

Number and algebra

International Baccalaureate · IB Diploma · Mathematics:
Applications and Interpretation · SL

Scope and prerequisites

This handout covers selected SL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Exact arithmetic and estimation

■ Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** : A prime number that divides the integer exactly.

■ The method

- Prime factors reveal shared structure.
- Use the smallest common prime powers for the HCF and the largest for the LCM.
- Estimate before calculating; use brackets to preserve the order of operations.

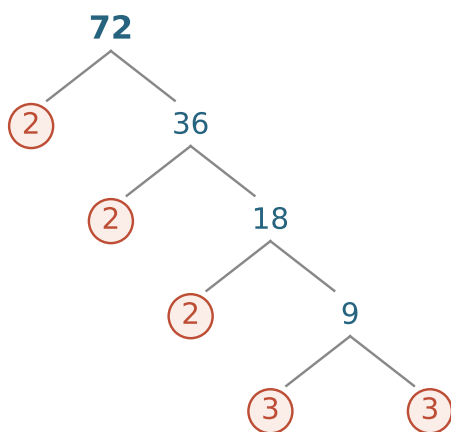
■ A worked case

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

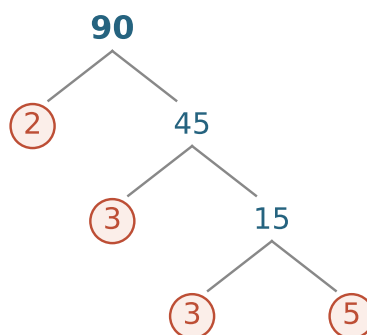
Example:

$72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$. Their HCF is $2 \times 9 = 18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5 = 360$.

Factor trees: $\text{HCF}(72, 90) = 2 \times 3^2 = 18$



$$72 = 2^3 \times 3^2$$



$$90 = 2 \times 3^2 \times 5$$

Factor trees: $72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$, so $\text{HCF} = 2 \times 3^2 = 18$

■ Where it goes wrong

- The HCF divides both numbers; the LCM is a multiple of both.
- They answer different questions.
- A decimal estimate is not an exact fraction.

Not true: The HCF of two positive integers is always larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

■ Applying it

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

■ In your course

- Applications and Interpretation SL, the course first assessed in 2021.

Key:

- **Prime factor:** A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

■ Write from an HCF of 14

- Find the HCF and LCM of 28 and 42.
- The HCF is smaller than both integers.

Percentages, ratio and proportional reasoning

■ Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
 - This lesson studies **multiplier** : A factor that performs a percentage change in one multiplication.
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■ The method

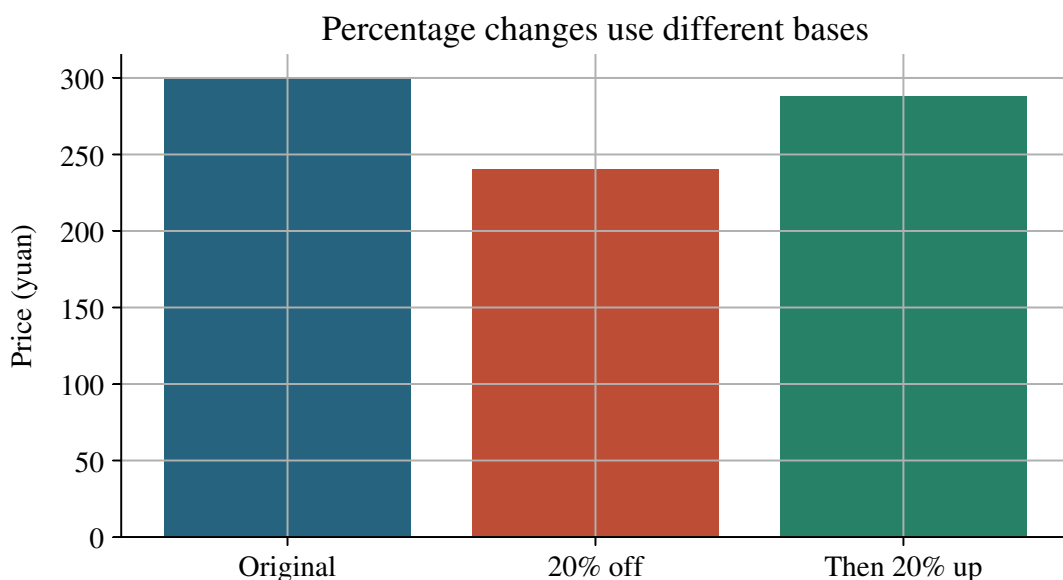
- A p% increase has multiplier $1 + p/100$; a decrease has multiplier $1 - p/100$.
 - Reverse a percentage by dividing by the multiplier.
 - In a ratio, first find the total number of parts.
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■ A worked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100} \right)$$

Example:

Let the original price be P. The model is sale price = 0.8P. Hence $P = 240 / 0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.



