

Exercise walk-through

5.2

Galilean and special relativity (full A.5, HL)

International Baccalaureate · IB Diploma · Physics · HL

Relativity · 2.1

Reasoning

In special relativity, inertial observers measure the same vacuum light speed. Time dilation compares proper time measured where two events occur at one place with the interval in another inertial frame. proper time: Time between events measured at one location in a frame; inertial frame: A frame in which a free body moves at constant velocity.

Relativity · 3.1

Reasoning

The Lorentz factor is 1 divided by the square root of 1 minus speed squared over light speed squared. Galilean velocity addition is an approximation for speeds much smaller than light speed. Label which frame measures proper time or proper length before substituting. Use event coordinates consistently. Do not combine lengths from one frame with time intervals from another without transformation.

Choose the proper measurement

- 1 Set the model** An object moves at $0.60c$ relative to the laboratory. Its rest length is 8.0 m .
- 2 Calculate** $\gamma = 1/\sqrt{1 - 0.60^2} = 1.25$. Simultaneous laboratory measurements give $L = L_0/\gamma = 6.4\text{ m}$.

Decision

Proper length is measured in the object's rest frame. Simultaneity must be specified when comparing the endpoints.

Exercises 2.1-2.3

2.1 Calculate γ for $v = 0.28c$. [2]

2.2 State the two postulates of special relativity. [2]

2.3 Write the Galilean transformations and state when they are adequate. [2]

Exercises 2.1·2.3

2.1 Calculate γ for $v = 0.28c$. [2]

2.1 $\gamma = 1/\sqrt{1 - 0.0784} = 1/0.960 = 1.04$.

2.2 State the two postulates of special relativity. [2]

2.2 The laws of physics are the same in all inertial reference frames; the speed of light in vacuum is the same for all inertial observers, regardless of the source's motion.

2.3 Write the Galilean transformations and state when they are adequate. [2]

2.3 $x' = x - vt$, $t' = t$; adequate when all speeds are much less than c .

Exercises 2.4·3.1

2.4 A rod of proper length 45 m moves at $0.60c$. Calculate its contracted length. [2]

2.5 A particle's world line makes 45° with the ct axis. Calculate its speed. [2]

3.1 A spacecraft passes Earth at $0.90c$. An observer on the spacecraft measures a journey between two stars as lasting 4.0 years (proper time). Calculate γ , the journey time measured from Earth, and the distance Earth observers say the stars are apart if the spacecraft moves the whole time. [5]

Exercises 2.4·3.1

2.4 A rod of proper length 45 m moves at $0.60c$. Calculate its contracted length. [2]

2.4 $\gamma = 1/\sqrt{1 - 0.36} = 1.25$; $L = 45/1.25 = 36$ m.

2.5 A particle's world line makes 45° with the ct axis. Calculate its speed. [2]

2.5 $\tan 45^\circ = 1 = v/c$, so $v = c$ – a photon's world line.

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3.1 $\gamma = 1/\sqrt{1 - 0.81} = 1/0.436 = 2.29$; Earth time $\Delta t = \gamma \Delta t_0 = 2.29 \times 4.0 = 9.2$ years; distance $= vt = 0.90c \times 9.2$ years $= 8.3$ light-years.

Exercises 3.2-3.3

3.2 A spaceship moving at $0.80c$ fires a probe forward at $0.70c$ relative to the ship. Calculate the probe's speed relative to Earth using relativistic velocity addition, and compare it with the Galilean answer. [4]

3.3 Muons created in the upper atmosphere have a proper lifetime of $2.2 \mu\text{s}$ and travel at $0.995c$ ($\gamma = 10.0$). Explain, using BOTH time dilation and length contraction, how they reach the ground – and state what would happen in a purely Galilean calculation. [5]

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3.2 With the ship moving at $v = 0.80c$ and the probe at $u' = 0.70c$ relative to it, Earth measures $u = (u' + v)/(1 + u'v/c^2) = 1.50c/1.56 = 0.96c$. Galilean: $0.70c + 0.80c = 1.50c$ – faster than light, impossible; the relativistic denominator prevents the excess.

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3.3 Earth frame: lifetime dilated to $\Delta t = 10.0 \times 2.2 \mu\text{s} = 22 \mu\text{s}$; distance travelled $= 0.995c \times 22 \mu\text{s} \approx 6.6 \text{ km}$ – far enough to reach the ground. Muon frame: its lifetime stays $2.2 \mu\text{s}$, but the atmosphere's thickness contracts by $\gamma = 10.0$, so the same few hundred km becomes tens of km – also far enough. Galilean: only $0.995c \times 2.2 \mu\text{s} \approx 0.66 \text{ km}$ – the muons would decay high up; the observed ground flux is evidence for relativity.