

Exercise walk-through

# 16.2

## Gravitational fields (full D.1, HL)

---

International Baccalaureate · IB Diploma · Physics · HL

# Gravitation · 3.1

## Reasoning

For a point mass or outside a spherical mass, field strength follows an inverse-square distance dependence. Use distance from the centre, not height above the surface alone. State the circular-orbit approximation and ignore atmospheric drag only when justified. Draw the force toward the central body and velocity tangential to the orbit. Do not add an outward force merely because the path is circular.

## Distance is measured from the centre

- 1 Set the model** Earth has  $M = 6.0 \times 10^{24} \text{ kg}$  and radius  $6.4 \times 10^6 \text{ m}$ . Use  $G = 6.67 \times 10^{-11}$  in SI units.
- 2 Calculate**  $g = GM/r^2 = 9.77 \text{ m/s}^2$ . At twice the centre distance,  $g$  is one quarter as large:  $2.44 \text{ m/s}^2$ .

### Decision

Inverse square uses centre distance, not altitude. Kepler's period law compares orbits about the same central mass.

## Escape and circular orbit speeds

- 1 Set the model** For Earth use  $GM = 4.0 \times 10^{14} \text{ m}^3/\text{s}^2$  at  $r = 6.4 \times 10^6 \text{ m}$ . Ignore atmosphere and rotation.
- 2 Calculate**  $v_{\text{orb}} = \sqrt{GM/r} = 7.91 \text{ km/s}$ ;  $v_{\text{esc}} = \sqrt{2GM/r} = 11.2 \text{ km/s}$ .

### Decision

Circular orbit energy is negative. Escape with zero final speed at infinity requires total energy to reach zero.

## Exercises 2.1·2.3

---

**2.1** State Newton's law of gravitation, naming the quantities in the equation. [2]

---

**2.2** Mars has mass  $6.4 \times 10^{23}$  kg and radius  $3.4 \times 10^6$  m. Calculate the gravitational field strength at its surface ( $G = 6.67 \times 10^{-11}$ ). [3]

---

**2.3** State Kepler's third law, and calculate the orbital period of a planet at 9.5 AU, in Earth years. [3]

## Exercises 2.1·2.3

---

**2.1** State Newton's law of gravitation, naming the quantities in the equation. [2]

**2.1** The attractive force between two point masses is proportional to the product of the masses and inversely proportional to the square of their separation:  $F = Gm_1m_2/r^2$ ;  $G$  the gravitational constant,  $m_1, m_2$  the masses,  $r$  the separation between centres.

---

**2.2** Mars has mass  $6.4 \times 10^{23}$  kg and radius  $3.4 \times 10^6$  m. Calculate the gravitational field strength at its surface ( $G = 6.67 \times 10^{-11}$ ). [3]

**2.2**  $g = GM/r^2 = (6.67 \times 10^{-11} \times 6.4 \times 10^{23})/(3.4 \times 10^6)^2 = 4.3 \times 10^{13}/1.2 \times 10^{13} = 3.7 \text{ m/s}^2$ .

---

**2.3** State Kepler's third law, and calculate the orbital period of a planet at 9.5 AU, in Earth years. [3]

**2.3** The square of the orbital period is proportional to the cube of the orbital radius,  $T^2 \propto r^3$ ;  $T = 9.5^{3/2} = 29$  Earth years.

## Exercises 2.4·3.1

---

**2.4** Describe two features of the gravitational field-line pattern around a spherical planet.[2]

---

**2.5** A satellite orbits Earth ( $M = 6.0 \times 10^{24}$  kg) at  $r = 4.2 \times 10^7$  m. Calculate its orbital speed and its escape speed from that orbit. [3]

---

**3.1** Calculate the gravitational field strength at a point a distance  $2R$  from Earth's centre, where  $R = 6.4 \times 10^6$  m is Earth's radius and surface  $g = 9.8$  m/s<sup>2</sup>. [3]

## Exercises 2.4·3.1

---

**2.4** Describe two features of the gravitational field-line pattern around a spherical planet.[2]

**2.4** Any two: the lines point radially inward toward the centre; they never cross; they are equally spaced over a sphere at any radius and crowd nearer the surface where the field is stronger.