300 → 240 → 288 yuan: 20% off then 20% up does not return to 300, because the second 20% is of 240

■ Where it goes wrong

- A percentage uses a stated base.
- Subtracting the percentages loses that base.
- For compound change, multiply the multipliers; do not add the percentages.

Not true: A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100 \times 0.8 \times 1.2 = 96$, not 100.

■ Applying it

- For direct proportion use $y=kx$; for inverse proportion use $y=k/x$.
- Calculate k from a known pair before using a new value.
- State what you held constant.

■ In your course

- Applications and Interpretation SL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Multiplier:** A factor that performs a percentage change in one multiplication.

- A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$.
- Watch for: A percentage uses a stated base.

■ Write from a final price of 480

- A price of 500 falls by 20 percent and then rises by 20 percent.
- Find the final price. The two changes do not cancel.

Sequences, series and recurrence

■ Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
 - This lesson studies **common ratio** : The constant multiplier between consecutive terms of a geometric sequence.
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■ The method

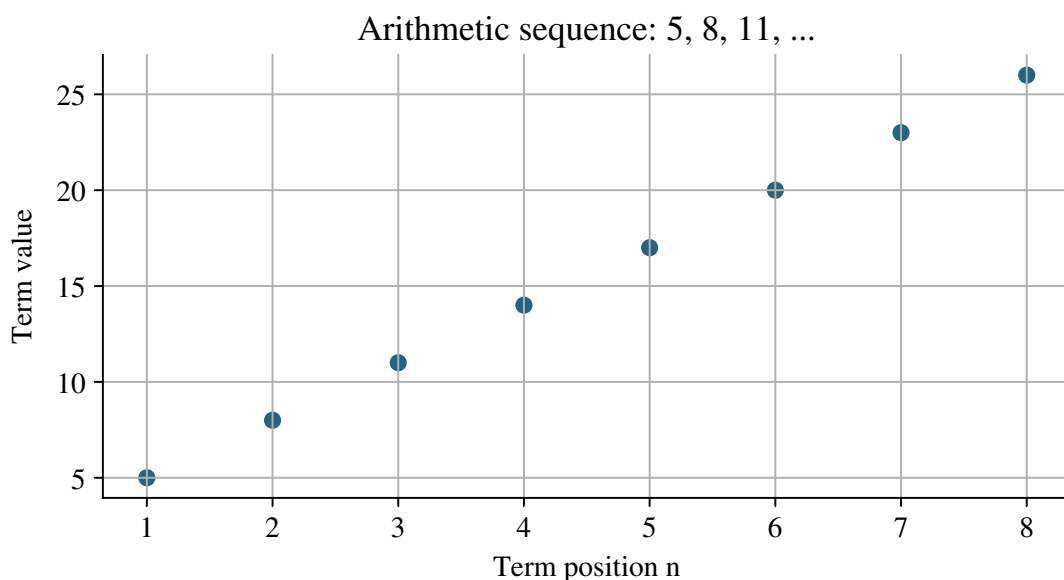
- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$.
 - For a geometric progression, $u_n = ar^{n-1}$ and $S_n = a(1-r^n)/(1-r)$.
 - An infinite geometric sum exists only if $|r| < 1$.
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■ A worked case

$$u_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

Example:

For $a=5, d=3, n=8$, $u_8 = 5 + 7 \times 3 = 26$ and $S_8 = 8(10+21)/2 = 124$. For $a=12, r=1/2$, $S_{\infty} = 12/(1-1/2) = 24$. For $u_{n+1} = 2u_n + 1$ with $u_1 = 1$, the next terms are 3, 7, 15.



$u_n = 5 + 3(n - 1)$: equal steps of 3 put the terms on a straight line

■ Where it goes wrong

- The first term has index 1, so the exponent is $n-1$.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

Not true: Every geometric series has a finite sum to infinity. A sum to infinity exists only when $-1 < r < 1$; for $r=2$ the terms grow without limit.

■ Applying it

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

■ In your course

- Applications and Interpretation SL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Common ratio:** The constant multiplier between consecutive terms of a geometric sequence.

- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$.
- Watch for: the first term has index 1, so the exponent is $n-1$.

■ Write from the 5th term 23

- An arithmetic sequence has first term 7 and common difference 4.
- Find the 5th term. An infinite geometric series needs a common ratio with size below 1.

Repeated-deposit sequence models and timing

■ Which timeline does the sum describe?

