5.2 Galilean and special relativity (full A.5, HL)

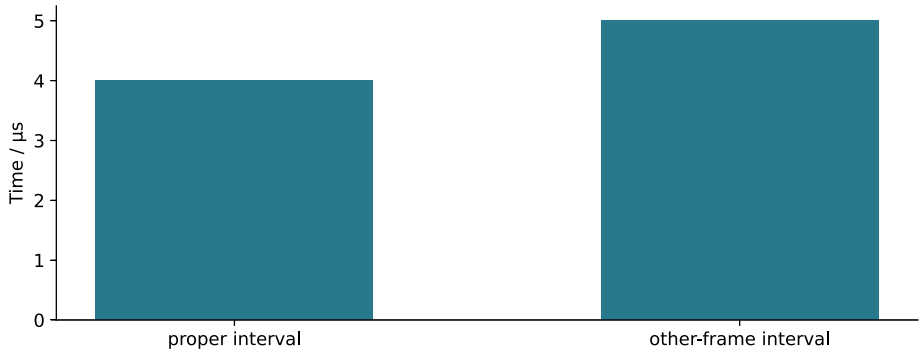
Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS reference frame • time dilation • length contraction

Objective

Practise the full guide A.5 toolkit (HL only): inertial frames and the Galilean transformations, the two postulates, the Lorentz transformations with γ, relativistic velocity addition, the invariant space–time interval, proper time/length, time dilation and length contraction, relativity of simultaneity, and space–time diagrams with world lines at tan θ = v/c. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ **The relativity equation set**

$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad u' = \frac{u - v}{1 - uv/c^2}$$

$$(\Delta s)^2 = (c\,\Delta t)^2 - (\Delta x)^2 \qquad \Delta t = \gamma\,\Delta t_0 \qquad L = \frac{L_0}{\gamma} \qquad \tan\theta = \frac{v}{c}$$

Galilean limits: $x' = x - vt$, $t' = t$, $u' = u - v$ (valid when $v \ll c$).

Worked check. At $v = 0.80c$: $\gamma = 1/\sqrt{1 - 0.64} = 1/0.60 = 1.67$; a 30 m proper length contracts to $L = 30/1.67 = 18$ m; a 1.0 s proper time dilates to $\Delta t = 1.67$ s.

2 Practice

2.1 Calculate γ for v = 0.28c. [2]

2.2 State the two postulates of special relativity. [2]

2.3 Write the Galilean transformations and state when they are adequate. [2]

2.4 A rod of proper length 45 m moves at 0.60c. Calculate its contracted length. [2]

2.5 A particle's world line makes 45° with the ct axis. Calculate its speed. [2]

3 Exam-style questions

3.1 A spacecraft passes Earth at 0.90c. An observer on the spacecraft measures a journey between two stars as lasting 4.0 years (proper time). Calculate γ, the journey time measured from Earth, and the distance Earth observers say the stars are apart if the spacecraft moves the whole time. [5]

3.2 A spaceship moving at 0.80c fires a probe forward at 0.70c relative to the ship. Calculate the probe's speed relative to Earth using relativistic velocity addition, and compare it with the Galilean answer. [4]

3.3 Muons created in the upper atmosphere have a proper lifetime of 2.2 µs and travel at 0.995c ($\gamma = 10.0$). Explain, using BOTH time dilation and length contraction, how they reach the ground – and state what would happen in a purely Galilean calculation. [5]

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 $\gamma = 1/\sqrt{1 - 0.0784} = 1/0.960 = 1.04$.

2.2 The laws of physics are the same in all inertial reference frames; the speed of light in vacuum is the same for all inertial observers, regardless of the source's motion.

2.3 $x' = x - vt$, $t' = t$; adequate when all speeds are much less than c.

2.4 $\gamma = 1/\sqrt{1 - 0.36} = 1.25$; $L = 45/1.25 = 36$ m.

2.5 $\tan 45° = 1 = v/c$, so $v = c$ – a photon's world line.

3.1 $\gamma = 1/\sqrt{1 - 0.81} = 1/0.436 = 2.29$; Earth time $\Delta t = \gamma \Delta t_0 = 2.29 \times 4.0 = 9.2$ years; distance $= vt = 0.90c \times 9.2$ years $= 8.3$ light-years.

3.2 With the ship moving at $v = 0.80c$ and the probe at $u' = 0.70c$ relative to it, Earth measures $u = (u' + v)/(1 + u'v/c^2) = 1.50c/1.56 = 0.96c$. Galilean: $0.70c + 0.80c = 1.50c$ – faster than light, impossible; the relativistic denominator prevents the excess.

3.3 Earth frame: lifetime dilated to $\Delta t = 10.0 \times 2.2$ µs $= 22$ µs; distance travelled $= 0.995c \times 22$ µs ≈ 6.6 km – far enough to reach the ground. Muon frame: its lifetime stays 2.2 µs, but the atmosphere's thickness contracts by $\gamma = 10.0$, so the same few hundred km becomes tens of km – also far enough. Galilean: only $0.995c \times 2.2$ µs ≈ 0.66 km – the muons would decay high up; the observed ground flux is evidence for relativity.