Exercise walk-through

14.2
Standing waves and resonance (full C.4)

International Baccalaureate · IB Diploma · Physics · HL

1 The method

Method — The C.4 rule set

Node–node or antinode–antinode ends: $\lambda_n = 2L/n$, $f_n = nv/2L$. Node–antinode ends (closed–open pipe, fixed–free string): $\lambda_n = 4L/(2n-1)$, $f_n = (2n-1)v/4L$ – odd harmonics only. A fixed end or closed pipe end is a displacement node; a free end or open pipe end is an antinode. Points between neighbouring nodes oscillate in phase; points either side of a node are in antiphase.

Method — Worked example: one pipe, two configurations

A 0.50 m pipe with air at 340 m/s. Open at both ends:
$f_1 = v/2L = 340/1.0 = 340\ \mathrm{Hz}$. Closed at one end:
$f_1 = v/4L = 340/2.0 = 170\ \mathrm{Hz}$, and only odd harmonics – 170, 510, 850 Hz – can sound. Two habits: write the boundary condition at each end before choosing the formula, and check the harmonic list against the pattern (every n, or odd n only).

Exercises 2.1·2.3

2.1 State how a standing wave forms, and what is meant by a node. [2]

2.2 State the phase difference between two points on a standing wave that lie on the same side of a node, and between two points on opposite sides of a node. [2]

2.3 A string fixed at both ends is 0.80 m long and carries waves at 160 m/s. Calculate the first and second harmonic frequencies. [3]

Exercises 2.1·2.3

2.1 State how a standing wave forms, and what is meant by a node. [2]

2.1 Two identical waves travelling in opposite directions superpose; a node is a point of zero displacement where the two always cancel.

2.2 State the phase difference between two points on a standing wave that lie on the same side of a node, and between two points on opposite sides of a node. [2]

2.2 Same side of a node: in phase, 0°; opposite sides: antiphase, 180°.

2.3 A string fixed at both ends is 0.80 m long and carries waves at 160 m/s. Calculate the first and second harmonic frequencies. [3]

2.3 $f_1 = v/2L = 160/1.6 = 100$ Hz; $f_2 = 200$ Hz.

Exercises 2.4·3.2

2.4 State two changes damping makes to the resonance curve of a driven oscillator. [2]

3.1 A pipe closed at one end is 0.34 m long; the speed of sound is 340 m/s. Calculate the first harmonic frequency and the next harmonic frequency above it, and explain why no harmonic exists between them. [4]

3.2 A guitar string is fixed at both ends. State the boundary condition at each end, and explain why only certain wavelengths can form standing waves on the string. [4]

Exercises 2.4·3.2

2.4 State two changes damping makes to the resonance curve of a driven oscillator. [2]

2.4 Any two of: the maximum amplitude falls; the peak broadens; the resonant frequency shifts slightly below the natural frequency.

3.1 A pipe closed at one end is 0.34 m long; the speed of sound is 340 m/s. Calculate the first harmonic frequency and the next harmonic frequency above it, and explain why no harmonic exists between them. [4]

3.1 $f_1 = v/4L = 340/1.36 = 250$ Hz; the next harmonic is the third, $f_3 = 750$ Hz; a node must exist at the closed end and an antinode at the open end, so only an odd number of quarter-wavelengths fits – the second harmonic cannot meet both conditions.

3.2 A guitar string is fixed at both ends. State the boundary condition at each end, and explain why only certain wavelengths can form standing waves on the string. [4]

3.2 A displacement node at each fixed end; a standing wave must fit a whole number of half-wavelengths between the nodes, $\lambda_n = 2L/n$, so only those wavelengths reinforce after reflection – all others destructively interfere away.

Exercises 3.3·3.4

3.3 A car's suspension oscillates at a natural frequency of about 1.5 Hz. Describe what happens to the amplitude as the car is driven over regular bumps at increasing speed, and explain the role of the shock absorbers in this context. [4]

3.4 A pipe of length 0.50 m is first tested open at both ends and then closed at one end (air speed 340 m/s). Calculate both first harmonic frequencies and state the ratio between them. [4]

Exercises 3.3·3.4

3.3 A car's suspension oscillates at a natural frequency of about 1.5 Hz. Describe what happens to the amplitude as the car is driven over regular bumps at increasing speed, and explain the role of the shock absorbers in this context. [4]

3.3 As bump frequency approaches 1.5 Hz the amplitude grows sharply – resonance; at higher driving frequencies the amplitude falls again; shock absorbers damp the oscillation, lowering the resonant peak and broadening it, so the car cannot build up a large amplitude at any speed.

3.4 A pipe of length 0.50 m is first tested open at both ends and then closed at one end (air speed 340 m/s). Calculate both first harmonic frequencies and state the ratio between them. [4]

3.4 Open–open: $f_1 = v/2L = 340/1.0 = 340$ Hz; closed–open: $f_1 = v/4L = 340/2.0 = 170$ Hz; ratio 2:1 – an octave apart (stated on either line).