

5.2 Galilean and special relativity (full A.5, HL)

Name: _____ Class: _____ Date: _____

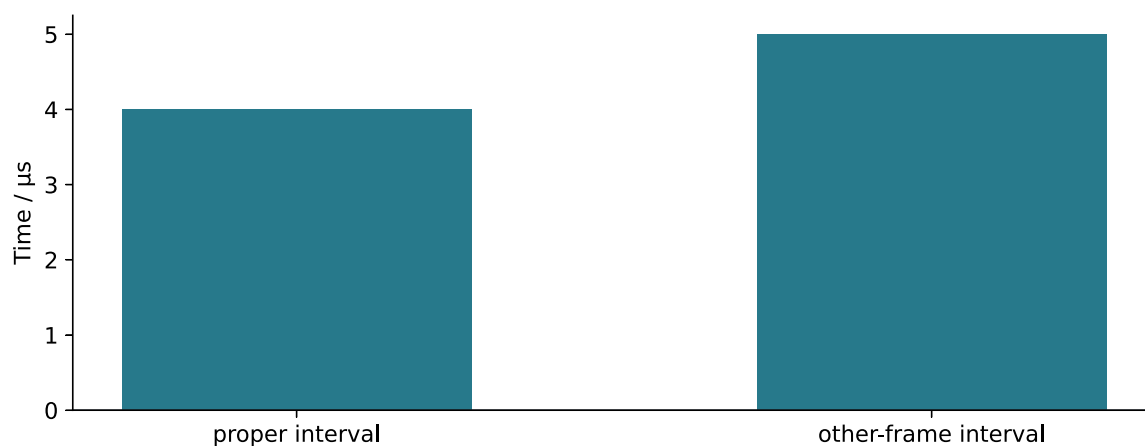
Total: 24 marks

KEY WORDS reference frame • time dilation • length contraction

Objective

Practise the full guide A.5 toolkit (HL only): inertial frames and the Galilean transformations, the two postulates, the Lorentz transformations with γ , relativistic velocity addition, the invariant space–time interval, proper time/length, time dilation and length contraction, relativity of simultaneity, and space–time diagrams with world lines at $\tan \theta = v/c$. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ The relativity equation set

$$x' = \gamma(x - vt) \quad t' = \gamma\left(t - \frac{vx}{c^2}\right) \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad u' = \frac{u - v}{1 - uv/c^2}$$

$$(\Delta s)^2 = (c \Delta t)^2 - (\Delta x)^2 \quad \Delta t = \gamma \Delta t_0 \quad L = \frac{L_0}{\gamma} \quad \tan \theta = \frac{v}{c}$$

Galilean limits: $x' = x - vt$, $t' = t$, $u' = u - v$ (valid when $v \ll c$).

Worked check. At $v = 0.80c$: $\gamma = 1/\sqrt{1 - 0.64} = 1/0.60 = 1.67$; a 30 m proper length contracts to $L = 30/1.67 = 18$ m; a 1.0 s proper time dilates to $\Delta t = 1.67$ s.

2 Practice

2.1 Calculate γ for $v = 0.28c$. [2]

2.2 State the two postulates of special relativity. [2]

2.3 Write the Galilean transformations and state when they are adequate. [2]

2.4 A rod of proper length 45 m moves at $0.60c$. Calculate its contracted length. [2]

2.5 A particle's world line makes 45° with the ct axis. Calculate its speed. [2]

3 Exam-style questions

3.1 A spacecraft passes Earth at $0.90c$. An observer on the spacecraft measures a journey between two stars as lasting 4.0 years (proper time). Calculate γ , the journey time measured from Earth, and the distance Earth observers say the stars are apart if the spacecraft moves the whole time. [5]

3.2 A spaceship moving at $0.80c$ fires a probe forward at $0.70c$ relative to the ship. Calculate the probe's speed relative to Earth using relativistic velocity addition, and compare it with the Galilean answer. [4]

3.3 Muons created in the upper atmosphere have a proper lifetime of $2.2\ \mu\text{s}$ and travel at $0.995c$ ($\gamma = 10.0$). Explain, using BOTH time dilation and length contraction, how they reach the ground – and state what would happen in a purely Galilean calculation. [5]

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 $\gamma = 1/\sqrt{1 - 0.0784} = 1/0.960 = 1.04$.

2.2 The laws of physics are the same in all inertial reference frames; the speed of light in vacuum is the same for all inertial observers, regardless of the source's motion.

2.3 $x' = x - vt$, $t' = t$; adequate when all speeds are much less than c .

2.4 $\gamma = 1/\sqrt{1 - 0.36} = 1.25$; $L = 45/1.25 = 36$ m.

2.5 $\tan 45^\circ = 1 = v/c$, so $v = c$ – a photon's world line.

3.1 $\gamma = 1/\sqrt{1 - 0.81} = 1/0.436 = 2.29$; Earth time $\Delta t = \gamma \Delta t_0 = 2.29 \times 4.0 = 9.2$ years; distance $= vt = 0.90c \times 9.2$ years $= 8.3$ light-years.

3.2 With the ship moving at $v = 0.80c$ and the probe at $u' = 0.70c$ relative to it, Earth measures $u = (u' + v)/(1 + u'v/c^2) = 1.50c/1.56 = 0.96c$. Galilean: $0.70c + 0.80c = 1.50c$ – faster than light, impossible; the relativistic denominator prevents the excess.

3.3 Earth frame: lifetime dilated to $\Delta t = 10.0 \times 2.2 \mu\text{s} = 22 \mu\text{s}$; distance travelled $= 0.995c \times 22 \mu\text{s} \approx 6.6$ km – far enough to reach the ground. Muon frame: its lifetime stays $2.2 \mu\text{s}$, but the atmosphere's thickness contracts by $\gamma = 10.0$, so the same few hundred km becomes tens of km – also far enough. Galilean: only $0.995c \times 2.2 \mu\text{s} \approx 0.66$ km – the muons would decay high up; the observed ground flux is evidence for relativity.