4.2 Rigid body mechanics (full A.4, HL)

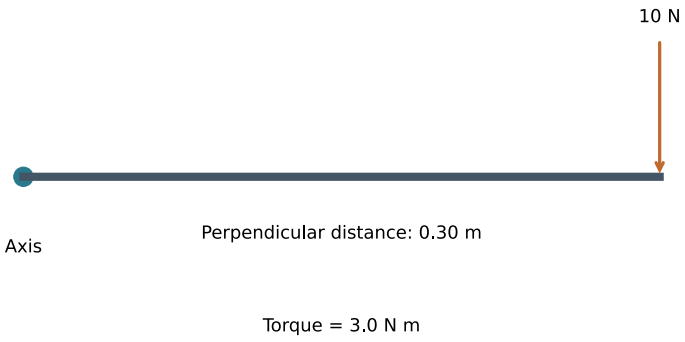
Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS torque • moment of inertia • angular momentum

Objective

Practise the full guide A.4 toolkit (HL only): torque and rotational equilibrium, the angular equations of motion, moment of inertia for point masses, Newton's second law for rotation, angular momentum conservation with changing moment of inertia, and rotational kinetic energy. Original practice; not an official IB paper.

1 Key knowledge and worked examples



The rotational analogues

$\tau = Fr\sin\theta \qquad \tau = I\alpha \qquad I = \sum mr^2 \qquad L = I\omega \qquad \Delta L = \tau\Delta t \qquad E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$

Equilibrium: resultant torque zero (state the clockwise/counter-clockwise sense). Angular equations of motion mirror the linear set with θ, ω, α in place of s, v, a; rolling without slipping: $v = \omega r$.

Worked check. A uniform angular acceleration from rest to 12 rad/s in 4.0 s: $\alpha = 3.0$ rad/s^2; $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 3.0 \times 16 = 24$ rad.

2 Practice

2.1 A 25 N force acts 0.40 m from an axis at 60° to the lever. Calculate the torque. [2]

2.2 A 500 g mass and a 300 g mass sit 0.50 m and 0.70 m from the axis. Calculate the moment of inertia. [3]

2.3 A wheel of $I = 0.20$ kg m^2 accelerates angularly at 4.0 rad/s^2. Calculate the torque. [2]

2.4 A disc of $I = 2.4$ kg m^2 spins at 8.0 rad/s. Calculate its angular momentum and rotational kinetic energy. [3]

2.5 State the rolling-without-slipping relation between linear and angular speed. [1]

3 Exam-style questions

3.1 A beam pivots at its centre. A 120 N weight hangs 0.80 m left of the pivot and a 90 N weight 0.60 m right. Determine whether the beam is in rotational equilibrium, stating senses; if not, calculate the mass needed 0.50 m further right to balance it (g = 9.8 m/s^2). [5]

3.2 A flywheel ($I = 3.0\ \mathrm{kg\,m^2}$) spins at 20 rad/s and is slowed by a constant friction torque to rest in 25 s. Calculate the angular deceleration, the friction torque, and the initial angular momentum. [5]

3.3 A dancer with $I = 5.0\ \mathrm{kg\,m^2}$ spins at 3.0 rad/s. She changes her shape to $I = 3.0\ \mathrm{kg\,m^2}$. Calculate her new angular speed and the change in rotational kinetic energy, and state where the extra energy comes from. [5]

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 $\tau = 25 \times 0.40 \times \sin 60° = 8.7$ N · m.

2.2 $I = 0.500 \times 0.50^2 + 0.300 \times 0.70^2 = 0.125 + 0.147 = 0.272$ kg m^2.

2.3 $\tau = I\alpha = 0.20 \times 4.0 = 0.80$ N · m.

2.4 $L = I\omega = 2.4 \times 8.0 = 19.2$ kg m^2/s; $E_k = \frac{1}{2} \times 2.4 \times 64 = 76.8 \approx 77$ J. Check $L^2/2I = 368.64/4.8 = 76.8$ J ✓.

2.5 $v = \omega r$.

3.1 Left: $120 \times 0.80 = 96$ N · m counter-clockwise; right: $90 \times 0.60 = 54$ N · m clockwise; not balanced – net 42 N · m counter-clockwise. Balance needs 42 N · m clockwise at 0.50 m further right: force = 84 N → mass = 84/9.8 = 8.6 kg.

3.2 $\alpha = (0 - 20)/25 = -0.80$ rad/s^2; $\tau = I\alpha = 3.0 \times 0.80 = 2.4$ N · m opposing the spin; $L_0 = 3.0 \times 20 = 60$ kg m^2/s.

3.3 No external torque: $L = 5.0 \times 3.0 = 15$ kg m^2/s conserved; $\omega' = 15/3.0 = 5.0$ rad/s. E_k: from $\frac{1}{2} \times 5.0 \times 9 = 22.5$ J to $\frac{1}{2} \times 3.0 \times 25 = 37.5$ J – up 15 J; her muscles do work pulling her mass inward.