

Calculus

International Baccalaureate · IB Diploma · Mathematics:
Applications and Interpretation · SL

Scope and prerequisites

This handout covers selected SL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Derivatives and stationary points

■ What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
 - This lesson studies **derivative** : The instantaneous rate of change, also the gradient of a tangent.
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■ The method

- For $y = ax^n$, $dy/dx = anx^{n-1}$.
 - A stationary point satisfies $dy/dx = 0$.
 - Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.
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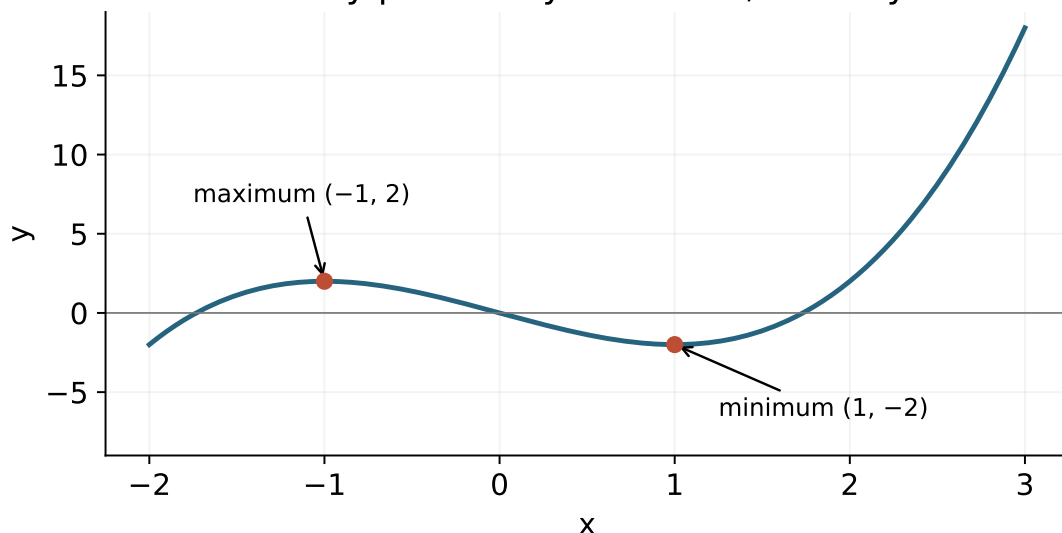
■ A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

Example:

For $y = x^3 - 3x$, $dy/dx = 3x^2 - 3$. At $x = 1$, the gradient is 0 and $y = -2$. The second derivative is $6x$, positive at $x = 1$, so this is a local minimum. At $x = -1$, $y = 2$ and the second derivative is negative, giving a local maximum.

Stationary points of $y = x^3 - 3x$, where $y' = 0$



$$y' = 3x^2 - 3 = 0 \text{ at } x = \pm 1: \text{ maximum } (-1, 2), \text{ minimum } (1, -2)$$

■ Where it goes wrong

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

Not true: Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

■ Applying it

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there.
- At advanced level, connect the derivative to rates and optimization with a valid domain.

■ In your course

- Applications and Interpretation SL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Derivative:** The instantaneous rate of change, also the gradient of a tangent.
- For $y = ax^n$, $dy/dx = anx^{n-1}$.
- Watch for: A zero derivative does not always mean a maximum or minimum: $y = x^3$ is stationary at 0 but continues increasing.

■ Write from a derivative of 4

- For $y = x^2 - 6x$, find the derivative at $x = 5$.
- Find the stationary value. A derivative of 0 is not always a turning point.

