16.2 Gravitational fields (full D.1, HL)

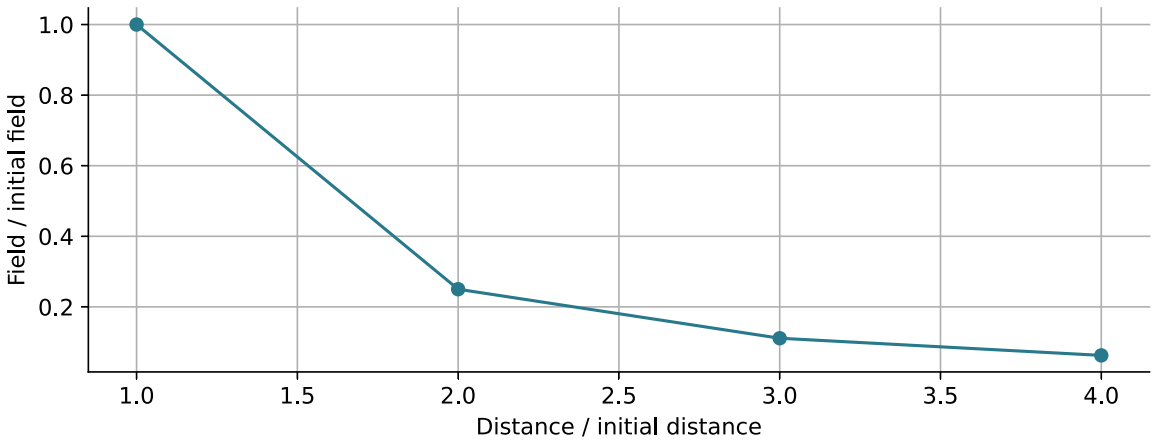
Name: _____ Class: _____ Date: _____ **Total: 31 marks**

KEY WORDS gravitational field strength • gravitational constant • Kepler's laws • gravitational potential • escape speed

Objective

Practise the full guide D.1 toolkit including AHL: Newton's law of gravitation $F = Gm_1m_2/r^2$ with the point-mass condition, gravitational field strength $g = GM/r^2$, field-line patterns, resultant fields along the line of two bodies, and Kepler's three laws including $T^2 \propto r^3$ in problems; at higher level also potential $V_g = -GM/r$, potential energy $E_p = -Gm_1m_2/r$, the potential gradient, equipotential surfaces, escape and orbital speeds, and satellite energetics. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ The D.1 rule set (HL)

$$F = \frac{Gm_1m_2}{r^2} \qquad g = \frac{GM}{r^2} \qquad V_g = -\frac{GM}{r} \qquad g = -\frac{\Delta V_g}{\Delta r} \qquad v_{esc} = \sqrt{\frac{2GM}{r}} \qquad v_{orbital} = \sqrt{\frac{GM}{r}}$$

Field lines point inward, never cross, and crowd where the field is stronger. A spherical body of uniform density acts as a point mass at its centre. Kepler: ellipses with the central body at a focus; equal areas in equal times; $T^2 \propto r^3$.

■ **Worked example: Kepler with real bodies**

Mars orbits at 1.52 AU: $T = 1.52^{3/2} = 1.87$ Earth years. Earth's surface: $g = GM/r^2 = (6.67 \times 10^{-11} \times 6.0 \times 10^{24})/(6.4 \times 10^6)^2 = 9.8$ m/s^2. Two habits: measure r from the centre of the mass, not from the surface; and for period comparisons cube the radius ratio before taking its square root.