---

**2.5** A satellite orbits Earth ( $M = 6.0 \times 10^{24}$  kg) at  $r = 4.2 \times 10^7$  m. Calculate its orbital speed and its escape speed from that orbit. [3]

$$\begin{aligned} \mathbf{2.5} \quad v_{\text{orbital}} &= \sqrt{GM/r} = \sqrt{(4.0 \times 10^{14})/(4.2 \times 10^7)} = \sqrt{9.5 \times 10^6} = 3.1 \times 10^3 \text{ m/s;} \\ v_{\text{esc}} &= \sqrt{2} v_{\text{orbital}} = 4.4 \times 10^3 \text{ m/s.} \end{aligned}$$

---

**3.1** Calculate the gravitational field strength at a point a distance  $2R$  from Earth's centre, where  $R = 6.4 \times 10^6$  m is Earth's radius and surface  $g = 9.8 \text{ m/s}^2$ . [3]

**3.1**  $g \propto 1/r^2$ ; doubling the distance divides the field by four:  $g = 9.8/4 = 2.5 \text{ m/s}^2$ .

## Exercises 3.2·3.3

---

**3.2** A geostationary satellite orbits at  $r = 4.2 \times 10^7$  m from Earth's centre ( $M = 6.0 \times 10^{24}$  kg). Use  $T^2 = 4\pi^2 r^3 / GM$  to show its period is about one day. [4]

---

**3.3** A comet's orbit has perihelion 0.59 AU and aphelion 35 AU. Estimate its orbital period using Kepler's third law, stating one assumption the estimate makes. [4]

## Exercises 3.2-3.3

---

**3.2** A geostationary satellite orbits at  $r = 4.2 \times 10^7$  m from Earth's centre ( $M = 6.0 \times 10^{24}$  kg). Use  $T^2 = 4\pi^2 r^3 / GM$  to show its period is about one day. [4]

**3.2**  $T^2 = 4\pi^2 r^3 / GM = (39.5 \times (4.2 \times 10^7)^3) / (6.67 \times 10^{-11} \times 6.0 \times 10^{24})$ ; numerator =  $39.5 \times 7.4 \times 10^{22} = 2.9 \times 10^{24}$ , denominator =  $4.0 \times 10^{14}$ , so  $T^2 = 7.3 \times 10^9$  s<sup>2</sup>;  $T = 8.5 \times 10^4$  s  $\approx$  24 h.

---

**3.3** A comet's orbit has perihelion 0.59 AU and aphelion 35 AU. Estimate its orbital period using Kepler's third law, stating one assumption the estimate makes. [4]

**3.3** Mean orbital radius  $\approx (0.59 + 35)/2 \approx 18$  AU;  $T = 18^{3/2} = 76$  years; assumes the comet's elliptical orbit is approximated by a circular orbit of that mean radius – the third law applied to the semi-major axis.

## Exercises 3.4·3.5

---

**3.4** Explain when a planet may be treated as a point mass for gravitation calculations, and describe how the field strength varies along a line moving away from its surface. [3]

---

**3.5** A 1200 kg satellite is raised from Earth's surface ( $r = 6.4 \times 10^6$  m) to a parking orbit at  $r = 1.28 \times 10^7$  m. Calculate the increase in its gravitational potential energy, and state what happens to the total energy of a bound circular orbit as its radius increases. [4]

## Exercises 3.4·3.5

---

**3.4** Explain when a planet may be treated as a point mass for gravitation calculations, and describe how the field strength varies along a line moving away from its surface. [3]

**3.4** When the body is spherically symmetric with (near-)uniform density, its gravity outside it is that of a point mass at its centre; equally when the separation is much larger than the body's size; the field strength falls as  $1/r^2$  with distance from the centre – quadrupling the distance reduces  $g$  by a factor of four.

---

**3.5** A 1200 kg satellite is raised from Earth's surface ( $r = 6.4 \times 10^6$  m) to a parking orbit at  $r = 1.28 \times 10^7$  m. Calculate the increase in its gravitational potential energy, and state what happens to the total energy of a bound circular orbit as its radius increases. [4]

**3.5**  $\Delta E_p = GMm(1/r_1 - 1/r_2) = (4.0 \times 10^{14} \times 1200)(1/6.4 \times 10^6 - 1/1.28 \times 10^7) = (4.8 \times 10^{17})(7.8 \times 10^{-8}) = 3.8 \times 10^{10}$  J; the total energy  $-GMm/2r$  becomes less negative – increases – as the radius grows.