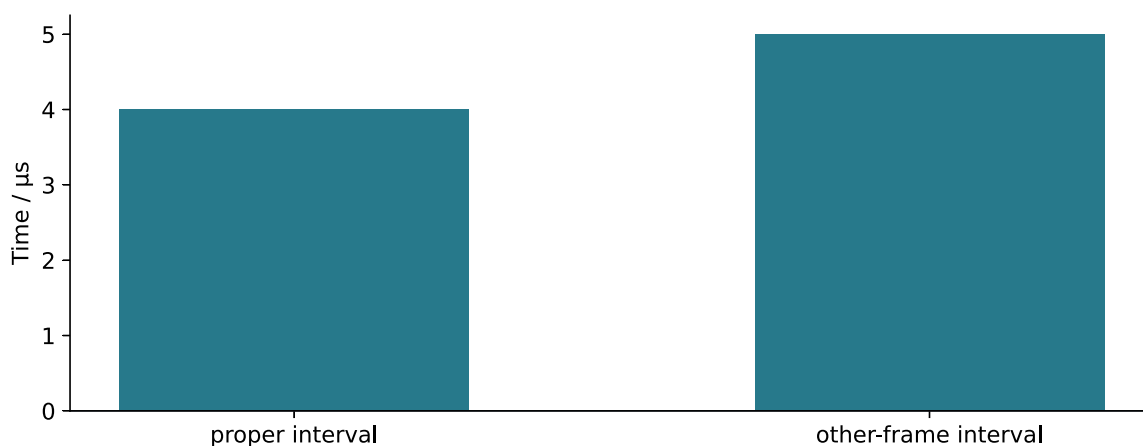
Total: 24 marks

KEY WORDS reference frame • time dilation • length contraction

Objective

Practise the full guide A.5 toolkit (HL only): inertial frames and the Galilean transformations, the two postulates, the Lorentz transformations with γ , relativistic velocity addition, the invariant space–time interval, proper time/length, time dilation and length contraction, relativity of simultaneity, and space–time diagrams with world lines at $\tan \theta = v/c$. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ The relativity equation set

$$x' = \gamma(x - vt) \quad t' = \gamma\left(t - \frac{vx}{c^2}\right) \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad u' = \frac{u - v}{1 - uv/c^2}$$

$$(\Delta s)^2 = (c \Delta t)^2 - (\Delta x)^2 \quad \Delta t = \gamma \Delta t_0 \quad L = \frac{L_0}{\gamma} \quad \tan \theta = \frac{v}{c}$$

Galilean limits: $x' = x - vt$, $t' = t$, $u' = u - v$ (valid when $v \ll c$).

Worked check. At $v = 0.80c$: $\gamma = 1/\sqrt{1 - 0.64} = 1/0.60 = 1.67$; a 30 m proper length contracts to $L = 30/1.67 = 18$ m; a 1.0 s proper time dilates to $\Delta t = 1.67$ s.

2 Practice

2.1 Calculate γ for $v = 0.28c$. [2]

2.2 State the two postulates of special relativity. [2]

2.3 Write the Galilean transformations and state when they are adequate. [2]

2.4 A rod of proper length 45 m moves at $0.60c$. Calculate its contracted length. [2]

2.5 A particle's world line makes 45° with the ct axis. Calculate its speed. [2]

3 Exam-style questions

3.1 A spacecraft passes Earth at $0.90c$. An observer on the spacecraft measures a journey between two stars as lasting 4.0 years (proper time). Calculate γ , the journey time measured from Earth, and the distance Earth observers say the stars are apart if the spacecraft moves the whole time. [5]

3.2 A spaceship moving at $0.80c$ fires a probe forward at $0.70c$ relative to the ship. Calculate the probe's speed relative to Earth using relativistic velocity addition, and compare it with the Galilean answer. [4]

3.3 Muons created in the upper atmosphere have a proper lifetime of $2.2\ \mu\text{s}$ and travel at $0.995c$ ($\gamma = 10.0$). Explain, using BOTH time dilation and length contraction, how they reach the ground – and state what would happen in a purely Galilean calculation. [5]
