Geometry and trigonometry

Topic 3 · SL

A map that stopped an epidemic

> Geometry measures what cannot be measured directly.

- London, 1854: cholera deaths clustered round one pump.
- Snow drew the area nearest that pump.
- This unit: 3D $\to$ triangles $\to$ sectors $\to$ Voronoi diagrams.

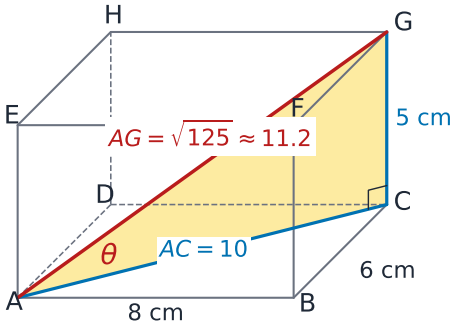
Solids

1. **Pyramid.** $V = \frac{1}{3}(36)(4) = 48$ cm^3
2. **Face height.** $\sqrt{4^2 + 3^2} = 5$ cm
3. **Surface.** $36 + 4 \times \frac{1}{2}(6)(5) = 96$ cm^2

Decision

Check curved surface or total

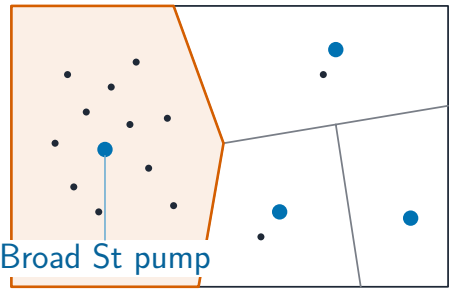
Line and plane



$\tan\theta = \frac{5}{10}$
$\theta \approx 26.6°$

- The angle is between the line and its shadow.
- $\tan\theta = \frac{5}{10}$, so $\theta \approx 26.6°$.

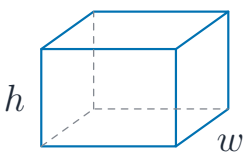
Nearest-pump cells



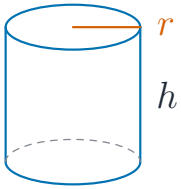
each cell: points nearest one pump; deaths (dots) cluster in one cell (schematic, not Snow's data)

A **Voronoi diagram** points to the source

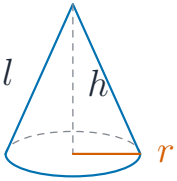
Volumes and surface areas of solids



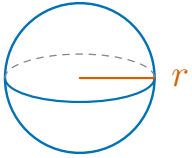
cuboid



cylinder



cone



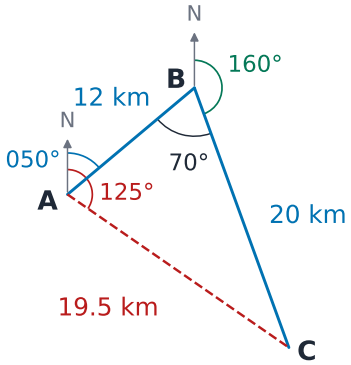
sphere

Use the formula booklet

Sine and cosine rules

Rule	Formula
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$
Area	$\frac{1}{2}ab\sin C$

Bearings



- Clockwise from north, three figures; $AC \approx 19.5$ km on a **bearing** of 125°.

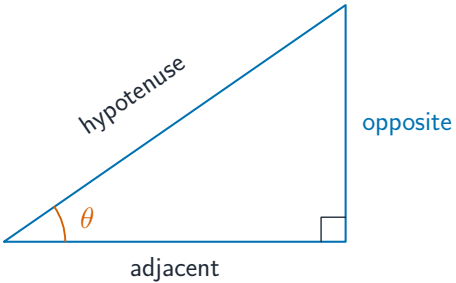
Check: which rule?

Your turn

Which rule finds the largest angle?

A. The cosine rule
B. The sine rule
C. $\frac{1}{2}ab\sin C$
D. Pythagoras' theorem

Naming the sides



SOH CAH TOA

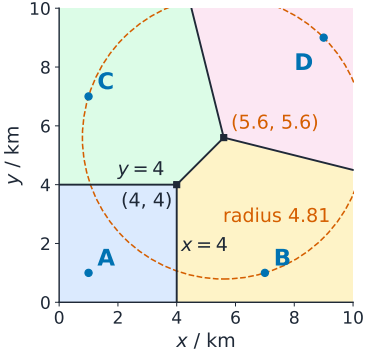
Sectors and bisectors

1. **Arc.** $\frac{40}{360} \times 30\pi \approx 10.5$ cm
2. **Area.** $\frac{40}{360} \times 225\pi \approx 78.5$ cm^2
3. **Bisector of** $(2, 1)$**,** $(6, 5)$**.** $y = -x + 7$

Decision

Every point on the bisector is equidistant

Voronoi diagrams



- Edges are perpendicular bisectors.
- A **vertex** is equidistant from three sites.

Using a Voronoi diagram

1. **Vertices.** $(4, 4)$ is 4.24 km; $(5.6, 5.6)$ is 4.81 km
2. **Boundary.** best about 4.61 km
3. **Choose.** $(5.6, 5.6)$, furthest from all wells

Decision

Compare vertices, boundary and corners

Exam tips

> Draw the diagram first, then calculate.

- Draw the right-angled triangle on its own.
- North line at every **bearing** point.
- Voronoi **edge** = **perpendicular bisector**.

Check yourself

Try these

Find the angle between a space diagonal of an 8 by 6 by 5 cuboid and its base.

Find the perpendicular bisector of $(2, 1)$ and $(6, 5)$.

A pizza slice has radius 15 cm and angle 40°. Find its area.