8.2 Gas laws and the kinetic model (full B.3)

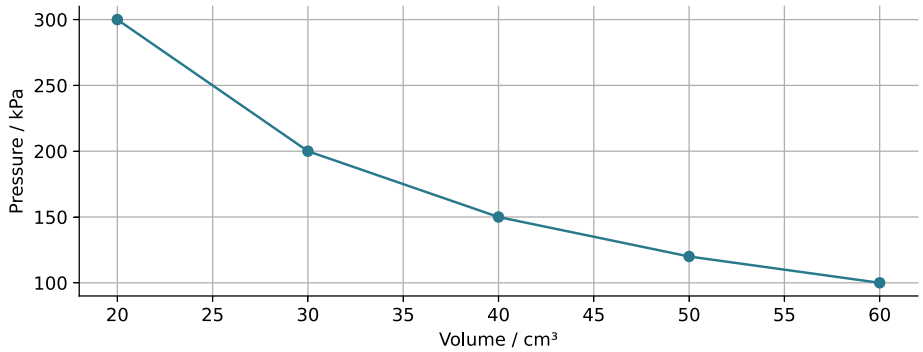
Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS ideal gas • Avogadro constant • monatomic

Objective

Practise the full guide B.3 toolkit: pressure as F/A, n = N/N_A, the ideal gas law in both forms (PV = Nk_BT and PV = nRT) with PV/T constant, the kinetic derivation link P = 1/3 $\rho\langle v^2\rangle$, internal energy of a monatomic ideal gas, pressure–volume diagrams, and the conditions where the ideal gas approximates a real gas. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ **The B.3 equation set**

$$P = \frac{F}{A} \qquad n = \frac{N}{N_A} \qquad \frac{PV}{T} = \text{constant} \qquad PV = Nk_BT = nRT$$

$$P = \tfrac{1}{3}\rho\langle v^2\rangle \qquad U = \tfrac{3}{2}Nk_BT = \tfrac{3}{2}nRT$$

Worked check. 0.50 mol of an ideal gas at 250 K in 8.0×10^{-3} m^3: P = nRT/V = $0.50 \times 8.31 \times 250/8.0\times10^{-3} = 1.3\times10^5$ Pa; U = $1.5 \times 0.50 \times 8.31 \times 250 = 1.6\times10^3$ J.

Ideal ≈ real at high temperature and low pressure; deviations grow near condensation.

2 Practice

2.1 State the two forms of the ideal gas equation, naming every symbol. [4]

2.2 A gas occupies 3.0×10^{-3} m^3 at 300 K and 1.2×10^5 Pa. It is compressed to 1.0×10^{-3} m^3 at constant temperature. Calculate the new pressure. [3]

2.3 Calculate n for N = 1.2×10^{24} molecules (N_A = 6.02×10^{23} mol^{-1}). [2]

2.4 State the internal energy formula for an ideal monatomic gas. [1]

2.5 Under what conditions does a real gas behave most ideally? [2]

3 Exam-style questions

3.1 A sealed cylinder holds 0.20 mol of an ideal monatomic gas at 200 K, with volume 6.0×10^{-3} m^3. Calculate the pressure and internal energy; the gas is then heated at constant volume to 400 K – calculate the new pressure. [6]

3.2 Explain, using momentum change at the walls, why gas pressure rises with temperature at constant volume, referencing $P = 1/3\ \rho\langle v^2\rangle$. [4]

3.3 Sketch or describe the pressure–volume diagram for an isothermal compression, and explain why the hyperbola shape follows from $PV/T = \text{constant}$. [3]

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 PV = Nk_BT (N molecules, Boltzmann constant) and PV = nRT (n moles, gas constant).

2.2 At constant T: $P_1V_1 = P_2V_2$; $P_2 = 1.2\times10^5 \times 3.0\times10^{-3}/1.0\times10^{-3} = 3.6\times10^5$ Pa.

2.3 n = N/N_A = $1.2\times10^{24}/6.02\times10^{23} = 2.0$ mol.

2.4 U = 3/2 Nk_BT = 3/2 nRT.

2.5 High temperature and low pressure (low density) – molecules far from condensing.

3.1 P = nRT/V = $0.20 \times 8.31 \times 200/6.0\times10^{-3} = 5.5\times10^4$ Pa; U = $1.5 \times 0.20 \times 8.31 \times 200 = 5.0\times10^2$ J; constant V, $P \propto T$: $P_2 = 5.5\times10^4 \times 400/200 = 1.1\times10^5$ Pa.

3.2 Molecules rebounding from a wall reverse momentum component, so each collision gives the wall an impulse; hotter molecules move faster, arriving more often and transferring more momentum per collision; since $P = 1/3\ \rho\langle v^2\rangle$ and ρ is fixed at constant V, the larger $\langle v^2\rangle$ raises P.

3.3 A smooth hyperbola falling from upper-left to lower-right ($P \propto 1/V$); from PV/T = constant with T fixed, PV is constant, so P = constant/V; points farther along the curve mark smaller V with proportionally larger P, all at the same T.