

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 $\gamma = 1/\sqrt{1 - 0.0784} = 1/0.960 = 1.04$.

2.2 The laws of physics are the same in all inertial reference frames; the speed of light in vacuum is the same for all inertial observers, regardless of the source's motion.

2.3 $x' = x - vt$, $t' = t$; adequate when all speeds are much less than c .

2.4 $\gamma = 1/\sqrt{1 - 0.36} = 1.25$; $L = 45/1.25 = 36$ m.

2.5 $\tan 45^\circ = 1 = v/c$, so $v = c$ – a photon's world line.

3.1 $\gamma = 1/\sqrt{1 - 0.81} = 1/0.436 = 2.29$; Earth time $\Delta t = \gamma \Delta t_0 = 2.29 \times 4.0 = 9.2$ years; distance $= vt = 0.90c \times 9.2$ years $= 8.3$ light-years.

3.2 With the ship moving at $v = 0.80c$ and the probe at $u' = 0.70c$ relative to it, Earth measures $u = (u' + v)/(1 + u'v/c^2) = 1.50c/1.56 = 0.96c$. Galilean: $0.70c + 0.80c = 1.50c$ – faster than light, impossible; the relativistic denominator prevents the excess.

3.3 Earth frame: lifetime dilated to $\Delta t = 10.0 \times 2.2 \mu\text{s} = 22 \mu\text{s}$; distance travelled $= 0.995c \times 22 \mu\text{s} \approx 6.6$ km – far enough to reach the ground. Muon frame: its lifetime stays $2.2 \mu\text{s}$, but the atmosphere's thickness contracts by $\gamma = 10.0$, so the same few hundred km becomes tens of km – also far enough. Galilean: only $0.995c \times 2.2 \mu\text{s} \approx 0.66$ km – the muons would decay high up; the observed ground flux is evidence for relativity.