

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 The attractive force between two point masses is proportional to the product of the masses and inversely proportional to the square of their separation: $F = Gm_1m_2/r^2$; G the gravitational constant, m_1, m_2 the masses, r the separation between centres.

2.2 $g = GM/r^2 = (6.67 \times 10^{-11} \times 6.4 \times 10^{23})/(3.4 \times 10^6)^2 = 4.3 \times 10^{13}/1.2 \times 10^{13} = 3.7 \text{ m/s}^2$.

2.3 The square of the orbital period is proportional to the cube of the orbital radius, $T^2 \propto r^3$; $T = 9.5^{3/2} = 29$ Earth years.

2.4 Any two: the lines point radially inward toward the centre; they never cross; they are equally spaced over a sphere at any radius and crowd nearer the surface where the field is stronger.

2.5 $v_{\text{orbital}} = \sqrt{GM/r} = \sqrt{(4.0 \times 10^{14})/(4.2 \times 10^7)} = \sqrt{9.5 \times 10^6} = 3.1 \times 10^3 \text{ m/s}$;
 $v_{\text{esc}} = \sqrt{2} v_{\text{orbital}} = 4.4 \times 10^3 \text{ m/s}$.

3.1 $g \propto 1/r^2$; doubling the distance divides the field by four: $g = 9.8/4 = 2.5 \text{ m/s}^2$.

3.2 $T^2 = 4\pi^2 r^3/GM = (39.5 \times (4.2 \times 10^7)^3)/(6.67 \times 10^{-11} \times 6.0 \times 10^{24})$; numerator $= 39.5 \times 7.4 \times 10^{22} = 2.9 \times 10^{24}$, denominator $= 4.0 \times 10^{14}$, so $T^2 = 7.3 \times 10^9 \text{ s}^2$; $T = 8.5 \times 10^4 \text{ s} \approx 24 \text{ h}$.

3.3 Mean orbital radius $\approx (0.59 + 35)/2 \approx 18 \text{ AU}$; $T = 18^{3/2} = 76$ years; assumes the comet's elliptical orbit is approximated by a circular orbit of that mean radius – the third law applied to the semi-major axis.

3.4 When the body is spherically symmetric with (near-)uniform density, its gravity outside it is that of a point mass at its centre; equally when the separation is much larger than the body's size; the field strength falls as $1/r^2$ with distance from the centre – quadrupling the distance reduces g by a factor of four.

3.5 $\Delta E_p = GMm(1/r_1 - 1/r_2) = (4.0 \times 10^{14} \times 1200)(1/6.4 \times 10^6 - 1/1.28 \times 10^7) = (4.8 \times 10^{17})(7.8 \times 10^{-8}) = 3.8 \times 10^{10} \text{ J}$; the total energy $-GMm/2r$ becomes less negative – increases – as the radius grows.