22.2 Radioactive decay (full E.3, HL)

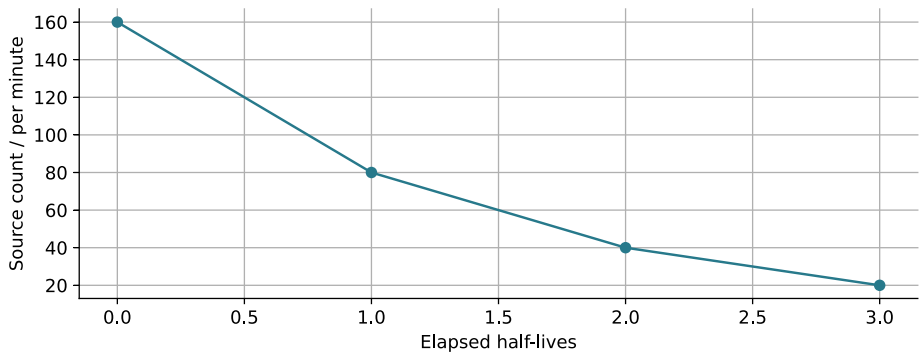
Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS isotope • binding energy • half-life • background radiation • decay constant • neutrino

Objective

Practise the full guide E.3 toolkit including AHL: isotopes, mass defect and binding energy with $E = \Delta mc^2$, the binding-energy-per-nucleon curve, the strong nuclear force, the random spontaneous nature of decay, alpha, beta-minus, beta-plus and gamma decay equations with neutrinos, penetration and ionizing ability, activity, count rate, background radiation and integer half-lives; at higher level also the N/Z stability band, saturated binding above A = 60, discrete nuclear levels against beta's continuous spectrum, the neutrino, and the decay law $N = N_0 e^{-\lambda t}$ with $A = \lambda N$ and $T_{1/2} = \ln 2/\lambda$. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ **The E.3 rule set (HL)**

$$E = \Delta mc^2 \qquad N = N_0 e^{-\lambda t} \qquad A = \lambda N \qquad T_{1/2} = \frac{\ln 2}{\lambda}$$

Alpha: $A-4$, $Z-2$. Beta-minus: $Z+1$ with an electron and antineutrino. Beta-plus: $Z-1$ with a positron and neutrino. Gamma: A, Z unchanged. Penetration falls and ionizing power rises in the order gamma, beta, alpha; subtract background from any count rate.

■ **Worked example: counting through background**

A sample gives 5200 counts/min with a 200 counts/min background: the source alone is 5000. After three half-lives it contributes $5000/8 = 625$, and the meter reads 825. Two habits: strip the background before halving anything, and check decay equations conserve both A and Z.

2 Practice

2.1 Define an isotope, and state what isotopes of an element share. [2]

2.2 State what is meant by the mass defect of a nucleus. [1]

2.3 Write the full nuclear equation for beta-minus decay of ${}^{14}_{6}C$. [2]

2.4 Place alpha, beta and gamma radiation in order of increasing penetrating ability. [1]

2.5 A nuclide has decay constant 0.138 day^{-1}. Calculate its half-life, and the fraction remaining after 10 days. [3]

3 Exam-style questions

3.1 State the nuclear changes in A and Z for alpha decay, and write the equation for ${}^{226}_{88}Ra$ decaying by alpha emission. [3]

3.2 Two protons and two neutrons have total mass 4.0319 u; a helium-4 nucleus has mass 4.0026 u. Calculate the binding energy in MeV (1 u = 931.5 MeV/c^2) and the binding energy per nucleon. [4]

3.3 Explain why a medical tracer isotope is chosen with a short half-life and gamma emission, referring to penetration and ionizing ability. [3]

3.4 A source reads 4200 counts/min above a background of 200 counts/min. Calculate the count rate after two half-lives, and state one assumption. [3]

3.5 Explain how the continuous spectrum of beta decay points to the neutrino, while the line spectra of alpha and gamma radiation point to discrete nuclear energy levels. [4]

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 Nuclides of the same element with different neutron numbers; they share the same proton number Z (and chemical symbol).

2.2 The difference between the total mass of the separate nucleons and the mass of the assembled nucleus – the mass converted to binding energy.

2.3 ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e + \bar{\nu}$.

2.4 Alpha, beta, gamma.

2.5 $T_{1/2} = \ln 2/\lambda = 0.693/0.138 = 5.0$ days; fraction $= e^{-\lambda t} = e^{-(0.138\times 10)} = e^{-1.38} = 0.25$.

3.1 A falls by 4 and Z falls by 2; ${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}\alpha$.

3.2 $\Delta m = 4.0319 - 4.0026 = 0.0293\ u$; $E = 0.0293 \times 931.5 = 27.3$ MeV; per nucleon: $27.3/4 = 6.8$ MeV.

3.3 Gamma penetrates body tissue to reach detectors outside while ionizing least along its path, minimizing damage; a short half-life means activity – and dose – fall quickly once the scan is done.

3.4 Source alone: $4200 - 200 = 4000$ counts/min; after two half-lives: $4000/4 = 1000$; the meter reads $1000 + 200 = 1200$ counts/min, assuming the background rate stays constant.

3.5 If beta decay had only two products, conservation of energy and momentum would fix the electron's energy exactly – one value; the observed electrons span a continuous range, so a third, neutral, barely-interacting particle – the neutrino – must share the energy unpredictably; alpha and gamma photons instead appear with single sharp energies, meaning the emitting nucleus moves between definite energy levels, exactly as atomic line spectra mean atomic levels.