

Exercise walk-through

11.2

Simple harmonic motion (full C.1, HL)

International Baccalaureate · IB Diploma · Physics · HL

Oscillations · 2.1

Reasoning

Simple harmonic motion has acceleration proportional to displacement and directed toward equilibrium. Period is time per cycle; frequency is its reciprocal. Damping transfers energy away from the oscillating system. resonance : A large response to periodic forcing near a natural frequency. damping: Energy transfer out of an oscillating system.

Exercises 2.1·2.3

2.1 State the defining equation of simple harmonic motion and explain the significance of its minus sign. [2]

2.2 A mass–spring system has $m = 0.30$ kg and $k = 12$ N/m. Calculate its period and angular frequency. [3]

2.3 An oscillator with amplitude x_0 passes through equilibrium at $t = 0$. State its phase constant ϕ , and give its displacement and speed at $t = 0$ in terms of x_0 and ω . [3]

Exercises 2.1·2.3

2.1 State the defining equation of simple harmonic motion and explain the significance of its minus sign. [2]

2.1 $a = -\omega^2 x$; acceleration always opposes displacement – directed toward equilibrium, the restoring condition.

2.2 A mass–spring system has $m = 0.30$ kg and $k = 12$ N/m. Calculate its period and angular frequency. [3]

2.2 $T = 2\pi\sqrt{0.30/12} = 2\pi(0.158) = 0.99$ s; $\omega = 2\pi/T = 6.3$ rad/s.

2.3 An oscillator with amplitude x_0 passes through equilibrium at $t = 0$. State its phase constant ϕ , and give its displacement and speed at $t = 0$ in terms of x_0 and ω . [3]

2.3 $\phi = 0$; $x = 0$ at equilibrium; $v = \omega x_0$, its maximum.

Exercises 2.4·3.2

2.4 For the same oscillator, write the total energy and the potential energy at displacement x , and state where in the cycle each of kinetic, potential and total energy is greatest. [3]

3.1 A 0.25 kg mass on a spring oscillates with amplitude 0.060 m and angular frequency 8.0 rad/s. Calculate the total energy, the maximum speed, and the speed at $x = 0.030$ m. [5]

3.2 A simple pendulum of length 0.90 m ($g = 9.8 \text{ m/s}^2$) is released from rest at a 5.0° angle, giving amplitude $x_0 = 0.079$ m. Calculate the period, and the maximum speed of the bob. [4]

Exercises 2.4·3.2

2.4 For the same oscillator, write the total energy and the potential energy at displacement x , and state where in the cycle each of kinetic, potential and total energy is greatest. [3]

2.4 $E_T = \frac{1}{2}m\omega^2x_0^2$; $E_p = \frac{1}{2}m\omega^2x^2$; kinetic greatest at equilibrium, potential greatest at the extremes ($x = \pm x_0$), total the same everywhere.

3.1 A 0.25 kg mass on a spring oscillates with amplitude 0.060 m and angular frequency 8.0 rad/s. Calculate the total energy, the maximum speed, and the speed at $x = 0.030$ m. [5]

3.1 $E_T = \frac{1}{2}(0.25)(64)(0.0036) = 2.9 \times 10^{-2}$ J; $v_{max} = \omega x_0 = 8.0 \times 0.060 = 0.48$ m/s;
 $v = \omega \sqrt{x_0^2 - x^2} = 8.0 \sqrt{0.0036 - 0.00090} = 8.0(0.0520) = 0.42$ m/s.

3.2 A simple pendulum of length 0.90 m ($g = 9.8$ m/s²) is released from rest at a 5.0° angle, giving amplitude $x_0 = 0.079$ m. Calculate the period, and the maximum speed of the bob. [4]

3.2 $T = 2\pi\sqrt{0.90/9.8} = 2\pi(0.303) = 1.9$ s; $v_{max} = \omega x_0$ with $\omega = 2\pi/1.9 = 3.3$ rad/s;
 $v_{max} = 3.3 \times 0.079 = 0.26$ m/s.

Exercise 3.3

3.3 A particle in SHM is described by $x = 0.050 \sin(4\pi t)$ m. State the amplitude, period and phase constant, and calculate the earliest time at which the speed is maximum. [4]

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3.3 Amplitude 0.050 m; $\omega = 4\pi$, so $T = 2\pi/\omega = 0.50$ s; $\phi = 0$. Speed $v = \omega x_0 \cos(4\pi t)$ is maximum when $\cos(4\pi t) = \pm 1$, first at $t = 0$ – the particle starts at equilibrium moving positive.