- Two equal payments can earn different amounts of interest if one is paid earlier.
 - This lesson studies **deposit**: An amount added to an account at a stated time.
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■ The method

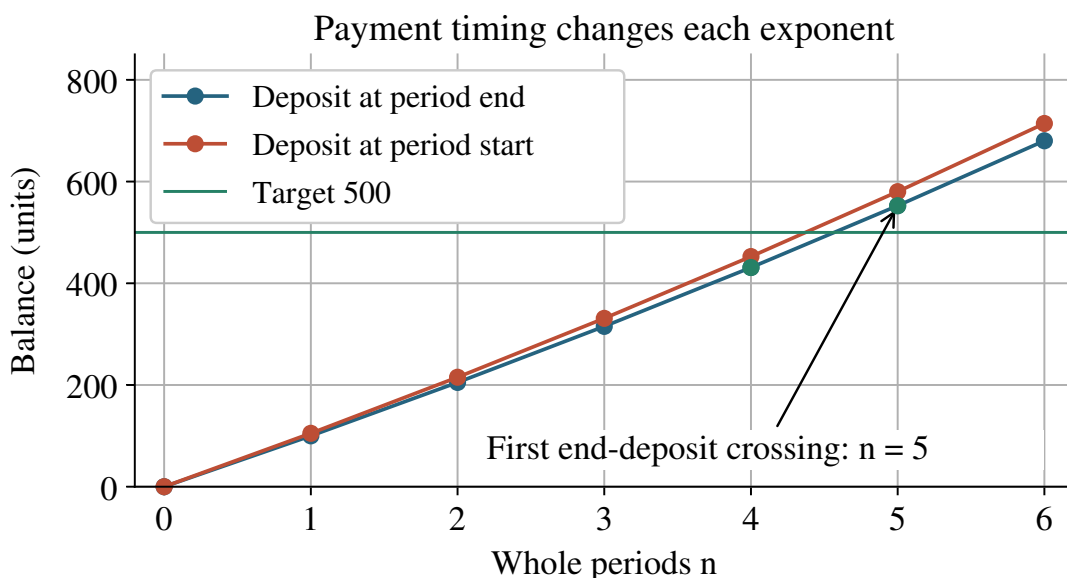
- Define the balance immediately after a stated event and give the initial balance.
 - With growth factor r and an end-of-period deposit d , $B_0 = 0$ and $B_n = rB_{n-1} + d$.
 - The n th balance is $d(1+r+\dots+r^{n-1}) = d(r^n - 1)/(r - 1)$ for $r \neq 1$.
 - For $r = 1$, use $B_n = nd$.
 - The period count n is a nonnegative integer.
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■ A worked case

$$B_0 = 0, \quad B_n = rB_{n-1} + d, \quad B_n = d \frac{r^n - 1}{r - 1} \quad (r \neq 1)$$

Example:

Use an illustrative growth rate of 5% per period and deposit 100 units at each period end. $B_1 = 100$, $B_2 = 1.05 \times 100 + 100 = 205$ and $B_3 = 1.05 \times 205 + 100 = 315.25$. The first deposit earns interest for $n-1$ periods, while the final one earns none at the time B_n is measured. Thus $B_n = 100(1.05^n - 1)/0.05$. For a target of at least 500, $B_4 = 431.0125 < 500$ and $B_5 = 552.563125 \geq 500$, so 5 periods first suffice. The balance exceeds the five deposits totalling 500 by 52.563125 units. If the same deposits are made at each period start and the balance is measured at the period end, $C_n = 1.05(B_n)$; hence $C_3 = 331.0125$, not 315.25.



Depositing at the start earns one more period of interest; the end-deposit balance first passes 500 at $n = 5$

■ Where it goes wrong

- Do not give the final end-of-period deposit an extra interest period.
- A balance includes both deposits and growth; it is not the interest alone.
- An exact target and an at-least target need different integer checks.
- A quoted annual rate cannot simply be used as a per-month factor without an appropriate conversion.

Not true: Beginning- and end-of-period deposits always give the same final balance. A deposit at the start of a period earns one more period of interest, so the final balance is larger.

■ Applying it

- The illustration assumes a fixed per-period rate, equal deposits, no fees or withdrawals and the stated compounding/payment times.
- If these change, refine the recurrence: $B_n = (1+i_n)B_{(n-1)} + d_n - w_n$ for end-of-period deposits/withdrawals.
- Keep a separate ledger of payments to identify growth.
- Compare adjacent period balances before claiming the first target crossing; here all balances are nonnegative and $B_n - B_{(n-1)} = 0.05B_{(n-1)} + 100 > 0$.

■ In your course

- Applications and Interpretation SL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Deposit:** An amount added to an account at a stated time.
- Define the balance immediately after a stated event and give the initial balance.
- Watch for: do not give the final end-of-period deposit an extra interest period.