

Geometry and trigonometry

International Baccalaureate · IB Diploma · Mathematics:
Applications and Interpretation · SL

Scope and prerequisites

This handout covers selected SL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Angle reasoning, similarity and mensuration

■ Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

■ The method

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 .
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

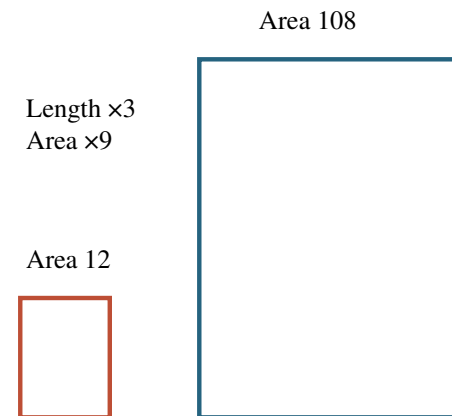
■ A worked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

Example:

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12 \times 3^2 = 108 \text{ cm}^2$ and a model volume of 8 cm^3 gives $8 \times 3^3 = 216 \text{ cm}^3$. A cylinder with $r=3, h=5$ has volume $\pi r^2 h = 45\pi$.

A length scale factor is squared for area



Length scale factor 3 gives area scale factor $3^2 = 9$: $12 \times 9 = 108$

■ Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

Not true: Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

■ Applying it

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
- A scale drawing is evidence only when the task permits measurement.

■ In your course

- Applications and Interpretation SL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Scale factor:** The multiplier that relates corresponding lengths in similar shapes.

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 .
- Watch for: equal angles alone establish similarity, not equal size.

■ Write from a volume factor of 64

- A length scale factor is 4.
- Find the area factor and the volume factor. Doubling lengths does not double the volume.

Right triangles and non-right triangles

■ Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.

■ The method

- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$ and $\tan\theta = \text{opposite/adjacent}$.
- For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.

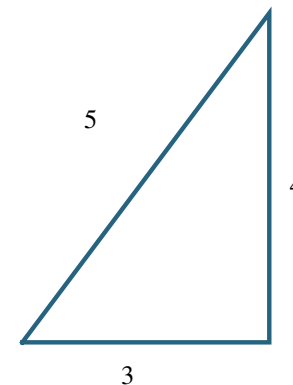
■ A worked case

$$a^2 + b^2 = c^2, \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Example:

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan\theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



Label the sides first: here 3, 4 and hypotenuse 5, so $\sin = 4/5$, $\cos = 3/5$, $\tan = 4/3$ at the bottom-left angle

■ Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

■ Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
- Explain why the chosen triangle contains the required length or angle.

■ In your course

- Applications and Interpretation SL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Hypotenuse:** The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2+b^2=c^2$; $\sin\theta=\text{opposite/hypotenuse}$, $\cos\theta=\text{adjacent/hypotenuse}$ and $\tan\theta=\text{opposite/adjacent}$.
- Watch for: label sides relative to the chosen angle.

■ **Write from a hypotenuse of 17**

- A right triangle has legs 8 and 15.
- Find the hypotenuse and the area. Pythagoras is only for right triangles.

■ **Write from a hypotenuse of 13**

- A right triangle has legs 5 and 12.
- The hypotenuse is root of 25 plus 144 = 13, and the area is half of $5 \times 12 = 30$.

■ **Write from an area of 14**

- A right triangle has legs 7 and 4.
- Area = 14; the hypotenuse is the root of 65 – keep it exact unless a decimal is asked.