Root finding and numerical integration

■ How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
 - This lesson studies **iteration** : A repeated update in which each new approximation is calculated from the previous one.
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■ The method

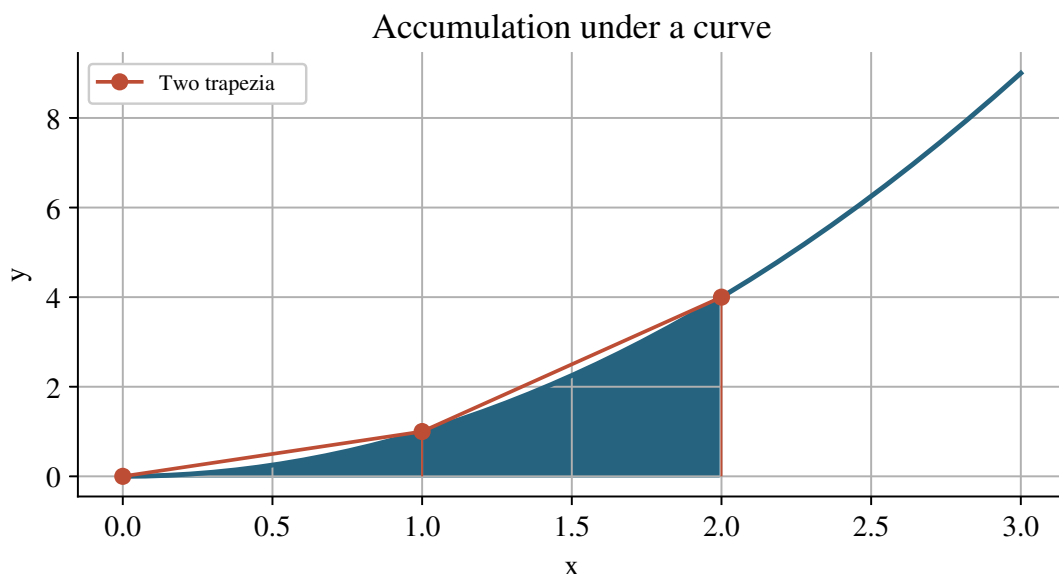
- A continuous function with opposite signs at two endpoints has a root between them.
 - Newton's method uses $x_{\text{next}} = x - f(x)/f'(x)$, with a nonzero derivative.
 - The trapezium rule approximates a definite integral using endpoint heights.
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■ A worked case

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Example:

For $f(x) = x^2 - 2$ and $x_0 = 1.5$, Newton gives $x_1 = 1.5 - (2.25 - 2)/3 = 1.4166667$. With $y = x^2$ on $[0, 2]$ and two equal strips, $h = 1$ and trapezium area $= (1/2)(0 + 2 \times 1 + 4) = 3$; the exact area is $8/3$.



Two trapezia give $0.5 + 2.5 = 3$; the exact area is $8/3$, so the rule overestimates this convex curve

■ Where it goes wrong

- A sign change across a discontinuity does not prove a root.
- Iteration can diverge or cycle.
- The trapezium rule's overestimate or underestimate depends on curvature, not just whether the function increases.

Not true: Every sign change proves a root, including one across a discontinuity. A sign change shows a root only if the function is continuous there: $1/x$ changes sign across $x=0$ but has no root.

■ Applying it

- Give a stopping criterion and report sensible accuracy.
 - Confirm the approximated root by a sign bracket around the stated rounded answer; explain any failure of the chosen iteration.
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■ In your course

- Applications and Interpretation SL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Iteration:** A repeated update in which each new approximation is calculated from the previous one.
- A continuous function with opposite signs at two endpoints has a root between them.
- Watch for: A sign change across a discontinuity does not prove a root.

■ **Write from a Newton step to 1.5**

- For x squared minus 2, take one Newton step from 2.
- A sign change across a discontinuity does not prove a root.

■ **Write from a second Newton value of 1.417**

- Continue the same computation: one more Newton step from 1.5.
- $x_2 = 1.5 \text{ minus } 0.25 \text{ over } 3 \approx 1.4167$ – near the true root of the square root of 2, each step roughly squares the accuracy.

■ **Write from a bisected interval**

- $f(x) = x^3 - x - 2$ changes sign between 1 and 2.
- The midpoint 1.5 gives minus 0.125, so the root lies in $(1.5, 2)$ – the midpoint's sign decides which half survives.