

16.2 Gravitational fields (full D.1, HL)

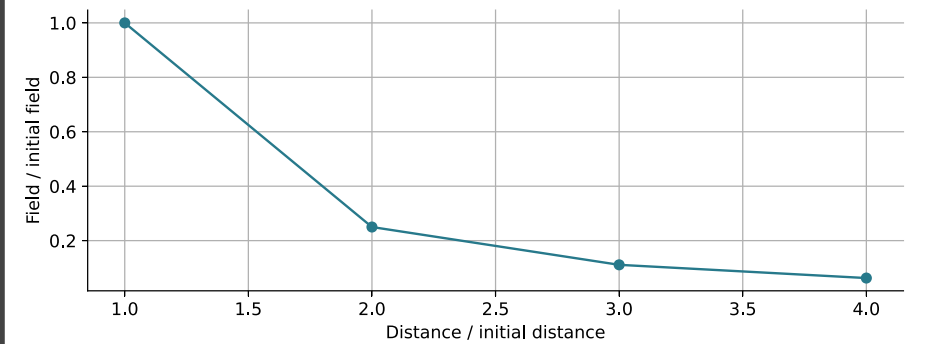
Name: _____ Class: _____ Date: _____ **Total: 31 marks**

KEY WORDS gravitational field strength • gravitational constant
• Kepler's laws • gravitational potential • escape speed

Objective

Practise the full guide D.1 toolkit including AHL: Newton's law of gravitation $F = Gm_1m_2/r^2$ with the point-mass condition, gravitational field strength $g = GM/r^2$, field-line patterns, resultant fields along the line of two bodies, and Kepler's three laws including $T^2 \propto r^3$ in problems; at higher level also potential $V_g = -GM/r$, potential energy $E_p = -Gm_1m_2/r$, the potential gradient, equipotential surfaces, escape and orbital speeds, and satellite energetics. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ The D.1 rule set (HL)

$$F = \frac{Gm_1m_2}{r^2} \quad g = \frac{GM}{r^2} \quad V_g = -\frac{GM}{r} \quad g = -\frac{\Delta V_g}{\Delta r} \quad v_{esc} = \sqrt{\frac{2GM}{r}} \quad v_{orbital} = \sqrt{\frac{GM}{r}}$$

Field lines point inward, never cross, and crowd where the field is stronger. A spherical body of uniform density acts as a point mass at its centre. Kepler: ellipses with the central body at a focus; equal areas in equal times; $T^2 \propto r^3$.

■ Worked example: Kepler with real bodies

Mars orbits at 1.52 AU: $T = 1.52^{3/2} = 1.87$ Earth years. Earth's surface: $g = GM/r^2 = (6.67 \times 10^{-11} \times 6.0 \times 10^{24}) / (6.4 \times 10^6)^2 = 9.8 \text{ m/s}^2$. Two habits: measure r from the centre of the mass, not from the surface; and for period comparisons cube the radius ratio before taking its square root.

2 Practice

2.1 State Newton's law of gravitation, naming the quantities in the equation. [2]

2.2 Mars has mass $6.4 \times 10^{23} \text{ kg}$ and radius $3.4 \times 10^6 \text{ m}$. Calculate the gravitational field strength at its surface ($G = 6.67 \times 10^{-11}$). [3]

2.3 State Kepler's third law, and calculate the orbital period of a planet at 9.5 AU, in Earth years. [3]

2.4 Describe two features of the gravitational field-line pattern around a spherical planet. [2]

2.5 A satellite orbits Earth ($M = 6.0 \times 10^{24} \text{ kg}$) at $r = 4.2 \times 10^7 \text{ m}$. Calculate its orbital speed and its escape speed from that orbit. [3]

3 Exam-style questions

3.1 Calculate the gravitational field strength at a point a distance $2R$ from Earth's centre, where $R = 6.4 \times 10^6$ m is Earth's radius and surface $g = 9.8$ m/s². [3]

3.2 A geostationary satellite orbits at $r = 4.2 \times 10^7$ m from Earth's centre ($M = 6.0 \times 10^{24}$ kg). Use $T^2 = 4\pi^2 r^3 / GM$ to show its period is about one day. [4]

3.3 A comet's orbit has perihelion 0.59 AU and aphelion 35 AU. Estimate its orbital period using Kepler's third law, stating one assumption the estimate makes. [4]

3.4 Explain when a planet may be treated as a point mass for gravitation calculations, and describe how the field strength varies along a line moving away from its surface. [3]

3.5 A 1200 kg satellite is raised from Earth's surface ($r = 6.4 \times 10^6$ m) to a parking orbit at $r = 1.28 \times 10^7$ m. Calculate the increase in its gravitational potential energy, and state what happens to the total energy of a bound circular orbit as its radius increases. [4]
