INTERNATIONAL BACCALAUREATE · IB DIPLOMA · PHYSICS · HL • EXERCISE SHEET

# 16.2 Gravitational fields (full D.1, HL)

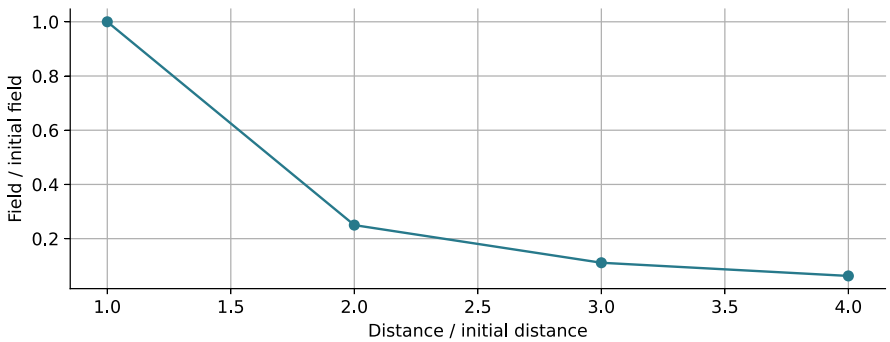
Name: _____ Class: _____ Date: _____ **Total: 31 marks**

**KEY WORDS** gravitational field strength • gravitational constant • Kepler's laws • gravitational potential • escape speed

## Objective

Practise the full guide D.1 toolkit including AHL: Newton's law of gravitation $F = Gm_1m_2/r^2$ with the point-mass condition, gravitational field strength $g = GM/r^2$, field-line patterns, resultant fields along the line of two bodies, and Kepler's three laws including $T^2 \propto r^3$ in problems; at higher level also potential $V_g = -GM/r$, potential energy $E_p = -Gm_1m_2/r$, the potential gradient, equipotential surfaces, escape and orbital speeds, and satellite energetics. Original practice; not an official IB paper.

## 1 Key knowledge and worked examples




### ■ The D.1 rule set (HL)

$$F = \frac{Gm_1m_2}{r^2} \qquad g = \frac{GM}{r^2} \qquad V_g = -\frac{GM}{r} \qquad g = -\frac{\Delta V_g}{\Delta r} \qquad v_{esc} = \sqrt{\frac{2GM}{r}} \qquad v_{orbital} = \sqrt{\frac{GM}{r}}$$

Field lines point inward, never cross, and crowd where the field is stronger. A spherical body of uniform density acts as a point mass at its centre. Kepler: ellipses with the central body at a focus; equal areas in equal times; $T^2 \propto r^3$.

### ■ Worked example: Kepler with real bodies

Mars orbits at 1.52 AU: $T = 1.52^{3/2} = 1.87$ Earth years. Earth's surface: $g = GM/r^2 = (6.67 \times 10^{-11} \times 6.0 \times 10^{24})/(6.4 \times 10^6)^2 = 9.8$ m/s$^2$. Two habits: measure $r$ from the centre of the mass, not from the surface; and for period comparisons cube the radius ratio before taking its square root.

## 2 Practice

**2.1** State Newton's law of gravitation, naming the quantities in the equation. [2]

**2.2** Mars has mass $6.4 \times 10^{23}$ kg and radius $3.4 \times 10^6$ m. Calculate the gravitational field strength at its surface ($G = 6.67 \times 10^{-11}$). [3]

**2.3** State Kepler's third law, and calculate the orbital period of a planet at 9.5 AU, in Earth years. [3]

**2.4** Describe two features of the gravitational field-line pattern around a spherical planet. [2]

**2.5** A satellite orbits Earth ($M = 6.0 \times 10^{24}$ kg) at $r = 4.2 \times 10^7$ m. Calculate its orbital speed and its escape speed from that orbit. [3]

## 3 Exam-style questions

**3.1** Calculate the gravitational field strength at a point a distance $2R$ from Earth's centre, where $R = 6.4 \times 10^6$ m is Earth's radius and surface $g = 9.8$ m/s$^2$. [3]

**3.2** A geostationary satellite orbits at $r = 4.2 \times 10^7$ m from Earth's centre ($M = 6.0 \times 10^{24}$ kg). Use $T^2 = 4\pi^2 r^3/GM$ to show its period is about one day. [4]

**3.3** A comet's orbit has perihelion 0.59 AU and aphelion 35 AU. Estimate its orbital period using Kepler's third law, stating one assumption the estimate makes. [4]

**3.4** Explain when a planet may be treated as a point mass for gravitation calculations, and describe how the field strength varies along a line moving away from its surface. [3]

**3.5** A 1200 kg satellite is raised from Earth's surface ($r = 6.4 \times 10^6$ m) to a parking orbit at $r = 1.28 \times 10^7$ m. Calculate the increase in its gravitational potential energy, and state what happens to the total energy of a bound circular orbit as its radius increases. [4]