2 Practice

2.1 State Newton's law of gravitation, naming the quantities in the equation. [2]

2.2 Mars has mass 6.4×10^{23} kg and radius 3.4×10^6 m. Calculate the gravitational field strength at its surface ($G = 6.67 \times 10^{-11}$). [3]

2.3 State Kepler's third law, and calculate the orbital period of a planet at 9.5 AU, in Earth years. [3]

2.4 Describe two features of the gravitational field-line pattern around a spherical planet. [2]

2.5 A satellite orbits Earth ($M = 6.0 \times 10^{24}$ kg) at $r = 4.2 \times 10^7$ m. Calculate its orbital speed and its escape speed from that orbit. [3]

3 Exam-style questions

3.1 Calculate the gravitational field strength at a point a distance $2R$ from Earth's centre, where $R = 6.4 \times 10^6$ m is Earth's radius and surface $g = 9.8$ m/s^2. [3]

3.2 A geostationary satellite orbits at $r = 4.2 \times 10^7$ m from Earth's centre ($M = 6.0 \times 10^{24}$ kg). Use $T^2 = 4\pi^2 r^3/GM$ to show its period is about one day. [4]

3.3 A comet's orbit has perihelion 0.59 AU and aphelion 35 AU. Estimate its orbital period using Kepler's third law, stating one assumption the estimate makes. [4]

3.4 Explain when a planet may be treated as a point mass for gravitation calculations, and describe how the field strength varies along a line moving away from its surface. [3]

3.5 A 1200 kg satellite is raised from Earth's surface ($r = 6.4 \times 10^6$ m) to a parking orbit at $r = 1.28 \times 10^7$ m. Calculate the increase in its gravitational potential energy, and state what happens to the total energy of a bound circular orbit as its radius increases. [4]

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 The attractive force between two point masses is proportional to the product of the masses and inversely proportional to the square of their separation: $F = Gm_1m_2/r^2$; G the gravitational constant, m_1, m_2 the masses, r the separation between centres.

2.2 $g = GM/r^2 = (6.67 \times 10^{-11} \times 6.4 \times 10^{23})/(3.4 \times 10^6)^2 = 4.3 \times 10^{13}/1.2 \times 10^{13} = 3.7$ m/s^2.

2.3 The square of the orbital period is proportional to the cube of the orbital radius, $T^2 \propto r^3$; $T = 9.5^{3/2} = 29$ Earth years.

2.4 Any two: the lines point radially inward toward the centre; they never cross; they are equally spaced over a sphere at any radius and crowd nearer the surface where the field is stronger.

2.5 $v_{orbital} = \sqrt{GM/r} = \sqrt{(4.0 \times 10^{14})/(4.2 \times 10^7)} = \sqrt{9.5 \times 10^6} = 3.1 \times 10^3$ m/s; $v_{esc} = \sqrt{2}\, v_{orbital} = 4.4 \times 10^3$ m/s.

3.1 $g \propto 1/r^2$; doubling the distance divides the field by four: $g = 9.8/4 = 2.5$ m/s^2.

3.2 $T^2 = 4\pi^2 r^3/GM = (39.5 \times (4.2 \times 10^7)^3)/(6.67 \times 10^{-11} \times 6.0 \times 10^{24})$; numerator $= 39.5 \times 7.4 \times 10^{22} = 2.9 \times 10^{24}$, denominator $= 4.0 \times 10^{14}$, so $T^2 = 7.3 \times 10^9$ s^2; $T = 8.5 \times 10^4$ s ≈ 24 h.

3.3 Mean orbital radius $\approx (0.59 + 35)/2 \approx 18$ AU; $T = 18^{3/2} = 76$ years; assumes the comet's elliptical orbit is approximated by a circular orbit of that mean radius – the third law applied to the semi-major axis.

3.4 When the body is spherically symmetric with (near-)uniform density, its gravity outside it is that of a point mass at its centre; equally when the separation is much larger than the body's size; the field strength falls as $1/r^2$ with distance from the centre – quadrupling the distance reduces g by a factor of four.

3.5 $\Delta E_p = GMm(1/r_1 - 1/r_2) = (4.0 \times 10^{14} \times 1200)(1/6.4 \times 10^6 - 1/1.28 \times 10^7) = (4.8 \times 10^{17})(7.8 \times 10^{-8}) = 3.8 \times 10^{10}$ J; the total energy $-GMm/2r$ becomes less negative – increases – as the radius grows.