5.2 Galilean and special relativity (full A.5, HL)

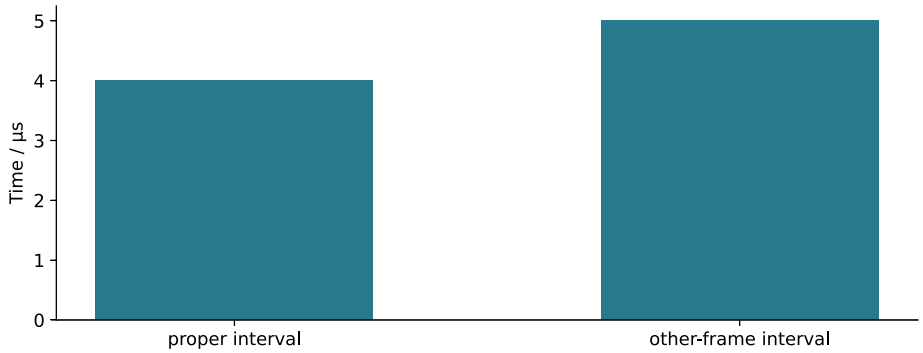
Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS reference frame • time dilation • length contraction

Objective

Practise the full guide A.5 toolkit (HL only): inertial frames and the Galilean transformations, the two postulates, the Lorentz transformations with γ, relativistic velocity addition, the invariant space–time interval, proper time/length, time dilation and length contraction, relativity of simultaneity, and space–time diagrams with world lines at tan θ = v/c. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ **The relativity equation set**

$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad u' = \frac{u - v}{1 - uv/c^2}$$

$$(\Delta s)^2 = (c\,\Delta t)^2 - (\Delta x)^2 \qquad \Delta t = \gamma\,\Delta t_0 \qquad L = \frac{L_0}{\gamma} \qquad \tan\theta = \frac{v}{c}$$

Galilean limits: $x' = x - vt$, $t' = t$, $u' = u - v$ (valid when $v \ll c$).

Worked check. At $v = 0.80c$: $\gamma = 1/\sqrt{1 - 0.64} = 1/0.60 = 1.67$; a 30 m proper length contracts to $L = 30/1.67 = 18$ m; a 1.0 s proper time dilates to $\Delta t = 1.67$ s.

2 Practice

2.1 Calculate γ for v = 0.28c. [2]

2.2 State the two postulates of special relativity. [2]

2.3 Write the Galilean transformations and state when they are adequate. [2]

2.4 A rod of proper length 45 m moves at 0.60c. Calculate its contracted length. [2]

2.5 A particle's world line makes 45° with the ct axis. Calculate its speed. [2]

3 Exam-style questions

3.1 A spacecraft passes Earth at 0.90c. An observer on the spacecraft measures a journey between two stars as lasting 4.0 years (proper time). Calculate γ, the journey time measured from Earth, and the distance Earth observers say the stars are apart if the spacecraft moves the whole time. [5]

3.2 A spaceship moving at 0.80c fires a probe forward at 0.70c relative to the ship. Calculate the probe's speed relative to Earth using relativistic velocity addition, and compare it with the Galilean answer. [4]

3.3 Muons created in the upper atmosphere have a proper lifetime of 2.2 µs and travel at 0.995c ($\gamma = 10.0$). Explain, using BOTH time dilation and length contraction, how they reach the ground – and state what would happen in a purely Galilean calculation. [5]