11.2 Simple harmonic motion (full C.1, HL)

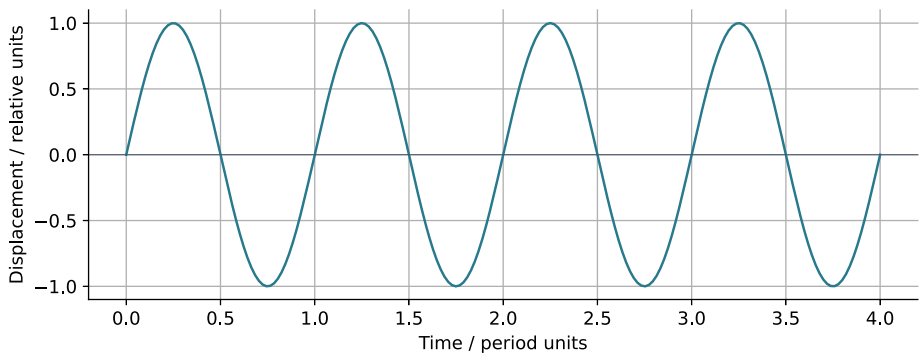
Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS simple harmonic motion • phase angle • angular frequency • amplitude

Objective

Practise the full guide C.1 toolkit including AHL: the defining condition $a = -\omega^2 x$, $T = 1/f = 2\pi/\omega$, mass–spring and pendulum periods, and the higher-level equation set – phase angle, $x = x_0 \sin(\omega t + \phi)$, $v = \omega x_0 \cos(\omega t + \phi)$, $v = \pm\omega\sqrt{x_0^2 - x^2}$, and quantitative energy $E_T = \frac{1}{2}m\omega^2 x_0^2$, $E_p = \frac{1}{2}m\omega^2 x^2$. Original practice; not an official IB paper.

1 Key knowledge and worked examples



■ **The C.1 equation set (HL)**

$$a = -\omega^2 x \qquad T = \frac{1}{f} = \frac{2\pi}{\omega} \qquad T = 2\pi\sqrt{\frac{m}{k}} \qquad T = 2\pi\sqrt{\frac{l}{g}}$$

$$x = x_0 \sin(\omega t + \phi) \qquad v = \omega x_0 \cos(\omega t + \phi) \qquad v = \pm\omega\sqrt{x_0^2 - x^2}$$

$$E_T = \tfrac{1}{2}m\omega^2 x_0^2 \qquad E_p = \tfrac{1}{2}m\omega^2 x^2$$

Phase in radians: $\phi = 0$ starts at equilibrium moving positive; $\phi = \pi/2$ starts at rest at $+x_0$.

■ **Worked example: energy bookkeeping at a displacement**

$m = 0.20$ kg, $x_0 = 0.040$ m, $\omega = 5.0$ rad/s. Total $E_T = \frac{1}{2}(0.20)(25)(0.0016) = 4.0 \times 10^{-3}$ J. At $x = 0.020$ m: $E_p = \frac{1}{2}(0.20)(25)(0.00040) = 1.0 \times 10^{-3}$ J, so $E_k = 3.0 \times 10^{-3}$ J, and $v = 5.0\sqrt{0.0016 - 0.00040} = 0.17$ m/s – consistent with $\frac{1}{2}mv^2 = 2.9 \times 10^{-3}$ J $\approx E_k$. Two habits: compute E_T once from the amplitude, then take E_p at the stated displacement and let kinetic energy be the remainder; check the speed equation against both limits.

2 Practice

2.1 State the defining equation of simple harmonic motion and explain the significance of its minus sign. [2]

2.2 A mass–spring system has $m = 0.30$ kg and $k = 12$ N/m. Calculate its period and angular frequency. [3]

2.3 An oscillator with amplitude x_0 passes through equilibrium at $t = 0$. State its phase constant ϕ, and give its displacement and speed at $t = 0$ in terms of x_0 and ω. [3]

2.4 For the same oscillator, write the total energy and the potential energy at displacement x, and state where in the cycle each of kinetic, potential and total energy is greatest. [3]

3 Exam-style questions

3.1 A 0.25 kg mass on a spring oscillates with amplitude 0.060 m and angular frequency 8.0 rad/s. Calculate the total energy, the maximum speed, and the speed at $x = 0.030$ m. [5]

3.2 A simple pendulum of length 0.90 m ($g = 9.8$ m/s^2) is released from rest at a 5.0° angle, giving amplitude $x_0 = 0.079$ m. Calculate the period, and the maximum speed of the bob. [4]

3.3 A particle in SHM is described by $x = 0.050 \sin(4\pi t)$ m. State the amplitude, period and phase constant, and calculate the earliest time at which the speed is maximum. [4]

Solutions

Original-task marking: M1 method, A1 accuracy, B1 reasoning.

2.1 $a = -\omega^2 x$; acceleration always opposes displacement – directed toward equilibrium, the restoring condition.

2.2 $T = 2\pi\sqrt{0.30/12} = 2\pi(0.158) = 0.99$ s; $\omega = 2\pi/T = 6.3$ rad/s.

2.3 $\phi = 0$; $x = 0$ at equilibrium; $v = \omega x_0$, its maximum.

2.4 $E_T = \frac{1}{2}m\omega^2 x_0^2$; $E_p = \frac{1}{2}m\omega^2 x^2$; kinetic greatest at equilibrium, potential greatest at the extremes ($x = \pm x_0$), total the same everywhere.

3.1 $E_T = \frac{1}{2}(0.25)(64)(0.0036) = 2.9 \times 10^{-2}$ J; $v_{max} = \omega x_0 = 8.0 \times 0.060 = 0.48$ m/s; $v = \omega\sqrt{x_0^2 - x^2} = 8.0\sqrt{0.0036 - 0.00090} = 8.0(0.0520) = 0.42$ m/s.

3.2 $T = 2\pi\sqrt{0.90/9.8} = 2\pi(0.303) = 1.9$ s; $v_{max} = \omega x_0$ with $\omega = 2\pi/1.9 = 3.3$ rad/s: $v_{max} = 3.3 \times 0.079 = 0.26$ m/s.

3.3 Amplitude 0.050 m; $\omega = 4\pi$, so $T = 2\pi/\omega = 0.50$ s; $\phi = 0$. Speed $v = \omega x_0 \cos(4\pi t)$ is maximum when $\cos(4\pi t) = \pm 1$, first at $t = 0$ – the particle starts at equilibrium moving positive.